CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE SECONDARY EDUCATION CERTIFICATE EXAMINATION

MAY/JUNE 2010

MATHEMATICS
GENERAL PROFICIENCY

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. This was the first examination of the revised Mathematics syllabus effective for the May/June 2010 examinations.

There was a candidate entry of approximately 88,400 in May/June 2010. Forty-one per cent of the candidates achieved Grades I to III. The mean score for the examination was 76.59 out of 180 marks.

DETAILED COMMENTS

Paper 01 – Multiple Choice

Paper 01 consisted of 60 multiple choice items. This year, 54 candidates each earned the maximum available mark of 60. Fifty-seven per cent of the candidates scored 30 marks or more.

Paper 02 – Structured Questions

Paper 02 consisted of two sections. Section I comprised eight compulsory questions totalling 90 marks. Section II comprised three optional questions: one each from (i) Algebra and Relations, Functions and Graphs; (ii) Measurement, Geometry and Trigonometry and (iii) Vectors and Matrices. Candidates were required to answer any two questions from this section. Each question in this section was worth 15 marks.

This year, 20 candidates each earned the maximum available mark of 120 on Paper 02. Approximately 24 per cent of the candidates earned at least half of the maximum mark on this paper.

Compulsory Section

<u>Question 1</u>

This question tested candidates’ ability to

- subtract, multiply and divide fractions
- multiply, subtract and divide decimal numbers
- express a number correct to a given number of significant figures
- find the unit price, given the cost price and the number of items purchased
- determine the profit and profit percent, given the selling price and cost price

The question was attempted by 99 per cent of the candidates, 9.4 per cent of whom earned the maximum available mark. The mean mark was 7.0 out of 11.

Responses to this question were generally good. The majority of candidates were able to accurately perform the computations required with the use of a calculator.

In performing subtraction on fractions, some candidates failed to use a common denominator and incorrectly simplified $\frac{3}{2} - \frac{2}{5}$ as $\frac{1}{3}$ or $\frac{1}{-3}$.

In performing the division, some candidates mistakenly interpreted $\frac{11}{10} \div \frac{33}{10}$ as $\frac{33}{10} \div \frac{11}{10}$ and thus inverted the quotient instead of the divisor.

Squaring 2.5 also posed difficulties for some candidates, for example, 2.5^2 was interpreted as, 2.5×2 or $2^2 + 5^2$.

In computing $2.5^2 - \frac{2.89}{17}$ some candidates did not recognize that $\frac{2.89}{17}$ was a separate term and incorrectly subtracted 2.89 from 2.5^2, then divided by 17.

Expressing 6.08 correct to two significant figures also posed some difficulties for candidates. Common incorrect answers were 6.0, 60.8 and 61. In addition, a few candidates confused significant figures with scientific notation.

In Part (b), a few candidates were unable to differentiate between profit and profit percent. Surprisingly, a significant number of candidates could not express the profit as a percentage of the cost price. Some candidates did not multiply by 100 while others expressed the profit as a percentage of the selling price.

Solutions:

(a)	**(i)** $\mathbf{\frac{1}{3}}$	**(ii)**	**6.1**		
(b)	**(i) $12.80**	**(ii)**	**$2 998.50**	**(iii) $1 078.50**	**(iv) 56%**

Recommendations

Teachers should allow students to estimate the result of a computation prior to performing computations using calculators. In this way, they can determine if their answers are reasonable and make adjustments to their procedures.

Basic concepts in computation of fractions, percentages and decimals should be reviewed using conceptual rather than procedural approaches. Students should also be encouraged to investigate the use of calculators in performing multi-step computations. They should be allowed to check their results using different orders and verify the correct order based on the context of the problem.

In teaching approximations, a clear distinction must be made between significant figures, decimal places and standard form. The use of each type in real life situations must also be emphasized.

Question 2

This question tested candidates' ability to

- substitute numbers for algebraic symbols in simple algebraic expressions
- perform the four basic operations on directed numbers
- convert verbal phrases to algebraic expressions
- solve a pair of simultaneous linear equations, algebraically
- apply the distributive law to factorize or expand algebraic expressions
- factorize quadratic expressions

The question was attempted by 99.6 per cent of the candidates, 2 per cent of whom earned the maximum available mark. The mean mark was 4.6 out of 12.

Candidates performed reasonably well on substituting the numbers in the algebraic expressions. However, a small percentage had difficulty using directed numbers. Common errors were observed in simplifying $2^2 - (-3)^2$, which was often written as $2^2 - -3^2$ and equated to 4 + 9 =13.

In Part (b), a large number of candidates were unable to successfully use symbols to express a phrase as an algebraic expression. Further, they did not know when to use brackets and incorrectly wrote $7x + y$ instead of $7(x+y)$ in Part (b) (i).

Responses to Part (b)(ii) were generally below the required standard. Candidates either omitted this part or made unsuccessful attempts. Common incorrect responses included xy and $x < y$.

Many candidates displayed competence in solving simultaneous equations. Both the elimination and substitution methods were used, with the majority choosing the method of elimination. However, too many errors were made in simplifying algebraic terms, irrespective of the method used. Where the substitution method was used, many candidates were unable to express one variable in terms of the other. Quite a number of candidates arrived at the correct answers seemingly by trial and error and did not show working. A few candidates chose a matrix method but this approach was also fraught with errors and correct solutions were seldom obtained.

In factorizing the given expressions, the method of common factors and the difference of two squares was generally known by candidates. However, quite a number of them incorrectly interpreted $4y^2 - z^2$ as $4(y^2 - z^2)$; $(4y)^2 - z^2$ or $(2y - z)^2$.

Grouping terms to factorize also posed challenges for many candidates who did not strategize to obtain a second pair of common factors. For example, they often wrote:

$2ax - 2ay - bx + by = 2a(x - y) - b(x + y)$ and could go no further.

Some candidates correctly factorized the first step as $2a(x - y) - b(x - y)$ but failed to complete the factorization.

In (d)(iii), candidates also had difficulty factorizing the quadratic expression $3x^2 + 10x - 8$ and many ended up with incorrect factors such as $(3x - 2)(x - 4)$; $(3x + 2)(x - 4)$ or $(3x + 4)(x - 2)$. This suggests that they had no idea how to check their results to obtain the original expression.

Solutions:

(a) **(i)** **-2** **(ii)** **-5**

(b) **(i)** $\mathbf{7(x+y)}$ **(ii)** $\mathbf{y(y+1)}$

(c) $\mathbf{x = 3,\ y = 1}$

(d) **(i)** $\mathbf{(2y-z)(2y+z)}$ **(ii)** $\mathbf{(x-y)(2a-b)}$ **(iii)** $\mathbf{(x+4)(3x-2)}$

Recommendations

Teachers need to emphasize the importance of brackets when making substitutions. When indices are used in substitutions, they should note that the index must apply to what is inside the bracket only.

In solving simultaneous linear equations, teachers should also encourage students to determine the most efficient strategy to use. Both elimination and substitution methods should be taught and students must develop strategies to decide on which method is better in a given situation.

Teachers should also pay close attention to mathematical vocabulary so that students are familiar with basic terminology such as *solve*, *simplify* and *factorize*.

Question 3

This question tested candidates' ability to

- use a Venn diagram to solve practical problems involving two sets
- determine elements in the intersection, union and complement of a set
- solve a simple equation
- calculate the area of a compound shape
- use Pythagoras' theorem to find one side of a right-angled triangle
- solve geometric problems using properties of congruent triangles

The question was attempted by 99.2 per cent of the candidates, 3 per cent of whom earned the maximum available mark. The mean mark was 4.4 out of 12.

Performance was generally unsatisfactory with candidates having extremely poor responses to Part (b) in particular. In Part (a), the majority of candidates were able to identify the elements in the intersection, but only a small percentage of candidates could determine the elements in $A \cap B'$ and $B \cap A'$ correctly.

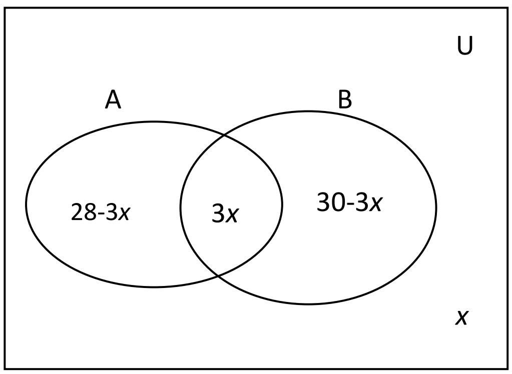
Although some candidates correctly placed $3x$ in the intersection, they subtracted x in their expressions to obtain $(28 - x)$ and $(30 - x)$ for $A \cap B'$ and $A' \cap B$ respectively. As a result, they obtained $x = -9$ but were unable to conclude that one cannot have a negative number of tourists. Far too many candidates produced an equation when asked for an expression.

In Part b (i) candidates generally applied Pythagoras' Theorem but many used it incorrectly, primarily using the incorrect side as the hypotenuse, instead of DE. Those who did not recognize that Pythagoras could be used applied methods such as the cosine rule or trigonometric ratios, which could not produce the intended solution.

Quite a few candidates assumed that ABCD was a square and gave the area as 36 cm^2 instead of 30 cm^2. Weaker candidates chose the formula for perimeter in computing the area. However, the majority of candidates knew that they had to add two separate areas to determine the area of the compound shape ABCDE.

Solutions:

(a) (i)



(a)	**(ii) 58 - 2x**	**(iii) x = 9**	
(b)	**(i) EF = 3 cm**	**(ii) DF = 4 cm**	**(iii) 42 cm^2**

Recommendations

Teachers should teach students to verify that the information recorded in their Venn diagram accurately represents the given data. This can be done by summing up the elements in the subsets and checking to see that the total for the universal set is obtained. Students should also devise strategies to check for unreasonableness of answers and make the adjustments where necessary.

In using Pythagoras' Theorem, students should first study the information given and then decide by inspection whether subtraction or addition is required. When diagrams are not drawn to scale, students should not assume that lengths look similar and are therefore equal.

Concepts of area and perimeter should be taught simultaneously and formulae should be introduced through guided investigative approaches.

Question 4

This question tested candidates' ability to

- solve problems involving direct variation
- construct a triangle given two angles and one side
- draw and measure line segments accurately
- construct angles of 60^{o} and 90^{o}
- measure the size of a given angle using a protractor

The question was attempted by 94.8 per cent of the candidates, 8 per cent of whom earned the maximum available mark. The mean mark was 4.4 out of 11.

Generally, performance was unsatisfactory in both parts of this question. In Part (a), although candidates were able to substitute correctly to determine the value of *k*, a significant number of them had difficulty transposing correctly after substituting and obtained incorrect values.

Candidates generally drew and measured the line segments accurately. However, the weaker candidates measured starting at one rather than zero. The construction of angles posed a problem for many candidates. They had more success in constructing 60^{o} than 90^{o}. It was also evident that some candidates used a pair of compasses in drawing the figure but erased their arcs afterwards thinking that this was the correct procedure. A small number of candidates also had problems labelling their diagram.

Solutions:

(a) (i) $k = 0.5$ (ii) $y = 450$
(b) (ii) (a) EF = 12 ± 0.1 cm (b) ∠EFG = 30^{o}±1

Recommendations

Candidates need to be reminded that they must use the required instruments when attempting questions on the construction of figures. They must also show all construction lines and refrain from answering these questions on graph paper.

Question 5

This question tested candidates' ability to

- interpret and use functional notations such as *f(x)* and *gf(x)*
- derive the inverse of a simple function
- determine the scale used on the axis of a graph
- use a graph to determine the value of one variable, given the equation
- state the range of a function from its graph

The question was attempted by 83.7 per cent of the candidates, 3 per cent of whom earned the maximum available mark. The mean mark was 4.0 out of 12.

Generally, performance on this question was unsatisfactory. A significant number of candidates did not attempt this question or scored zero.

In Part (a), candidates were able to substitute the value of *x* into the function *f(x)* and many obtained the correct answer for *f*(4). However, they were not as successful in evaluating *gf* (4). Many candidates could not interpret this notation and proceeded to evaluate *g*(4) instead of using the result obtained for *f*(4). Those candidates who went the route of first finding *gf(x)* experienced difficulties in squaring (2*x* - 5) and invariably ended up with an incorrect expression.

In determining $f^{-1}(x)$, many candidates knew that they had to interchange the variables, but far too many were unable to correctly carry out the other steps. A major difficulty arose when they had to transpose a term with a negative coefficient.

In Part (b), candidates generally knew that the scale for the *x*-axis was in the ratio 1:2 but few were able to write down the scale using the correct format. They were also able to use the graph to determine the value of *y* when $x = -1.5$ and to state at least one of the values of *x* for which $y = 0$. However, far too many candidates failed to follow instructions and used calculations instead of the graph in determining the unknown values.

Although candidates were able to state the range of values of *y*, many were unable to write the answer in the form $a \leq y \leq b$, as requested.

Solutions:

(a) **(i) (a) 3** **(b) 12** **(ii)** $\boldsymbol{f^{-1}(x) = \frac{x+5}{2}}$

(b) **(i) 2 cm represents 1 unit** **(ii)** $\boldsymbol{y = -3.8 \pm 0.1}$

(iii) $\boldsymbol{x = -3, 1}$ **(iv)** $\boldsymbol{-4 \leq y \leq 5}$

Recommendations

Teachers need to emphasize the role of language in teaching functional notation. In particular, students need to understand the meaning of *f(x)* and *gf(x).* With respect to the latter, they must allow students to make connections between notation in geometry (when representing combined transformations) and notation in algebra (for composite functions).

Students also need to understand the language used in interpreting graphs. Terms such as *range* and phrases such as *the value of x for which y = k* are not fully understood.

Question 6

This question tested candidates' ability to

- use appropriate theorems to determine the measure of given angles
- state the centre, angle and direction of the rotation, given a triangle and its image after a rotation
- state the geometric relationships between an object and its image after a rotation
- state its image after a translation by a given column vector, given a point

The question was attempted by 94.8 per cent of the candidates, 1 per cent of whom earned the maximum available mark. The mean mark was 3.3 out of 11.

This question was poorly done. Although some candidates were able to determine the measure of angles *x* and *y*, few were able to give correct reasons for their answers. A small number of candidates were familiar with alternate angles but co-interior angles were rarely mentioned.

In Part (b), describing the rotation proved particularly challenging for candidates. Some of them stated the centre correctly but they used informal language when describing the direction. Responses such as to the left, westward and south east were often given. It was evident that candidates did not know how to state a geometrical relationship between a triangle and its image. Many omitted this part of the question while others merely described the triangles. Candidates also had difficulty writing the image of the point, L, in coordinate form.

Solutions:

(a) **(i) 54°** **(ii) 65°**

(b) **(i) (a)(0, 0)** **(b) and (c) 90°, anticlockwise OR 270^0, clockwise**

(ii) Congruent or same size and shape **(iii) (2, 1)**

Recommendations

Students should have opportunities to express their ideas and to communicate effectively, orally and in writing, in the classroom. These experiences are necessary to develop mathematical vocabulary and proficiency in communication, not only in mathematics but in their daily experiences. Allowing students to orally state reasons for their answers can be a useful classroom strategy to assist them in improving their vocabulary and communication skills.

Question 7

This question tested candidates' ability to

- complete a frequency table from a set of raw scores
- determine the class boundaries for a given class interval
- construct a histogram from a set of data using given scales
- interpret data from a frequency distribution
- determine the probability of an event using data from a frequency table

The question was attempted by 71.9 per cent of the candidates, 4 per cent of whom earned the maximum available mark. The mean mark was 5.9 out of 11.

The performance on this question was generally good. The majority of candidates displayed a competence in completing the frequency table. However, a significant number of candidates were unable to state the lower boundary for the interval 20 – 29. Consequently, they did not use the class boundaries in drawing the histogram, but used the lower limits instead.

The majority of candidates used the correct scales and plotted the frequencies correctly. However, some of the weaker candidates drew a bar graph instead of a frequency polygon.

While many candidates were able to determine the number of students who threw the ball a distance of 50 metres or more, a significant number could not determine the probability.

Solutions:

(a)

Distance (m)	**Frequency**
20 – 29	**3**
20 – 39	**5**
40 – 49	**8**
50 – 59	**6**
60 – 69	**2**

(b) **19.5**

(d) **(i)** **8** **(ii)** $\frac{8}{24}$ **or** $\frac{1}{3}$

Recommendations

Teachers must ensure that students can differentiate between the statistical graphs and relate these differences to the type of data that is being represented.

When calculating probability, students should be reminded that results cannot be a whole number and this should be reinforced through reference to the probability scale.

Question 8

This question tested candidates' ability to

- recognize and extend a pattern in a sequence of diagrams
- calculate unknown terms in number sequences
- derive a formula connecting the variables in given sequences

The question was attempted by 91.6 per cent of the candidates, 16 per cent of whom earned the maximum available mark. The mean mark was 6.7 out of 10.

Performance on this question was quite good with the majority of candidates scoring more than 6 marks. Candidates were generally able to extend the pattern by drawing the fourth figure in the sequence and to identify the pattern in each number sequence. A small number of candidates had difficulty moving from the 5th to the 15th sequence. The major challenge for them was generalizing the formulae for the number sequence.

Solutions:

	Figure	**Area of Figure**	**Perimeter of Figure**
(i)	**4**	**16**	**22**
(ii)	**5**	**25**	**28**
(iii)	**15**	**225**	**88**
(iv)	**n**	$\mathbf{n^2}$	**6n - 2**

Recommendation

Teachers need to utilize real life situations that give rise to simple patterns, be it shapes or numbers. They also need to give students opportunities to investigate different ways of making generalizations from the patterns they observe.

Optional Section

Question 9

This question tested candidates' ability to

- interpret a speed-time graph
- write inequalities to represent given constraints
- use a given scale to represent three given inequalities on a graph
- use linear programming techniques

The question was attempted by 27.5 per cent of the candidates, 1 per cent of whom earned the maximum available mark. The mean mark was 4.1 out of 15.

Candidates demonstrated competence in using the graph to determine the maximum speed and the interval for which the speed was constant. However, only a few candidates were able to correctly evaluate the area under the curve to obtain the distance travelled. Many candidates knew the intervals for the different stages of the journey but were unable to express it in the desired format.

In Part (b), candidates were generally able to translate verbal statements into algebraic inequalities and used the given scale in drawing their graphs. However, a significant number of candidates were unsuccessful in drawing the lines $y = x$, $y + x = 12$ and $y = 3$. In many cases, the line $x = 3$ was drawn for $y = 3$. Furthermore, they were unable to successfully show the regions satisfied by the inequalities. Consequently, only the very able candidates were successful in obtaining the required region satisfying all inequalities.

The minimum values of *x* and *y* proved to be a difficult task for most candidates.

Solutions:

(a) **(i) (a) 12 m/s** **(b) 4 sec** **(c) 102 m**

(ii) (a) $0 \le t \le 6$ **(b)** $10 \le t \le 13$ **(c)** $6 \le t \le 10$

(b) **(i)** $x + y \le 12$ **(iv)** $x = 3$ $y = 3$

Recommendations

Emphasis must be placed on the use of the mathematical notation, $a \le x \le b$, when describing an interval and students should also be able to distinguish between the equation of lines of the form $y = k$ and $x = k$. Students should be encouraged to use points to test their solutions when solving inequalities in two variables.

Question 10

This question tested candidates' ability to

- solve geometric problems using the properties of circles and circle theorems
- solve practical problems involving heights and distances in three dimensional situations
- use the cosine rule in the solution of problems involving non-right-angle triangles
- use trigonometric ratios in the solution of right-angled triangles
- solve practical problems in 3-D situations involving angles of elevation

The question was attempted by 36.8 per cent of the candidates, 2 per cent of whom earned the maximum available mark. The mean mark was 2.3 out of 15.

Many candidates were able to correctly state the size of at least one angle. However, they were not able to state the reasons for their answers. A major weakness observed was that candidates made incorrect assumptions, for example, they assumed that triangle TPR was isosceles, that PR bisected angle TPR and some candidates even used the line TR as a diameter.

In Part (b), the cosine rule was stated correctly by the majority of candidates and substitution of the correct values was also seen. However, many candidates had difficulty following through to the end successfully.

Some of the weaker candidates used incorrect or inefficient strategies in calculating the required lengths. For example, Pythagoras' Theorem was used to calculate the length EG although the triangle was not right-angled. Where trigonometric ratios could have been used to calculate unknown sides of right-angled triangles, many applied the sine rule which resulted in lengthy calculations.

Only the very able candidates calculated the angle of elevation correctly.

Solutions:

(a) (i) 46° **(ii) 56°** **(iii) 124°**

(b) (i) 8.57 m **(ii) 12.2 m** **(iii) 46.9°**

Recommendations

When teaching Circle Geometry, teachers must remind students that assumptions cannot be made about the size of an angle or the lengths of sides since the figures are not drawn to scale. In preparing students for solving problems in trigonometry, clear distinctions must be made between strategies for solving right-angled triangles and non-right-angled triangles. Attention must also be paid to the efficiency of the strategies.

Question 11

This question tested candidates' ability to

- perform matrix multiplication
- evaluate the determinant of a 2 × 2 matrix
- obtain the inverse of a non-singular matrix
- given a matrix equation in *x* and *y*, use a matrix method to solve for *x* and *y*
- combine vectors and write expressions for given vectors
- determine the geometrical relationship between two vectors

The question was attempted by 21.3 per cent of the candidates, 1 per cent of whom earned the maximum available mark. The mean mark was 3.2 out of 15.

The performance on this question was generally poor with only a few candidates showing mastery in the skills tested. A significant number of candidates were unable to correctly multiply two matrices. Instead of multiplying row by column, many candidates multiplied the corresponding elements. Further, a large number of candidates did not recognize that the inverse of matrix B was matrix A and proceeded to compute the inverse. Those who chose this method often encountered further problems in calculating the determinant.

Many candidates did not use the inverse obtained in Part (ii) to solve for *x* and *y* in Part (iii). In addition, writing $\begin{pmatrix} x \\ y \end{pmatrix}$ as a product of two matrices was particularly challenging. Some candidats used the incorrect order by placing the 2 × 1 matrix on the left of the 2 × 2 matrix.

The vector component was equally challenging. Although many students placed M and N correctly on JK and JL respectively, they were not able to conceptualize 'one third' thereby placing M and N at random positions. Further, many of the candidates were unable to use the triangular law of vectors to determine the resultant vectors. Although there were correct attempts at stating routes, candidates had difficulty with the directions, often omitting the negative signs.

Many candidates could not state the relationship between MN and KL. There was a tendency to use words like 'collinear' and 'parallel' without referring to the actual findings. Those candidates who drew the diagram were able to determine that the vectors were parallel. However, they were not able to state the relationship between the vectors KL and MN in terms of their lengths.

Solutions

(a) **(i)** $\mathbf{AB} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$ **(ii)** $\begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{2} & \mathbf{5} \end{pmatrix}$ **(iii)** $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{2} & \mathbf{5} \end{pmatrix} \begin{pmatrix} \mathbf{2} \\ \mathbf{3} \end{pmatrix}$ **(iv)** $x = 8$, $y = 19$

(b) **(ii)** **a)** $3u$ **(b)** $-u + v$ **(c)** $-3u + 3v$

(iii) **KL is parallel to MN**
KL = 3MN

Recommendations

Emphasis should be placed on the relationship between a matrix, its inverse and the identity matrix. Students must see the connection between solving matrix equations and solving simple equations in terms of the use of an identity element.

Greater emphasis should be placed on a matrix as a system in which order is important. The use of real life examples where matrices are used to represent authentic situations should enable students to appreciate the meaning of operations on matrices.

Prior to the teaching of vectors, basic concepts in fractions and geometry must be reviewed. Vocabulary associated with vectors can be reinforced through the use of practical examples in locating points on a line segment or describing relationships between line segments.