CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

JANUARY 2014

MATHEMATICS
GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. There was a candidate entry of approximately 11, 600 in January 2014 and 45 per cent of the candidates earned Grades I–III. The mean score for the examination was 74 out of 180 marks.

DETAILED COMMENTS

Paper 01 − Multiple Choice

Paper 01 consisted of 60 multiple-choice items. It was designed to provide adequate coverage of the content with items taken from all sections of the syllabus. Approximately 76 per cent of candidates earned acceptable grades on this paper; the mean score was 34 out of 60 marks. This year 16 candidates each earned the maximum available score of 60. Sixty per cent of the candidates scored 30 marks or more.

Paper 02 – Structured Questions

Paper 02 consisted of two sections. Section I comprised eight compulsory questions for a total of 90 marks. Section II comprised three optional questions: one each from Algebra, Relations, Functions and Graphs; Measurement, Trigonometry and Geometry; and Vectors and Matrices. Candidates were required to answer any two out of the three questions from Section II. Each question was worth 15 marks. The mean score for this paper was 41 out of 120 marks. This year, no candidate earned the maximum available mark of 120. Approximately 17 per cent of candidates earned at least 60 marks.

Section I: Compulsory Questions

<u>Question 1</u>

This question tested candidates' ability to

- perform the four basic operations on common fractions
- compute the square and positive square root of decimals
- divide decimals
- round off to a given number of decimal places
- solve problems involving cost price, taxes, profit and percentage profit.

The question was attempted by 99 per cent of candidates, 8.2 per cent of whom earned the maximum available mark. The mean mark was 7.03 out of 11.

The performance of candidates was satisfactory. In Part (a), some candidates knew the algorithms for subtracting, dividing and adding fractions although several erred by not following the order of operations. In Part (b), some candidates multiplied by 2 instead of squaring 1.31. Further, a number of candidates rounded off too early in their computations. In Part (c), some candidates made the error of adding money to the number of bracelets, while some obtained a percentage profit that was not logical in the context of the problem. Also, many candidates used an incorrect value for the cost price when calculating the percentage profit.

Solutions

(a) $2\frac{7}{8}$
(b) 2.40
(c) (i) \$ 8 160 (ii) a) \$3 200.25 b) 39%

Recommendations

Teachers should provide students with opportunities to use the calculator to perform basic arithmetic operations on rational numbers. Attention should also be given to squaring and finding the square root, converting improper fractions to decimals, and estimating the answer and the order in which operations should be performed.

<u>Question 2</u>

This question tested candidates' ability to

- solve a linear in-equation in one unknown and to graphically represent its solution set
- factorize quadratic functions by grouping and by difference of squares
- find the product of two binomial expressions
- solve a pair of simultaneous equations.

The question was attempted by 98 per cent of candidates, 4.8 per cent of whom earned the maximum available mark. The mean mark was 4.84 out of 12.

Candidate performance was unsatisfactory. In Part (a), candidates experienced difficulty working with the inequality sign. It was a common feature to see the $\leq$ sign incorrectly

replaced by the $\geq$ sign or the $=$ sign. Further, a large number of candidates who obtained the solution $x \leq 4$ could not represent the solution set on a number line.

In Part (b), candidates were proficient at factorizing the difference of squares and while they chose the correct strategy of factorizing by grouping in Part (b) (i), they encountered difficulty finding the second of the two factors. In Part (c), candidates knew that they needed to use the distributive property but could not complete the process. Generally, candidates encountered difficulty with both the multiplication and addition of directed numbers.

Solutions

(a) (i) $x \leq 4$
(b) (i) $(x - 2y)(3 + a)$ (ii) $(p - 1)(p + 1)$
(c) $2k^2 - 7k + 6$

Recommendations

Teachers should focus students' attention on the algebra of directed numbers and on the use of the number line to represent the solution of linear inequalities.

<u>Question 3</u>

This question tested candidates' ability to

- draw a Venn diagram to illustrate the relationship between two sets
- determine the elements in the intersection, union and complement of two sets
- describe the relationship between two sets using set notation
- formulate and solve a linear equation in one unknown
- determine the area of a composite plane figure bounded by straight lines.

The question was attempted by 99 per cent of candidates, 2.2 per cent of whom earned the maximum available mark. The mean mark was 3.42 out of 10.

The performance of candidates was unsatisfactory. In Part (a) (i), while most candidates were able to draw the Venn diagram, entering the correct value in each subset provided a considerable challenge. In Part (a) (ii), most candidates were able to determine the number of students who studied Spanish. However, they were unable to write, in set notation, the relationship between F and S.

In Part (b), candidates were generally able to derive an expression for the length of the floor. They were nevertheless unable to formulate an expression for the area of the floor and to derive an equation which could be solved to determine the value of x.

Solutions

(a) (ii) $n(S \cap F^{/}) = 12$ (iii) $F \subset S$
(b) (i) $l = 3x + 5$ (ii) a) $x = 5$ b) Area of floor $= 95\ m^2$

Recommendations

Teachers are advised to constantly review all concepts of set theory. Students should be taught to incorporate algebraic expressions in solving problems in measurement such as finding the perimeter and area.

<u>Question 4</u>

This question tested candidates' ability to

- determine the Cartesian equations of three different lines
- determine the gradient of a line
- identify, by shading, the region which represents the solution set of a linear equation
- derive a system of inequalities which represent a given solution set on a graph
- determine the equation of a line which passes through the origin and is perpendicular to a given line.

The question was attempted by 88 per cent of candidates, less than one per cent of whom earned the maximum available mark. The mean mark was 1.99 out of 11.

Candidate performance was unsatisfactory. In Part (a), candidates recognized the line with equation $y = 2$, but not the lines with equations $y = x$ and $y = x + 2$. In Part (d), candidates demonstrated little proficiency at writing a system of inequalities to represent the solution set shown on the graph. In cases where candidates were able to correctly obtain two of the three inequalities, $x \geq 0$ was invariably omitted. In Part (e), candidates were unable to apply the gradient relationship between perpendicular lines and the significance of a line passing through the origin to write the equation of the required line.

Solutions

(a) (i) $line\ 1: y = x + 2$ (ii) $line\ 2: y = x$ (iii) $line\ 3: y = 2$
(d) $y \leq 2; \quad y \geq x; \quad x \geq 0$
(e) $y = -x$

Recommendations

Teachers should reinforce the basic concepts of inequalities in the senior classes. The use of test points to check the validity of inequalities should also be emphasized.

<u>Question 5</u>

This question tested candidates' ability to

- construct, using ruler and compasses, a triangle and a kite
- use a protractor to determine the measure of an angle
- determine the area of a trapezium
- calculate the volume of a prism
- convert from kilograms to grams.

The question was attempted by 94 per cent of the candidates, 3.2 per cent of whom earned the maximum available mark. The mean mark was 4.15 out of 12. The performance of candidates was generally unsatisfactory. In Part (a), while candidates were able draw lines of given lengths, they lacked the skill of using a pair of compasses to construct a triangle whose three sides were given. Moreover, many candidates experienced difficulty with obtaining, through measurement, the size of one of the angles of the triangle drawn. Further, a large number of candidates could not complete the kite $BACD$ using the triangle BAC which was already constructed or drawn.

In Part (b), candidates performed below expectations at finding the area of a trapezium. However, they were able to use the area calculated to find the volume of the prism. Many candidates could not calculate the mass, in grams, of 1 cm^3 of the metal, as they experienced difficulty with unit conversions.

Solutions

(a) $A\hat{B}C = 53° \pm 1°$
(b) (i) $135\ cm^2$ (ii) $405\ cm^3$ (iii) $3.7\ g$

Recommendations

Teachers should provide students with more opportunities to use mathematical instruments to construct plane shapes. They should also insist that students state the units when giving the results of measurements.

Question 6

This question tested candidates' ability to

- determine the measure of angles using the properties of parallel lines and transversals
- state the coordinates of a point in the plane
- state the length of a line segment drawn in the plane
- describe a transformation in the plane given an object and its image
- determine the location of the image of an object after it has undergone a simple transformation.

The question was attempted by 94 per cent of the candidates, less than one per cent of whom earned the maximum available mark. The mean mark was 3.35 out of 12.

Candidate performance was unsatisfactory. In Part (a), candidates recognized that x and $28°$ were alternate angles and correctly solved that part of the problem. They also recognized that there are two equal angles in an isosceles triangle and used this knowledge to determine the value of y. However, they were less competent at determining the value of z. Some candidates who correctly stated the value of z could not give the reason for their answer.

In Part (b), candidates were able to state the coordinates of a given point on the graph. While a number of candidates correctly stated the length of $K'L'$ as 2 units, there was a large number who gave 3 units as the measure. In addition, many candidates who correctly described the given transformation as a reflection were unable to state the correct mirror line. Further, the majority of candidates was unable to correctly use the given translation vector to derive the image of a point in the plane, and in many instances when they proceeded to add the vectors, they did not give the response as coordinates.

Solutions

(a) (i) $x = 28°$ (ii) $y = 104°$ (iii) $z = 104°$
(b) (i) $J\left(-4, 1\right)$ (ii) $K'L' = 2$ units (iii) A reflection in the line $y = x$
(iv) $J''\ (1, -2)$; $K''\ (4, -2)$; $L''\ (4, 0)$

Recommendations

Teachers should encourage students to use mathematical terms to describe the relationship between angles formed when parallel lines are crossed by a transversal. Candidates should be drilled in the practice of stating the reason or reasons for answers derived from the geometry of plane figures. Attention should also be given to describing transformations in the plane.

<u>Question 7</u>

This question tested candidates' ability to

- complete a grouped-frequency table to show mid-interval values and frequencies
- determine the size of the sample from information given in a grouped frequency table
- identify class limits, class boundaries and class width
- construct a frequency polygon.

The question was attempted by 94 per cent of the candidates, one per cent of whom earned the maximum available mark. The mean mark was 5.50 out of 12.

Candidates demonstrated proficiency in completing the table to show the midpoints and the class intervals. In addition, they were able to determine, from the table, the number of seedlings in the sample. However, they were less competent at constructing the frequency polygon, finding the class width, and stating the class limits and boundaries. Some candidates experienced difficulty differentiating between the vertical and horizontal axes. Moreover, a significant number of candidates were unable to correctly use the given scales to plot the points associated with the polygon.

Solutions

(a) 85 seedlings

(b) (i) lower class limit is 8 (ii) upper class boundary is 12.5 (iii) class width is 5

(c)

Height (in cm)	Midpoint	Frequency
18–22	20	16
23–27	25	22
28–32	30	18
33–37	35	14
38–42	40	0

Recommendations

Teachers should approach the construction of histograms and frequency polygons separately. Emphasis should be placed on the definition of a polygon and the shared properties of line graphs and frequency polygons. The correct use of scales in plotting points on graphs is an important concept which should be emphasized in all related areas of mathematics.

Question 8

This question tested candidates' ability to

- draw the fourth figure in a sequence of shapes in which the first three figures in the sequence are given
- determine the terms of a sequence for which the values of the function are given
- derive algebraic expressions related to the general term of a sequence.

The question was attempted by 97 per cent of the candidates, 10 per cent of whom earned the maximum available mark. The mean mark was 6.53 out of 10.

The performance of candidates was good. They demonstrated competence in answering most of the questions. In Part (a), obtaining the fourth diagram in the sequence proved challenging; many candidates drew triangles instead of trapezia. In Part (b) (iv), when writing the general formulae based on the given sequences, candidates resorted to $\times 4$ and $\times 4 + 2$ or 4^n and $4^n + 2$ instead of $4n$ and $4n + 2$.

Solution

(b)

No. of Trapezoids	Triangles	Dots
4	16	18
10	40	42
16	64	66
n	4n	4n+2

Recommendations

Teachers should provide experiences for students translating verbal statements to algebraic representations and vice versa. Further, students need practice in exploring patterns in the real world and representing these patterns algebraically.

Section II: Optional Questions

Question 9

This question tested candidates' ability to

- substitute numbers for symbols expressed as functions and composite functions
- determine the inverse of a rational function
- derive the composite of two linear functions
- interpret information given on the graph of a quadratic function
- determine the roots of a quadratic equation, its intercept on the *y*-axis and the coordinates of its minimum point.

The question was attempted by 70 per cent of the candidates, 1.5 per cent of whom earned the maximum available mark. The mean mark was 3.62 out of 15.

Candidates performed unsatisfactorily. In Part (a), they were generally able to determine the image of a domain member of a function and the composite of functions. While they were aware that they needed to reverse the mapping to find the inverse function, they lacked the algebraic skills to follow through with the process. Further, the inability to correctly transpose to find the inverse was a common feature.

In Part (b), a significant number of candidates were unfamiliar with the term *roots* of a function and were unable to interpret information shown on the graph.

Solutions

(a) (i) $g(4) = 10$; $h\big(g(4)\big) = -2$

(ii) $h^{-1}(x) = \frac{10}{x+3}$; $g\big(g(x)\big) = 9x - 8$

(b) (i) $x = -1, 5$ (ii) $c = -5$; (iii) $x = 2,\ y = -9$

Recommendations

Teachers should provide students with more opportunities to transpose formulae. Attention needs to be given to the association between the solutions of a quadratic equation and the term *roots* of the function. The pictorial representation of 3-set mapping diagrams should be utilized when introducing composite functions.

<u>Question 10</u>

This question tested candidates' ability to

- use the properties of circles, parallel lines, transversals and circle theorems to determine the measure of angles
- use the cosine rule to determine the length of the side of a triangle
- calculate the area of a triangle given two sides and the included angle
- solve problems related to bearings.

The question was attempted by 46 per cent of candidates, less than one per cent of whom earned the maximum available mark. The mean mark was 2.35 out of 15.

Generally, candidates performed unsatisfactorily. In Part (a), several candidates demonstrated that they knew that the angle at the circumference in a semicircle is a right angle and used this fact to calculate angle $\angle$FAW. However, the majority of candidates attempting this question failed to identify relationships associated with transversals crossing parallel lines, such as co-interior and alternate angles in the given diagram.

In Part (b), candidates selected the cosine formula as the appropriate rule in calculating QR. They nevertheless were unable to use the angle and bearing given to calculate the

bearing of R from P. Further, finding the intermediate angles required to compute the bearing was problematic for most candidates.

Solutions

(a) (i) $F\hat{A}W = 36°$ (ii) $S\hat{K}F = 126°$ (iii) $A\hat{S}W = 64°$

(b) (i) a) $61.3\ km$ b) $3516.6\ km^2$ (ii) $95°$

Recommendations

Teachers should expose their students to higher order geometric problems which require the use of basic concepts such as alternate angles, co-interior angles and bearings.

Question 11

This question tested candidates' ability to

- invert a two by two matrix
- determine the coordinates of the pre-image of a point given a transformation in the form of a matrix and the coordinates of the image
- solve problems in geometry using vectors.

The question was attempted by 48 per cent of the candidates, less than one per cent of whom earned the maximum available mark. The mean mark was 3.35 out of 15.

Candidates performed unsatisfactorily on this question. While they were generally able to invert the two by two matrix, they could not apply this inverse to determine the coordinates of the point which was mapped onto a given point by the original matrix. In vector geometry, candidates were proficient in sketching the triangle and inserting the point L two-thirds the distance along MN. However, they encountered difficulty attempting to use vector algebra to solve the problem. The major challenge was expressing $\overrightarrow{ML}$ in terms of m and n.

Solutions

(a) (i) $T^{-1} = \frac{1}{7}\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ (ii) $a = 3,\ b = 2$

(b) (ii) a) $\overrightarrow{\mathrm{MN}} = -\mathrm{m} + \mathrm{n}$ b) $\overrightarrow{\mathrm{ML}} = \frac{2}{3}(-\mathrm{m} + \mathrm{n})$

(b) (iii) $\overrightarrow{\mathrm{OL}} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$

Recommendations

Teachers should reinforce students' knowledge of geometric concepts through the use of manipulatives and authentic tasks. They should consider revisiting ratios when teaching vector geometry. Some attention should be given to the application of the inverse matrix in the solution of problems involving linear equations in two unknowns.