C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE SECONDARY EDUCATION CERTIFICATE EXAMINATION JANUARY 2007

MATHEMATICS

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GENERAL PROFICIENCY EXAMINATION

JANUARY 2007

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. The Basic Proficiency is offered in May/June only.

There was a candidate entry of approximately 12 650 in January 2007. This year, forty-four per cent of the candidates achieved Grades I – III. The mean percentage for the examination was 44.

DETAILED COMMENTS

The examination consists of two papers. Paper 01 consists of 60 multiple choice items. The highest mark was 59 and approximately 61 per cent of the candidates scored at least half the available marks for this paper.

Paper 02 consists of two sections. Section I comprised eight compulsory questions totalling 90 marks. Section II comprised six optional questions: two each in Relations, Functions and Graphs; Trigonometry and Geometry; Vectors and Matrices. Candidates were required to choose any two questions. Each question was worth 15 marks.

This year, two candidates earned the maximum available marks on Paper 02 and approximately 26 per cent of the candidates earned at least half the maximum mark on this paper.

Question 1

This question tested candidates' ability to

- perform basic operations with decimals
- approximate a decimal number to the nearest whole number
- solve problems involving ratio, rates and proportion.

The question was attempted by 99 per cent of the candidates, 29 per cent of whom scored the maximum available mark. The mean score was 9 out of 12.

Generally, performance on this item was good, particularly in part (a) where many candidates demonstrated competence in performing the operations on decimals. Incorrect answers were mainly due to careless errors and in some cases $(1.5)^2$ was computed as 2 x 1.5.

In part (b), the majority of candidates demonstrated a sound understanding of ratio although many did not recognize that $60.00 represented two shares

Using the given rate to calculate the cost of 3 litres of gasoline in part (c) was well done by the majority of candidates. However, a significant number of candidates could not calculate the number of litres which could be bought with $50.00.

Many candidates had difficulty completing the approximation, since they were unable to round their answers to the nearest whole number.

Solutions

(a) (i) 12.2092 (ii) 2.1

(b) $210.00

(c) (i) $17.33 (ii) 14 litres

Recommendations

Teachers need to encourage candidates to show all working by writing their solutions to sub-parts of a question. In addition, the candidates need to be exposed to problems which could be solved using ratio or proportion. There should also be emphasis on approximation, especially rounding to whole numbers.

Question 2

This question tested candidates' ability to

- perform operations involving directed numbers
- substitute numbers for algebraic symbols in simple algebraic expressions
- solve linear equations in one unknown
- simplify algebraic fractions
- solve a simple linear inequality in one unknown.

The question was attempted by 99 per cent of the candidates, 6 per cent of whom scored the maximum available mark. The mean score was 6.15 out of 12.

Candidates generally demonstrated proficiency in the substitution in part (a) although some of the weaker candidates could not multiply negative numbers accurately.

In attempting to simplify the algebraic fraction in part (b), a significant number of candidates used an LCM of 5 instead of 6 while others incorrectly cross-multiplied and obtained
$3x + 2x = 5$. Solving the in-equation proved challenging for candidates. Some treated it as an equation, while others solved for $-x$ instead of x.

In part (c), candidates were able to use symbols to represent the cost of 5 muffins although some incorrectly wrote $5m + m$ or $5m \times m$. Stating the cost of 6 cupcakes given the cost of three, was more challenging and some candidates wrote $6c$, ignoring the information given. The majority of candidates were successful in writing an equation to represent the total cost of the muffins and cupcakes.

Solutions:

(a) (i) 6 (ii) –12
(b) (i) x = 6 (ii) x e” –9

(c) (i) a) $5m b) $4m
(ii) 5m + 4m = $31.50

Recommendations

Teachers need to teach candidates to simplify and perform the basic operations on algebraic expressions. Candidates must recognise that the solution to an in-equation is different from the solution of an equation.

Question 3

The question tested candidates’ ability to

- use set notation to describe a given set
- draw Venn diagrams to represent information
- list the members of the union, intersection and complement of a set
- determine the number of elements in the union, intersection and complement of a set.

The question was attempted by 99 per cent of the candidates, less that 1 per cent of whom scored the maximum available mark. The mean mark was 4.15 out of 10.

Candidates generally were able to describe the shaded region in part (a)(i), but parrts (ii) and (iii) were poorly done with some candidates describing the un-shaded regions instead of the shaded regions.

In part (b), candidates were able to identify odd and even numbers. However, the candidates had difficulty identifying the prime numbers, with many candidates listing 1 as a prime number.

In part (c), a number of candidates interpreted the numbers in each region on the Venn diagram as elements instead of the number of elements. For example, the number of elements in AUB was given as {10, 4, 3} or 3 instead of 17.

Solutions:

(i) $A \cup B$ (ii) $(A \cup B)'$ or $A' \cap B'$ (iii) A

(c) (i) 17 (ii) 4 (ii) 21 (iv) 25

Recommendations

Teachers need to emphasise the importance of set notation in the teaching of sets. In addition, candidates must be given opportunities to (i) represent practical situations using Venn diagrams and (ii) shade given regions. Describing these regions in words is also necessary if a full understanding of the concept is to be attained.

Question 4

(a) The question tested candidates' ability to
- draw line segments
- construct a triangle given the lengths of the sides
- measure angles in a polygon
- calculate the gradient of a straight line given two points
- calculate the coordinates of the mid-point given two points.

The question was attempted by 95 per cent of the candidates, 8 per cent of whom scored the maximum available mark. The mean mark was 5.1 out of 11.

Candidates were generally able to draw the triangle with the prescribed dimensions. However, many candidates did not show the appropriate construction lines. Many candidates experienced difficulty in locating the point D and hence could not proceed to measure AD. In some instances candidates measured angle BAC instead of angle ABC.

Part (b) was generally well done with candidates showing strengths in the choice of formula for finding the gradient and mid-point of a line. However, some of the candidates experienced problems in substituting values in the formula because of failure to define or set up their coordinates in terms of (x_1, x_2) and (y_1, y_2). Also writing the mid-point as a coordinate was often omitted.

Solutions:

(iii) a) 7.4 cm b) 68^0

(i) 1.5 (ii) (4, 7)

Recommendations

Teachers need to teach candidates to develop proper techniques to measure line-segments and angles and to use the appropriate instruments to construct plane figures. Candidates also need to become familiar with the language and terms used in geometry.

Question 5

The question tested candidates' ability to

- find the output of a function given an input value
- find the input of a function given the output value
- locate the position of the mirror line given an object and its image
- locate the image of an object under a rotation
- describe a transformation given an object and its image.

The question was attempted by 94 per cent of the candidates, 3 per cent of whom scored the maximum available mark. The mean mark was 3.83 out of 11.

Candidates were able to evaluate g(3) and f(-2) correctly. They, however, experienced difficulty with working with inverse functions. In many cases, candidates were unable to complete the procedure for finding the inverse. For example, after interchanging x and y, they did not proceed to make y the subject. Those candidates who chose to equate f(x) to 11 and then solve for x, experienced more success.

Locating the mirror line was well done although a number of candidates drew the mirror line but did not write its equation.

Part (b) was poorly done with the majority of candidates being unable to perform a rotation about a given point. Many candidates mistakenly used (5,4) as an image point instead of the centre of rotation. Describing the transformation which mapped the original figure onto its final image also posed serious problems for candidates. Some candidates used the rotation matrix to determine the coordinates of the image, not recognising that this matrix may only be used when the centre of rotation is (0,0).

Solutions:

(a) **(i)** $\frac{1}{6}$ **(ii)** **-10** **(iii)** **1**

(b) **(i)** **x = 5**

(ii) **A″(1,2) B″ (3,2) C″ (3,-1)**

(iii) **Reflection in the line y = 4**

Recommendations

It is recommended that candidates understand how to perform geometrical transformations before they are exposed to matrix methods. Further, they should perform transformations using actual shapes on plane paper or using geo-boards, before performing transformations on the Cartesian plane.

<u>Question 6</u>

The question tested candidates' ability to

- construct a cumulative table for a given set of data
- draw a cumulative frequency curve
- use a cumulative frequency curve to estimate the median
- calculate the probability of an event using data in a frequency distribution.

The question was attempted by 92 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean mark was 4.57 out of 12.

Candidates were able to calculate the cumulative frequencies, use the given scale, plot points correctly and draw the curve. Most of the candidates did not use the boundaries to plot the cumulative frequency curve, but used mid-points or upper limits instead. Many candidates omitted to find the median and those who attempted to do so often made errors in interpreting the scale.

Although it was generally understood that the probability must be expressed as a fraction, some candidates had difficulty calculating the number of candidates whose score was greater than 40 while others used 40 as the total number of candidates in the sample.

Solutions:

(a) (i) Cumulative frequencies [5, 23, 46, 68, 89, 100]

(iii) Median score: 36

(iv) $\frac{32}{100}$

<u>Recommendations</u>

Teachers should give candidates opportunities to draw graphs using a variety of scales so that they are familiar with interpreting graphs. The use of class boundaries in drawing cumulative frequency curves also needs to be emphasized.

<u>Question 7</u>

The question tested candidates' ability to calculate the

- volume of a cuboid
- total surface area of a cuboid
- length of an arc
- perimeter of a sector
- area of a sector.

The question was attempted by 88 per cent of the candidates, 8 per cent of whom scored the maximum available mark. The mean mark was 4.26 out of 12.

Candidates generally knew how to select and apply the formula to calculate the volume of the cuboid. The perimeter of the sector was also well done. Many candidates were unable to select the appropriate formula to calculate the surface area of the cuboid. In some cases, candidates calculated the volume instead of the surface area.

In computing the area of the sector and the arc length, there was a tendency to omit the fraction in the formula. A few candidates opted to use $r\theta$ but failed to convert to radians.

Solutions:

(a) (i) 4320 cm^2
(ii) 1728 cm^2

(b) (i) 11.78 cm
(iii) 41.78 cm
(iii) 88.31 cm^2

<u>Recommendations</u>

The poor choice of formulae in finding surface area, arc length and area of the sector suggests that candidates are using measurement formulae without understanding how they are derived. Teachers should spend more time in developing formulae using practical approaches and emphasising the units of the attribute being measured.

<u>Question 8</u>

The question tested candidates’ ability to

- create and use number patterns
- determine the nth term in a number sequence.

The question was attempted by 81 per cent of the candidates, 2 per cent of whom scored the maximum available mark. The mean mark was 2.95 out of 10.

This question was poorly done by most of the candidates. Although some candidates were able to recognize a pattern, they did not test their pattern to see if it applied to all the cases. For example, candidates gave 256 as the result for n = 3, assuming the pattern to be n^2. Those who recognized the correct pattern were unable to calculate the value for n = 6, or the power to which 4 must be raised to give a result of 65 536. A limited number of candidates were able to compute the result of the mth term

Solutions

(i) **64**
(ii) **4 096**
(iii) **8**
(iv) **4^m**

Recommendations

Teachers should prepare candidates for questions of this type by allowing candidates to generate their own number sequences and have other candidates predict the general rule. Candidates must also be given opportunities to hypothesize and test conjectures so that they can observe instances when the pattern breaks down.

Teachers should also expose candidates to problem solving activities regularly, in an effort to develop the analytical skills of the candidates.

Question 9

The question tested candidates' ability to

- factorise a quadratic expression
- use common factors to factorise an algebraic expression
- expand an expression of the form $(x + a)^2 (x + b)$
- express a quadratic equation in the form $a(x + b)^2 + c$ and state the axis of symmetry and the minimum point
- sketch the graph of a quadratic function.

The question was attempted by 51 per cent of the candidates, less than 1 per cent of whom scored the maximum available mark. The mean mark was 2.32 out of 15.

Although this question was the most popular among the optional questions, candidates performed poorly with 41 per cent scoring no marks. The candidates were able to expand $(x+3)^2$ correctly and many knew the procedure for completing the square. Some candidates were also able to sketch the curve although they did not show the maximum point nor the axis of symmetry on the diagram.

Candidates had difficulty factorising, especially the difference of two squares, and expanding $(x^2 +6x+9)(x-4)$. Identifying the minimum point from the expression $2(x+1)^2 -7$ and stating the axis of symmetry were also poorly done.

Solutions:

(a) **(i)** **$(2p – 1) (p – 3)$**

(ii) **$(p + q) (5 + p – q)$**

(b) $\mathbf{x^3 + 2x^2 - 15x - 36}$

(c) **(i)** $\mathbf{2(x+1)^2 - 7}$

(ii) **Axis of symmetry x** $= -1$

(iii) **Minimum point (– 1, – 7)**

Recommendations

Teachers need to link the use of algebraic and graphical methods in solving quadratic equations. Candidates should be able to match expressions of the form $a(x+b)^2 + c$ with the graph of the function and interpret the meanings of the constants.

Question 10

The question tested candidates' ability to

- write inequalities from worded statements
- draw graphs of linear inequalities in one or two variables
- determine the solution of a set of inequalities
- use linear programming techniques to determine the maximum value of an expression.

The question was attempted by 40 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean mark was 4.44 out of 15.

Most candidates were able to write the inequalities $x \geq 3$ and $x + y \leq 10$. They were also able to obtain the profit expression and to draw the line $x = 3$. Candidates experienced problems in describing the inequality $5x + 2y \leq 35$ in their own words, the most common statement being: "the total of 5 pens and 2 pencils is less than 35". Candidates omitted to mention the words *cost* and *dollars* in their explanations.

Candidates also had difficulty drawing the line $x+y=10$, identifying the region common to all the inequalities and extracting information from the graph to obtain the maximum number of pencils that could be bought.

Solutions:

(a) **(i)** $\mathbf{x \geq 3}$

(ii) $\mathbf{x + y \leq 10}$

(iii) **The total cost of x pens and y pencils is no more than \$35.00**

(b) (ii) (3, 0) (7, 0) (5, 5) (3, 7)

(c) (i) Profit : 1.5x + y

(ii) The maximum profit: $12.50

(iii) Maximum number of pencils: 6

Recommendations

Teachers need to encourage candidates to test points in order to identify the correct regions and to use a consistent system of shading to define the wanted/unwanted region. In finding the maximum or minimum, the use of the fundamental theorem of testing points at the vertices of the feasible region must also be reinforced.

Question 11

The question tested candidates' ability to

- use theorems in circle geometry to calculate the measure of angles
- use the properties of circles to perform simple calculations
- recognize and use Pythagoras' theorem to calculate the length of a side of a right-angled triangle.

The question was attempted by 14 per cent of the candidates, less than 1 per cent of whom scored the maximum available mark. The mean mark was 4.54 out of 15.

Generally, candidates were successful in stating the lengths of PQ and PN. Many did not recognize that RS= NQ and hence were unable to proceed to use Pythagoras' theorem to calculate NQ. Providing reasons for answers to part (a) posed major problems for candidates who merely described the information shown in the diagram without supporting their answers by referring to circle theorems.

Candidates were able to calculate the angles in part (b) although they continued to experience difficulty in giving reasons to support their answers.

Solutions:

(a)

(i) a) PQT is a straight line because T is the common point of contact of two circles and a common tangent drawn through T will be perpendicular to both PT and TQ. Hence, angle PTQ = $90^o + 90^o = 180^0$

b) PQ = 7cm

c) PS is parallel to QR because PSR = QRS = 90^o (Tangent XY is perpendicular to both radii). The corresponding angles are equal; hence the lines must be parallel.

(b) (i) angle MNL = 55^o
angle LMO = 35^o

Recommendations

Candidates should be given more opportunities in the classroom to justify their solutions using properties of polygons and the relevant theorems. In addition, teachers should expose candidates to the vocabulary associated with the concepts and principles in geometry through the use of strategies which incorporate oral exercises.

Question 12

The question tested candidates' ability to

- draw a diagram to represent information involving bearings and distances
- calculate distances using the sine and cosine rule
- solve problems involving bearings.

The question was attempted by 33 per cent of the candidates, 2 per cent of whom scored the maximum available mark. The mean mark was 5.43 out of 15.

Candidates were generally able to sketch and label the diagram to illustrate the journey of the ship. There was some difficulty illustrating the bearings of 135^0 and 60^0 with 135^0 drawn as an acute angle in some cases. Quite a significant number of candidates used scale drawings and these were mainly well done yielding accurate results.

Part (b) of the question was poorly done. Candidates generally chose to use trigonometric ratios associated with right-angled triangles. In several instances where they chose the correct rule for solving the problem they proceeded to substitute values and angles incorrectly. For example, in using the cosine rule, many substituted 60^0 instead of 105^0 .

Solutions:

(b) (i) AC = 18.7 km
(ii) angle BCA = 50.8^0
(iii) Bearing of A from C is 290.8^0

Recommendations

Teachers should encourage candidates to construct drawings which include distances and bearings, where the focus would be on realistic representation of line segments and angles. Such diagrams should lead to more reasonable solutions to the problems.

Question 13

The question tested candidates' ability to

- locate points on a diagram given information
- add vectors
- use vectors to solve a problems in geometry.

The question was attempted by 13 per cent of the candidates, 1 per cent of whom scored the maximum available mark. The mean mark was 3.36 out of 15.

Candidates were generally able to approximate the position of X on the diagram and state a correct route for the vectors and . However, they were unable to locate the point Q given the instructions. The handling of fractions also posed major challenges for candidates in their attempts to express vectors in terms of *r* and *s*. Few candidates attempted part (c) where a proof was required.

Solutions:

(b) **(i)**

(ii) $\frac{1}{3}\underline{s} - \frac{2}{3}\underline{r}$

(iii) $\frac{5}{6}\underline{r} - \frac{7}{12}\underline{s}$

Recommendations

The responses to this question suggest that candidates are not familiar with applying basic concepts of ratio and fractions to locate points on line segments when drawing diagrams. These types of exercises must be addressed prior to the teaching of vectors using strategies which involve actual measurements and the use of squared paper.

Question 14

The question tested candidates' ability to:

- identify a singular matrix
- calculate the determinant of a 2×2 matrix
- find the inverse of a 2×2 matrix
- use matrix methods to solve a system of 2 linear equations

The question was attempted by 34 per cent of the candidates, 2 per cent of whom scored the maximum available mark. The mean mark was 7 out of 15.

Most candidates performed well on this question, especially solving the linear equation by a matrix method. In part (a), candidates generally knew that a singular matrix had a determinant of zero but only the stronger candidates were able to solve the quadratic equation 4 “ $9p^2$ = 0 to obtain two roots. Quite a few candidates found only the positive root.

In part (b), candidates knew how to convert the pair of simultaneous equations into matrix form and how to find the determinant and inverse of the matrix. Some errors were made in obtaining the adjunct with candidates changing the signs along the leading diagonal. Candidates were generally able to solve for x and y, although performing operations on directed numbers was a major weakness demonstrated by some candidates.

Solutions:

(a) **p =**

(b) **(i)** $\begin{pmatrix} 2 & 5 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$

(ii) **a)** **Determinant : “ 7**

c) $\mathbf{x} = \frac{16}{7}$**,** $\mathbf{y} = \frac{2}{7}$

<u>Recommendations</u>

Teachers should emphasise the processes for finding the determinant and the inverse of a matrix. Candidates should also be exposed to more problems where linear equations are solved by the matrix method.