CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

MAY/JUNE 2007

PURE MATHEMATICS

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INTRODUCTION

This is the third year and the final time that the current syllabus has been examined. **A revised syllabus will be examined in 2008**. There has been a significant increase in the number of candidates writing the examinations with approximately 5021 writing Unit 1 compared to 4 430 in 2006 and 2 521 compared to 1 500 for Unit 2. Performances varied across the entire spectrum of candidates with an encouraging number obtaining excellent grades, but there continues to be a large number of candidates who seem unprepared to write the examinations particularly for Unit 1; a more effective screening process needs to be instituted to reduce the number of ill-prepared candidates.

GENERAL COMMENTS

UNIT 1

The overall performance in this Unit was satisfactory with a number of candidates excelling in such topics as the Factor/Remainder Theorem, Coordinate Geometry, Basic Differential and Integral Calculus and Surds. However, many candidates continue to find Indices, Limits, Continuity/ Discontinuity and Inequalities challenging. These topics should be given special attention if improvements in performance are to be achieved. Other areas needing consolidation are general algebraic manipulation of simple terms; expressions and equations; substitution, either as a substantive topic in the syllabus or as a convenient tool for problem solving.

DETAILED COMMENTS

UNIT 1
PAPER 01
SECTION A
(Module 1: Basic Algebra and Functions)

Question 1

Specific Objective(s): (c) 1, 2, 3, 4, 6.

This question tested the use of the factor theorem to find factors and to evaluate an unknown coefficient in a polynomial given one of the factors. This question was attempted by almost all candidates. The majority of the candidates performed well on this question.

(a) The correct answer was obtained by the majority of candidates, however, a few of the candidates substituted incorrectly using $x = -1$ instead of $x = 1$ for the factor $x - 1$.

(b) This part of the question was generally well done by candidates, with various methods being used to determine the remaining factors such as long division, factor theorem, comparing coefficients and synthetic division. However, many candidates solved (for roots) after factorising and expressed the factors as $x = -1, -2$. Also, a small percentage of candidates expressed the remaining factors in quadratic form, failing to factorise completely.

Some candidates did not recognize that a non-zero remainder after division implied that no factors existed.

Answer(s):

(a) $p = 2$

(b) $(x + 1), (x + 2)$

Question 2

Specific Objective(s): (e) 1, 2; (f) 2.

This question examined the candidates' ability to solve an equation as well as to perform operations involving surds.

The question was generally well done by the many candidates (approximately 90 per cent) who attempted it, with approximately half of them scoring the maximum mark.

(a) Most candidates were able to express 27 as 3^3. However, approximately 25 per cent of the candidates were unable to express $(3^x)^2$ as 3^{2x}, expressing it instead as 3^{x+2} or, as 3^{x^2}, in a few cases. Despite the errors made with the indices, most candidates showed competence in their ability to equate the indices and to solve for x.

(b) Most candidates (approximately 75%) recognized the need to rationalize the denominator of the fraction by using its conjugate. However, some careless errors were made in the expansion, grouping and simplification of the surds, hence preventing some candidates from obtaining full marks in this part of the question. There were errors with fundamental operations such as $\sqrt{3}\sqrt{3} = \sqrt{6}$ and $\sqrt{3}\sqrt{3} = 9$.

Answer(s):

(a) $x = 6$ (b) $x = 13,\ y = -7$

Question 3

Specific Objective(s): (d) 1, 3, 9.

This question tested the ability of candidates to obtain coordinates of a given point after translations of the graph $y = f(x)$ and applying the principles of one-to-one functions in solving an equation involving composite functions.

This question was popular among the majority of candidates.

(a) Candidates were aware that a translation was involved. However, many failed to effect the correct translation; applying it to the wrong coordinate in some cases and to the wrong direction in other cases.

(b) The proof that $f(x)$ is a one-to-one function was poorly done by the majority of candidates. Much practice is needed with these proofs from first principles, that is, the general proof (instead of the proof for specific/particular values of x which was popular among candidates). Some candidates used proof by counter example for NOT one-to-one.

In the second part, the more confident candidates opted for the "hence" approach, while the majority used the "otherwise" approach in solving for x. The composite functions posed a challenge for the weaker candidates, with many simplifying and substituting incorrectly, for example, replacing x with $(3x - 2)$ to work out the composite as in $f(x - 3) = 3x - 2 - 3$ instead of the **correct approach $f(x - 3) = 3(x - 3) - 2$**.

Answer(s):

(a) (i) $A'(1, 1)$

(ii) $A''(-2, 3)$

(b) (ii) $x = -5$

Question 4

Specific Objective(s): (a) 6; (f) 5.

This question examined the candidates' ability to obtain the solution sets of inequalities involving the modulus function and to express a quadratic function in the form of a completed square.

The response to this question was good with approximately 80 per cent of the candidates scoring at least 6 marks.

(a) Some candidates encountered problems in performing the operations necessary for the removal of the modulus sign. A common error observed was $(x - 4)^2 - 6 > 0$ instead of the correct inequality $(x - 4)^2 > 6^2$ (or equivalent) which is obtained by squaring both sides.

Final answers were given as equations (roots) in some instances and not as a solution set in inequality form. A number of the candidates who used a graphical method to obtain their answer had difficulty expressing their final answer as an inequality.

A number of **errors** occurred with transpositions and sign changes, for example:-
$(x - 4) - 6 > 0 \quad \Rightarrow \quad x - 4 > 6 \quad \Rightarrow \quad x > 2$ instead of $x > 10$.

(b) The question on completing the square was better manipulated than Part (a), however, a few errors were made with simple fraction addition such as $w = 2 + \frac{1}{12} = \frac{1}{6}$. Some candidates equated the coefficients and **incorrectly** obtained $w = 2$.

Answer(s): (a) $x > 10$ or $x < -2$ (b) $u = -3,\ v = \frac{1}{6},\ w = \frac{25}{12}$

Question 5

Specific Objective(s): (f) 1, 7(i), 8(i), 8(ii), 10.

This question tested the candidates' ability to solve simultaneous equations in two unknowns, one being quadratic and one being linear.

The majority of the candidates attempted this question and a large percentage of them earned at least 5 out of 7 marks. The candidates who used the substitution $y = 5 - 3x$ were more successful than the ones who used $x = \frac{5 - y}{3}$ which presented a challenge to weaker candidates in terms of simplifying to obtain the correct quadratic equation.

A few candidates tried the elimination method without much success and in most cases, could not obtain the correct quadratic equation. Some candidates who obtained the quadratic equation in the form $-2x^2 + 5x - 2 = 0$ had difficulty factorizing it. Some candidates incorrectly solved $-2x + 1 = 0$ as $x = -\frac{1}{2}$. Also, a significant number of candidates solved for x and forgot to solve for y to complete the solutions.

Simple multiplication errors such as $3 \times \frac{1}{2} = 3\frac{1}{2}$ are not expected at the CAPE level.

Several errors in factorizing quadratic equations were noted as well, the most popular being:-

$-2x^2 + 5x = 2 \quad \Rightarrow \quad x(-2x + 5) = 2 \quad \Rightarrow \quad x = 2$ or $x = \frac{3}{2}$

Answer(s): $x = \frac{1}{2}, y = 3\frac{1}{2}$ and $x = 2, y = -1$

SECTION B
(Module 2: Plane Geometry)

Question 6

Specific Objective(s): (a) 1, 2, 7 (i), 8, 9

The question tested the candidates' ability to solve problems in coordinate geometry dealing with equations of lines, points of intersection and perpendicularity of lines.

Approximately 85 per cent of the candidates attempted the question with the majority earning 4 or 5 marks out of the maximum 9 marks. A common mistake made by candidates related to treating M as the mid-point of one or both of the lines AC or BD. A few candidates also used BD perpendicular to AC and obtained incorrect solutions.

Answer(s): (a) (i) Equation of AC is $3y = x + 2$

(ii) Equation of BD is $y = 2x - 6$

(b) $M \equiv (4, 2)$

Question 7

Specific Objective(s): (b) 1, 5, 18, 20

This question was a test of the trigonometric form $R\cos(\theta + \alpha)$.

Many candidates attempted the question but there was limited success in most cases.

(a) Some candidates ignored the form $R\cos(\theta + \alpha)$ in attempting to manipulate $\cos\theta - \sin\theta$ within the parameters specified. A few performed well, nevertheless.

(b) Candidates who were familiar with the form in Part (a) performed well in Part (b). However, some were unable to obtain the general solution which suggests that this form of the solution needs further practice.

Answer(s): (a) $\cos\theta - \sin\theta = \sqrt{2}\cos\left(\theta + \frac{\pi}{4}\right)$

(b) $\theta = 2n\pi, \quad 2n\pi - \frac{\pi}{2}, \ n \in Z$

Question 8

Specific Objective(s): (b) 3, 6

Knowledge of the area of a sector and of the area of a triangle was required to solve this problem.

In general, the question was popular with the candidates. However, many seemed unwilling to use radian measure, thus reducing the maximum mark they could achieve on the question. Several candidates experienced difficulties in using the formulae for the respective areas, in terms of π, of the sector and the triangle.

Answer(s): (a) π cm (b) $\left(3\pi - \frac{9}{4}\right)\text{cm}^2$.

Question 9

Specific Objective(s): (c) 4, 5, 7

The question tested the candidates' ability to obtain the conjugate of a complex number, to manipulate a complex number and its conjugate and to find the modulus of a complex number.

Approximately 90 per cent of the candidates responded to this question. Many of them were unable to earn the maximum seven marks, mainly because they seemed unfamiliar with the conjugate of a complex number. Several **incorrect** representations of the conjugate of $4 + 3i$ such as $-4 - 3i$, $3 - 4i$; $3i + 4$, $-4 + 3i$ and $\frac{1}{5}(4 + 3i)$ were seen.

A significant number of candidates did not express $\frac{\bar{z}}{z}$ in the form $a + bi$ as required; they left the answer as $\frac{7 - 24i}{25}$. Others who correctly obtained $\frac{7}{25} - \frac{24}{25}i$ disregarded the fractions for $a$and b, and found the modulus in Part (b) as $\sqrt{7^2 + 24^2}$.

Answer(s): (a) $\frac{\bar{z}}{z} = \frac{7}{25} - \frac{24}{25}i$ (b) $\left|\frac{\bar{z}}{z}\right| = 1$

Question 10

Specific Objective(s): (d) 1, 3, 5, 8, 9, 10

This question tested basic elements of vector algebra in terms of magnitude, position vectors and perpendicularity.

In manipulating the vectors in this question, candidates made several errors, many of which were numerical in nature. As a consequence, several candidates did not earn maximum marks for their efforts.

Answer(s): (a) (i) $\overrightarrow{AB} = -\boldsymbol{i} - 6\boldsymbol{j}$ (ii) $\left|\overrightarrow{AB}\right| = \sqrt{37}$ (iii) $\overrightarrow{OM} = \frac{8}{3}\boldsymbol{i}$

(b) $\overrightarrow{OA}$ is not perpendicular to $\overrightarrow{OB}$ since $\overrightarrow{OA} \cdot \overrightarrow{OB} \neq 0$.

SECTION C
(Module 3: Calculus I)

Question 11

Specific Objective(s): (a) 3, 4, 5, 6, 7

This question tested knowledge of limits and continuity or discontinuity of functions.

(a) The majority of candidates recognised the indeterminate form when $x = -2$ was substituted in the original expression. However, a large number of them experienced difficulty in factorizing the cubic expression in the numerator. Some candidates factorized and cancelled correctly but then encountered difficulties with substituting $x = -2$ in the rational function which remained after cancellation.

Some candidates used L'Hopital's rule with a few doing so satisfactorily.

(b) Most candidates associated "continuity" with putting the denominator equal to zero but some of these had difficulty in drawing the correct conclusion when the results $x \neq 6$ and $x \neq -3$ were obtained.

Answer(s): (a) $\frac{3}{2}$ (b) Continuous for all $x \neq 6$, $x \neq -3$

Question 12

Specific Objective(s): 4, 7, 8, 9, 10, 11

This question examined differentiation of rational functions and integration of a trigonometric function.

(a) About 95 per cent of the candidates attempted this part of the question and were able to apply the quotient rule for differentiation, however, many errors were made in simplifying the resulting algebraic expression to arrive at the correct answer.

Some candidates wrote $f(x)$ as $(x^2 - 4)(x^3 + 1)^{-1}$ and successfully applied the product rule to obtain the answer.

(b) Only about 60 per cent of the candidates attempted this part of the question which is based on Specific Objectives (c) 7 and 8. Many candidates did not seem to know how to differentiate $u = \sin 2x$ to obtain $du = 2 \cos 2x\, dx$, while others among those who did obtain the expression for du, failed to substitute correctly to transform the original integral to $\int \frac{1}{2} u\, du$; the point seemed to have been missed that in substituting $u = \sin 2x$ all occurrences of x in the integrand $\sin 2x \cos 2x$ and dx should be replaced with some function of u so that a new integrand appears under the integral sign expressed entirely in u and du.

Not many candidates used the "or otherwise" approach, but a few observed that $\sin 2x \cos 2x = \frac{1}{2} \sin 4x$ and proceeded to obtain the correct solution of $\frac{1}{4}$.

Answer(s): (a) $f'(x) = \dfrac{-x^4 + 12x^2 + 2x}{\left(x^3 + 1\right)^2}$

(b) $\frac{1}{4}$

Question 13

Specific Objective(s): (b) 2, 7, 8, 9, 25

This question required candidates to obtain constant coefficients in a cubic equation, given points on the curve and the gradient at one of the points.

(a) This part of the question was attempted by almost all of the candidates, not all of whom earned maximum marks because of weaknesses in the algebraic manipulation. Several candidates did not show that $r = 0$ since the curve passed through the origin and so complicated the calculations which depended on this fact. Other candidates failed to differentiate and could not make use of the gradient at P being equal to 8 in order to obtain a second equation.

(b) Many candidates did not use the fact that the gradient was given as 8 at P but derived it. Others substituted $x = 2$, $y = 1$ at P (1, 2) instead of $x = 1$ and $y = 2$ to find the equation of the normal.

Answer(s): (a) $p = 3,\ q = -1,\ r = 0$ (b) $8y + x = 17$

Question 14

Specific Objective(s): (b) 7, 8, 9, 16, 17, 19, 20, 21

The question tested candidates' ability to find coordinates of stationary points of a curve and to determine the nature of the stationary points.

(a) The majority of candidates who attempted this question knew that the function had to be differentiated, however, after finding the first derivative and equating the result to zero only 60 per cent of them were able to arrive at the correct solution to the quadratic equation obtained. Of those who got the x-values correct, a considerable number could not correctly find the y-values.

(b) Most candidates knew that the nature of the stationary points revolved around "something" being "positive" or "negative", but some were uncertain what that "something" was.

Approximately 20 per cent of the candidates gained full marks on this question.

Answer(s): (a) (2, 16) and (–2, –16)

(b) (2, 16), a maximum; (–2, –16), a minimum

Question 15

Specific Objective(s): (c) 3, 6 (ii), 7(ii), 10 (i)

The question tested the candidates' ability to use the integral calculus to find the area between two curves.

(a) Most candidates knew that the points of intersection of the straight line and the parabola had to be found but a few made mistakes in factorisation. Several candidates earned the maximum marks.

(b) There were many good answers to this part, however, a few candidates made mistakes in using the incorrect form of the integrand representing the difference between the equations of the straight line and the parabola. Some others used wrong limits and obtained incorrect results.

Answer(s): $P \equiv (-1, 1)$, $Q \equiv (3, 9)$ (b) $\frac{32}{3}$ units^2

UNIT 1

PAPER 02

SECTION A

(Module 1: Basic Algebra and Functions)

Question 1

Specific Objective(s): (a) 1, (c) 1, 5, (e) 2, (f) 8

The question tested knowledge of real roots and real factors. Overall, the question was attempted by more than 90 per cent of the candidates. However, only a few managed to earn maximum marks.

(a) (i) The majority of the candidates recognised that the expression $x^4 - 9$ is the difference of two squares and hence were successful in factorising to give $(x^2 - 3)(x^2 + 3)$. Many candidates failed to state the factors as $\left(x - \sqrt{3}\right)$, $\left(x + \sqrt{3}\right)$ and $(x^2 + 3)$.

(ii) It was evident that some candidates had difficulty with the terms 'real factors' and 'real roots'. Most of the candidates gave $\sqrt{3}$ as the only root.

(b) (i) Although all of the candidates attempted this question, some of them experienced difficulty squaring an equation involving a fraction. Instead of getting,

$u^2 = x^2 + 8 + \frac{16}{x^2}$, most candidates wrote $u^2 = x^2 + \frac{16}{x^2}$ or $u^2 = x^4 + 8x^2 + 16$.

A few candidates failed to simplify u^2, rather leaving it as $x^2 + \frac{4x}{x} + \frac{4x}{x} + \frac{16}{x^2}$.

(ii) Many candidates misinterpreted this question. They used the method of substitution to obtain $u^2 - 9u + 20 = 0$. In other words they used an 'or otherwise' approach rather than a deductive approach as suggested. Most of the candidates failed to recognise that since $x \neq 0$, then $x^2 \neq 0$, but rather ignored the presence of x^2 in $f(x)$.

(iii) A poor interpretation of Part (ii) led to much difficulty in solving Part (iii). Nevertheless, most candidates managed to utilize the factor theorem in order to solve for x when $f(x) = 0$. A number of candidates gave 0 as one of the possible values.

Answer(s): (a) (i) $\left(x + \sqrt{3}\right)$, $\left(x - \sqrt{3}\right)$ and $\left(x^2 + 3\right)$ (ii) $x = \sqrt{3}$, $x = -\sqrt{3}$

(b) (i) $u^2 = x^2 + 8 + \dfrac{16}{x^2}$ (iii) $x = 1, 2, 4$

Question 2

Specific Objective(s): (a) 1, 4, 10, (c) 2

This question tested the use of the summation notation, sums and products of quadratic equations and the principle of Mathematical Induction.

(a) Many candidates attempted to solve this question using the principle of mathematical induction. This misinterpretation led to incorrect solutions. Also, most candidates failed to equate correctly $3\,S_{2n}$ and $11\,S_n$ as a result of some careless mistakes such as:

(i) Incorrectly substituting n instead of $2n$ in finding S_{2n}

(ii) Inaccurately expanding brackets for instance, $3(2n^2 + n) = 6n^2 + 6n$ and $2\left(\frac{11}{2}\left(n^2 + n\right)\right) = 11n^2 + n$.

Several candidates had problems solving the quadratic equation $n^2 - 5n = 0$.

Many of the candidates who solved the quadratic equation $n^2 - 5n = 0$ correctly to obtain $n = 0$ and $n = 5$, failed to state that only $n = 5$ satisfies $n \in N$.

(b) Most of the candidates attempted this question and were able to earn maximum marks. However, a few failed to identify correctly the sums and roots of both equations. For example, instead of $(2\alpha + \beta) + (2\alpha - \beta) = 8$, they wrote $\alpha + \beta = 8$.

(c) Most of the candidates attempted this question. However, the majority performed extremely poorly. A number of candidates could not indicate all the steps clearly. The ability to use critical thinking and make logical deduction continues to be a major weakness.

Candidates preparing to write the examinations should find the "Pure Mathematics Resource Material" document recently published by CXC helpful in studying Mathematical Induction, and other topics as well.

Answer(s): (a) $n = 5$

(b) (i) $p = \alpha + \beta$, $q = 4\alpha^2 - \beta^2$ (ii) $\alpha = 2$, $\beta = 12$
(iii) $p = 14$, $q = -128$

SECTION B
(Module 2: Plane Geometry)

Question 3

Specific Objective(s): 1, 2, 5, 7 (ii), 8, 9, 10, 13, 14

This question dealt with the geometry of the circle and tested the candidates' ability to find the lengths of the radius and of the tangents, the equation of a circle and points of intersection of the circle with the x-axis from given data about the circle.

Approximately 80 per cent of the candidates attempted this question. There were several good solutions. Varying methods were used to obtain results, showing that candidates had a good grasp of the relevant material.

Answer(s): (a) (i) radius = 5 units (ii) $(x-5)^2 + (y+4)^2 = 5^2$
(iii) $A \equiv (2, 0)$, $B \equiv (8,0)$ (iv) $3x + 4y = 24$ (v) $P \equiv (0, 6)$

Question 4

Specific Objective(s): (b) 5, 7, 8, 12, 13, 14, 15, 16

This question challenged the candidates' ability to apply basic trigonometric ratios in establishing identities, and in evaluating angles in the context of a triangle.

Overall performance was below expectation. The main difficulty seems to stem from candidates' inability to recall information from CSEC level on basic trigonometric ratios. While many candidates readily used the identity $\cos 2\theta = \cos^2\theta - \sin^2\theta$ in Part (a) (i), many did not make the connection with Part (a) (ii). In Part (b), the use of the ratios for the triples (3, 4, 5) and (5, 12, 13) in right-angled triangles, the fact that the exterior angle r is equal to the sum of the interior opposite angles p and q, and the sum of the angles in the large triangle being 180 giving $p + t = 120 - q$, all proved difficult for the majority of candidates.

Answer(s): (b) (i) $4/5$ (ii) $12/13$ (iii) $63/65$ (iv) $\frac{3\sqrt{3} - 4}{10}$

SECTION C
(Module 3: Calculus 1)

Question 5

Specific Objective(s): (b) 8, 9, 10, 11, 12, 13

This question examined differentiation and relationships between first and second derivatives of a given function with a hint of mathematical modelling.

(a) (i) This question was generally well done by most candidates, however a few candidates demonstrated weaknesses in differentiation, especially in the concept of the chain rule.

(ii) Most candidates who got the right answer for Part (i) were able to follow through to show this part.

(iii) The overall performance on this part was poor. Candidates did not recognize **the use of the product rule**. Most candidates attempted to differentiate the expression for dy/dx to obtain d^2y/dx^2, but failed to obtain the correct expression as a result of the algebraic manipulation involved.

(b) Generally, this question was poorly done. A few candidates were able to attain full marks. Most candidates did not seem to understand the maximum and minimum concept of cosine.

Many weaknesses were seen in the differentiation of the expression $h = 2(1 + \cos(\pi t/450))$. A common error was $dh/dt = -2\sin(\pi t/450)$.

Answer(s): (a) (i) $\dfrac{dy}{dx} = \dfrac{5x}{\sqrt{5x^2 + 3}}$

(b) (i) 4 metres (ii) $t = 450$ min. (iii) $\dfrac{\pi}{450}$ m/min

Question 6

Specific Objective(s): (b) 23, (c) 7 (ii), (iii), 8, 9, 10 (ii)

This question focused on a fundamental principle of definite integrals and the use of substitution towards the evaluation of such integrals. Curve sketching and the calculation of volume is also included.

(a) As in the previous year, the result $\int_o^a f(x)dx = \int_o^a f(a - x)dx$ was not fully understood by the candidates.

(i) This question was poorly done by most candidates. Very few candidates recognized the relationship between the constants a and $\pi/2$.

(ii) A few candidates obtained the correct answer, but most candidates had difficulty obtaining the answer $\pi/4$. Not many candidates used the formula for cos $2x$ in terms of $\sin^2 x$ or $\cos^2 x$ as a means of calculating I.

(b) (i) This question was well done by most candidates, however, a few did not recognize that the graph was a parabola which concaved upwards.

(ii) This question was generally well done by the majority of candidates. Some candidates used $V = \pi \int y^2\, dx$ instead of $\pi \int x^2\, dy$. A few candidates calculated incorrect values for the limits.

Answer(s): (b) (ii) $\frac{\pi}{2}$ units^3

UNIT 1
PAPER 03/B (ALTERNATE TO INTERNAL ASSESSMENT)
SECTION A
(Module 1: Basic Algebra and Functions)

Question 1

Specific Objective(s): (a) 6, (c) 1, (d) 1, 3, 5, (g)

This question focused on properties of functions and the solution of inequalities.

(a) Most candidates were unable to substitute competently into the given function.

(i) Most candidates made simple errors such as: $h\left(\frac{1}{t}\right) = \frac{1}{\frac{1}{t} + t}$ and $h\left(t^2\right) = \left(t + \frac{1}{t}\right)^2$

(ii) A number of candidates failed to simplify $h(t)h\left(\frac{1}{t}\right)$ — $h(t^2)$

(b) The response to this question showed that most candidates had difficulty in solving inequalities. A number of candidates multiplied both sides of the inequality by $(x + 2)$ instead of the more direct approach of multiplying both sides by $(x + 2)^2$. A few candidates accurately ensured that zero was on one side of the equality. However, a few demonstrated poor algebraic manipulations in attempting to simplify the inequality.

(c) Most of the candidates demonstrated a poor understanding of key terms used in functions.

Answer(s): (a) (i) $\left(t + \frac{1}{t}\right)^2$ (ii) 2

(b) $-3 < x < -2$

(c) (i) range of $h = \{x \in \mathrm{R}: x \geq 2\}$ (iii) domain of $h^{-1} = \{x \in \mathrm{R}: x \geq 2\}$

SECTION B
(Module 2: Plane Geometry)

Question 2

Specific Objective(s): (a) 5, 6, 8, 9, 14, 16

The question focused on the ellipse represented in terms of parametric equations and by means of its Cartesian equation. The equation of a tangent to the ellipse was also examined.

(a) Several candidates attempted this question, many of whom recognised that the trigonometric identity $\cos^2\theta + \sin^2\theta = 1$ was the means to obtaining the desired equation. Some candidates who were not informed of the correction ($y = 2\sin\theta$) obtained instead the form $\frac{x^2}{9} + \frac{y^2}{16} = 1$; they were awarded full credit.

(b) Only a few candidates were able to solve correctly this equation. The majority of the candidates were unable to derive the gradient of the tangent which they could have obtained by using the chain rule $\left(\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}\right)$ or with knowledge of implicit functions. Candidates' inability to derive the equation of the tangent led to some problems in Part (c).

(c) (i) A number of candidates assumed a certain equation for the tangent. Consequently, a few of them earned a reasonable number of marks due to their knowledge of some basic geometric concepts (x and y intercepts, the area of a triangle, the length of a given line and perpendicular lines).

(ii) Those candidates who obtained the coordinates of Q and R were able to find the area of ΔQOR quite easily.

(iii) Candidates were able to calculate the length of QR by means of the formula for the distance between two points with known coordinates.

(iv) Candidates successfully utilized the formula for the length of perpendicular from O to QR as $\left|\frac{ax + by + c}{\sqrt{a^2 + b^2}}\right|$.

No candidate utilized the result of (c) (ii) (that is, the area of triangle $A = \frac{1}{2}bh$) to find the length of the perpendicular.

Answer(s): (b) $\frac{\sqrt{3}}{6}x + \frac{y}{4} = 1$

(c) (i) $Q \equiv \left(2\sqrt{3}, 0\right)$, $R \equiv (0, 2)$ (ii) Area of $\Delta QOR = 2\sqrt{3}$ units^2

(iii) $QR = 4$ units (iv) Perp. = $\sqrt{3}$ units in length

SECTION C
(Module 3: Calculus 1)

Question 3

Specific Objective(s): (a) 3, 4, 5, 6 (b) 9, 11, 15 - 17, 19 - 21

This question deals with limits, critical values of a given cubic function and differentiation leading to a maxima and minima.

(a) (i) The majority of the candidates who attempted this question were unable to factorise the expression $(x - 1)$ in terms of $x^{1/3}$.

(ii) A number of candidates attempted to solve this question using the "hence" approach, although their results to Part (i) were incorrect. As a result, only a few candidates were able to obtain maximum marks.

(b) (i) Most candidates attempted this question and were successful in obtaining full marks. Many candidates were confused with critical values of y and critical point; in most cases candidates gave the latter, that is, $\left(\frac{-1}{3}, \frac{32}{27}\right)$, (1, 0).

(c) (i) Most of the candidates were successful in showing that $S = 2\pi r^2 + \frac{20}{r}$. However, a few candidates were unable to show that S has a minimum value when $r^3 = 5/\pi$.

At least 90 per cent of the candidates attempted this question. With the exception of Part (a), the question appeared manageable to several candidates.

Answer(s): (a) (ii) $\frac{1}{3}$

(b) (i) $\left(-\frac{1}{3}, \frac{32}{27}\right)$ and (1, 0) (ii) $\left(-\frac{1}{3}, \frac{32}{27}\right)$ – a max.; (1, 0) – a min.

GENERAL COMMENTS

UNIT 2

In general, the performance of candidates in Unit 2 was of a high standard with a small number of candidates reaching an outstanding level of proficiency. However, there were some candidates who were inadequately prepared for the examination.

Topics in Calculus, Simple Probability, Approximation to Roots of Equations, and Series seemed well covered. Weaknesses continue to manifest themselves at the level of algebraic manipulation, including substitution, which frustrate the processes required to complete the problem-solving exercises posed in the questions. As indicated in previous years, extended practice on respective themes needs to be undertaken in order to eradicate such deficiencies and raise the level of performance in the areas of weakness identified.

However, the results on the whole were very encouraging.

DETAILED COMMENTS

UNIT 2
PAPER 01
SECTION A
(Module 1: Calculus II)

Question 1

Specific Objective(s): (a) 3, 10

This question examined the exponential function and the use of logarithms in solving equations.

(a) Almost all candidates attempted this question with approximately 30 per cent obtaining full marks, however, some candidates experienced difficulties in factorising the expressions $e^{2p} - 2e^{p}$ and $e^{-p} - 2e^{-2p}$.

(b) Candidates were asked to solve an equation of the form $a^x = b$.

Most candidates attempted this part of the question with a satisfactory degree of success.

Overall, about 35 per cent of the candidates scored between 6 and 8 marks on this question.

Answer(s): (a) $p = \ln 2$, $q = \frac{1}{2}$ (b) $x = \frac{\log 3}{2\log 2} = 0.792$

Question 2

Specific Objective(s): (b) 2, 3, 5

This question focused on parametric equations of a curve, gradients and differentiation of functions.

(a) There were several good responses from the candidates on this part of the question, nevertheless, a few candidates had difficulties differentiating $\frac{4}{t}$ while others seemed unaware of the chain rule and were unable to finish the question.

(b) The quality of responses to this question was mixed. Some candidates had difficulty differentiating one or both of $\tan^2(3x)$ and $\ln(x^3)$. A few candidates obtained full marks for Part (b).

Answer(s): (a) gradient = –1 (b) $\frac{dy}{dx} = 6 \tan 3x \sec^2 3x + \frac{3}{x}$

Question 3

Specific Objective(s): (c) 1, 3

This question involved the use of partial fractions in finding integrals.

(a) This question was well done with a success rate of approximately 96 per cent.

(b) Most candidates were aware that $\int \frac{1}{b+x}\, dx = \ln | b + x |$ but too many seemed unaware that $\int \frac{1}{b-x}\, dx = -\ln | b - x |$.

Some candidates did not include a 'constant of integration' in each case.

Answer(s): (a) $P = 1,\ Q = 1$ (b) $\ln | 3 + x | - \ln | 2 - x |$ + constant

Question 4

Specific Objective(s): (c) 4, 5, 6, 7

This calculus question covered the topics of integration by substitution and integration by parts.

(a) This part of the question was generally well done, however, many candidates who did not use the substitution completely were confronted with the task of manipulating an integrand which was a function of both u and x.

(b) Several candidates attempted this part but 80 per cent of them could only reach as far as $x \tan x - \int \tan x \, dx$, not being able to evaluate $\int \tan x \, dx$.

In both parts, the constant of integration was omitted.

Answer(s): (a) $\left(\frac{(2x - 5)}{24}\right)^6 + \frac{(2x - 5)^5}{4} + \text{const.}$ (b) $x \tan x + \ln(\cos x) + \text{const.}$

Question 5

Specific Objective(s): (c) 7, 9

This question models a manufacturing process by means of a first order differential equation. The use of integrating factors was tested.

Approximately 85 per cent of the candidates attempted this question with about 9 per cent obtaining full marks.

The major difficulty encountered by candidates was the failure in obtaining the correct integrating factor. Some candidates used 'c' as the constant of integration which confused their otherwise correct solution.

Answer: $c = 5x + \frac{205}{2} e^{-2x} - \frac{5}{2}$

SECTION B
(Module 2: Sequences, Series and Approximations)

Question 6

Specific Objective(s): (a) 1, 2

This question tested the candidates' knowledge of, and ability to manipulate, recurrence relations as they apply to sequences.

About 98 per cent of candidates, attempted this question and performed extremely well. Most scored full marks which indicated that they were well prepared in this area.

Candidates provided a variety of approaches to obtain the solution.

Answer(s): (b) $u_3 = 2^2, \; u_5 = 2^3$

Question 7

Specific Objective(s): (b) 4, 8

Geometric progressions were covered in this question.

Approximately 95 per cent of the candidates attempted this question. Many were able to generate the equations $a + ar^2 = 50$, $ar + ar^3 = 150$ connecting the first term a and common ratio r, but some encountered difficulties in solving these two equations to find a and r. Those candidates who obtained the correct values of a and r were able to complete the question and earn maximum marks.

Answer(s): (a) $r = 3$ (b) $a = 5$ (c) sum = 605

Question 8

Specific Objective(s): (c) 2, 3

This question examined the candidates' ability to extract the independent term in a binomial expansion.

Candidates found this question easy with about 95 per cent submitting good answers and approximately 90 per cent of those obtaining maximum marks.

Answer(s): ${}_{10}C_6\, 2^6(-5)^4$

Question 9

Specific Objective(s): (c) 1, 2, 3

This question on the binomial theorem tested the candidates' knowledge about binomial coefficients in terms of factorials.

Most candidates who attempted the question were able to do both parts of (a) but had considerable difficulty in obtaining the result in (c) which suggests that more practice is needed in the area of the manipulation of expressions involving factorials.

Answer(s): (a) (i) $\dfrac{(2n)!}{n!\,n!}$ (ii) $\dfrac{(2n-1)!}{n!\,(n-1)!}$

Question 10

Specific Objective(s): (b) 10

The topic covered the method of differences in the summation of series.

Approximately 80 per cent of the candidates attempted this question and of these 85 per cent obtained less than 5 marks while 15 per cent obtained between 5 and 8 marks.

Candidates were comfortable with the method of finding the solution to Part (a) but had some difficulties with substitution. A few candidates were unable to cope with the summation notation $\sum$.

Answer(s): (b) $\sum_{r=3}^{n} \frac{r+3}{r(r-1)(r-2)} = \frac{1}{2}\left[\frac{7}{2} - \frac{2n+3}{n(n-1)}\right]$

SECTION C
(Module 3: Counting, Matrices and Modelling)

Question 11

Specific Objective(s): (a) 2, 3, 4

This question tested arrangements of objects.

(a) Many candidates failed to deduce the number of combinations of the vowels. Some candidates confused combination with permutation.

(b) The question was generally well done by most candidates. Some candidates confused addition with multiplication. Candidates were unable to differentiate between AND and OR.

Answer(s): (a) $\frac{8!}{3!\ 2!}$ (b) 186

Question 12

Specific Objective(s): (a) 5, 6, 7, 8, 9

This question tested some basic properties of counting and probability.

(a) Many candidates wrote the number 36 instead of representing the answer as ordered pairs.

(b) (i) Many candidates did not subtract 1/36 from 12/36.

(ii) This part was well done by many candidates.

(iii) Many candidates did not subtract 1/36 from (11/36) + (6/36) and obtained the correct answer.

Answer(s): (a) $\{ (a, b) : a, b \in \mathrm{N}, 1 \le a, b \le 6\}$

(b) (i) $\frac{11}{36}$ (ii) $\frac{6}{36}$ (iii) $\frac{4}{9}$

Question 13

Specific Objective(s): (b) 1, 2

This question focused on the products, transposes and determinants of square matrices.

This question was reasonably well done by most candidates. A few candidates showed weaknesses in calculating the determinant.

Answer(s): (a) $|X| = -365$ (b) $Y^T X = \begin{pmatrix} 7 & 68 & 29 \\ 56 & 78 & 4 \end{pmatrix}$

Question 14

Specific Objective(s): (b) 7, 8

This question covers the calculation of the inverse of non-singular matrices and the solution of matrix equations.

(a) Most candidates who used row reduction were unable to attain the final answer for A^{-1}.

(b) Many candidates wrote $X = YA^{-1}$ instead of $X = A^{-1}Y$. This was a common error.

A few candidates attempted to derive the answer using simultaneous equations but experienced difficulties.

Answer(s): (a) $A^{-1} = \frac{1}{2}\begin{pmatrix} 5 & 6 & -15 \\ -7 & -8 & 21 \\ -1 & -2 & 5 \end{pmatrix}$ (b) $X = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$

Question 15

Specific Objective(s): (c) 1, 2

This question examined rate of increase in the context of mathematical modelling of properties of a sphere.

This question was generally well done by most candidates. A few students showed weaknesses in algebraic operations.

Incorrect differentiation was seen by a few candidates.

Answer(s): (a) $\frac{dr}{dt} = \frac{11}{16\pi}$ cm s^{-1} (b) $\frac{ds}{dt} = 55$ cm^2 s^{-1}

UNIT 2
PAPER 02
SECTION A
(Module 1: Calculus II)

Question 1

Specific Objective(s): (a) 6, 8, 10, 11

This question related to logarithmic and exponential functions, and their graphs.

All candidates attempted this question with varying degrees of success.

(a) About 90 per cent of the candidates attempted this part. Change of base and the value of x satisfying the equation $\log_8 x = -2$ presented some candidates with major challenges.

(b) The table was generally well done in (i). In (ii), most candidates changed the scale to accommodate the range of values for the graph while others did not plot the point at $x = 3$; in both instances, candidates were credited with maximum marks.

About 96 per cent of the candidates who attempted this part of the question obtained at least 4 of the 5 marks that were allocated.

In Part (iii) b), about 50 per cent of the candidates obtained the end points for the range and far less got the inequality completely correct.

Answer(s): (a) $x = 2, \frac{1}{64}$ (b) (iii) a) $x = 0,$ b) $-1 \leq x < 0$

Question 2

Specific Objective(s): (c) 7, 8

This question tested the candidates' ability to use reduction formulae.

(a) About 50 per cent of the candidates who attempted this question earned maximum marks. Many were familiar with the identity $\sec^2 x = 1 + \tan^2 x$ and applied it correctly.

(b) There were several good responses to this part, with many candidates obtaining full marks.

(c) (i) About 25 per cent of the many candidates who attempted this part of the question obtained full marks. The majority of candidates earned at least 4 of the 7 marks.

(ii) About 15 per cent of the candidates who attempted this part obtained full marks. Some candidates tried to use I_n for $n = 0$ even though it was stated earlier that $n \geq 2$.

Answer(s): (b) $n \tan^{n-1} x \sec^2 x$ (c) (ii) $I_4 = \frac{\pi}{4} - \frac{2}{3}$

SECTION B
(Module 2: Sequences, Series and Approximations)

Question 3

Specific Objective(s): (a) 2, 5; (b) 4, 8, 9; (c) 1

This question tested the candidates' ability to apply the principle of mathematical induction to factorials; identifying and obtaining the general term of a geometric series as well as its sum to infinity.

The majority of the candidates attempted this question. However, a minority was able to gain near maximum marks.

(a) The Mathematical Induction was very poorly done. It was very clear that most students lacked the understanding of the **process** of induction; not even being able to recognize what was to be proved.

The basic step of proving that $n! = 1! = u_1 = 1$(given) was hardly seen. Most candidates wrote a very clear and concise memorized conclusion even though a proper inductive step was absent. Many candidates added the $(k + 1)^{th}$ term to the k^{th} term although the question dealt with $n!$ – a clear lack of understanding.

Some other weaknesses observed were:

– The inductive step was attempted by some students who replaced k with $k + 1$ thus obtaining $u_{k+1} = (k + 1)!$ instead of $u_{k+1} = (k + 1)u_k = (k + 1)k! = (k + 1)!$

– Incorrect conclusions involving for all $n \in Z$, for all $n \in$ R, were frequently seen, instead of for all $n \in Z^+$, for all $n \in N$ or equivalent.

(b) In this part, many candidates failed to recognize that the question asked for the n^{th} term which should be simplified into a single fraction in this case. This prevented some candidates from obtaining full marks. Careless simplification errors were made, especially with the indices because of the lack of brackets, for example, $2 - (n - 1)$ was incorrectly written as $2 - n - 1 = 1 - n$.

In Part (ii), many candidates used a particular solution approach, when a general solution approach was necessary to show that the series S is a geometric progression.

Answer(s): (b) (i) $a_n = \frac{18}{3^n}$ **OR** $6\left(\frac{1}{3}\right)^{n-1}$ **OR** $2 \times 3^{2-n}$

(ii) $\frac{a_n}{a_{n-1}} = \frac{1}{3}$, a constant

(iii) $a = 6, \quad r = \frac{1}{3}$

(iv) $S_\infty = 9$

Question 4

Specific Objective(s): (b) 4, 7; (c) 1; (e) 1, 2, 4

This question examined the use of the Intermediate Value Theorem in testing for the existence of a root in an equation; the Newton-Raphson Method in finding successive approximations to a root in an equation and mathematical modelling involving an arithmetic series.

For this question, Part (b) proved more challenging to the candidates than Part (a).

(a) (i) The main difficulties candidates encountered with this part are as follows:–

- Not stating that the function is "continuous"
- Attempting to use the given formula to get the root
- Using the derivatives $f'(0)$ and $f'(1)$ in an attempt to show that $f(x)$ has a root in the interval (0,1).

However, many candidates correctly used $f(0)$. $f(1) < 0$ or the sign change criterion between $f(0)$ and $f(1)$, as well as mentioning the Intermediate Value Theorem, in this part of the question.

(ii) The main areas of concern regarding the candidates' approach were as follows:

- Writing the Newton-Raphson Formula incorrectly as

$x_2 = \frac{f(x_1)}{f'(x_1)}$ or $x_2 = -\frac{f(x_1)}{f'(x_1)}$ instead of the correct form $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$.

- Having difficulty in simplifying the expression $x_n - \dfrac{f(x_n)}{f'(x_n)}$ due mainly to the omission of the necessary brackets, that is, $\dfrac{x_1(4x_1^3 - 4) - x_1^4 - 4x + 1}{4x_1^3 - 4}$ instead of $\dfrac{x_1(4x_1^3 - 4) - (x_1^4 - 4x + 1)}{4x_1^3 - 4}$
- Substitution of particular values in the Newton-Raphson Formula in order to prove the given expression.

(b) (i)
- Not recognising the loan repayment as an A.P., hence writing **all 12** instalments, then summing.
- Using P = \$570 to work out the 12 payments then summing to prove that A = \$10 800 rather than using A = \$10 800 to prove that P = \$570.

(ii) It was noted that the students had more difficulty with (b) (ii) than with (b) (i).

- Writing the loan balance as (10 800 – nth instalment) instead of (10 800 – S_n) where S_n is the sum of the first n instalments.
- Using the sum of a G.P. instead of the sum of an A.P.
- Incorrect expansion and simplification of the brackets when forming an expression for the sum of the instalments.
- Using 10 800 = S_n instead of (10 800 – S_n) as an attempt to find an expression for the remaining debt.

Answer(s): (b) (ii) $D = 10\,800 - 540n - 30n^2$

SECTION C
(Module 3: Counting, Matrices and Modelling)

Question 5

Specific Objective(s): (a) 4, 7, 9, 10

This question tested the candidates' knowledge of simple counting principles and probability.

There were several good responses to this question with many candidates scoring between 12 and 20 marks. A significant number earned maximum marks.

(a) Some candidates experienced confusion between permutation and combination. More practice is recommended to clarify the distinction.

(b) A few candidates included the column totals in their calculation of the number of males and the number of females.

In Part (ii), some candidates did not recognise the problem as inclusive and so many found P (males) + P (news) rather than P (males) + P (news) – P (males and news).

Generally, attention should also be paid to computational correctness in problems of this kind.

Answer(s): (a) (i) 210 (ii) $\frac{55}{210} = 0.262$

(b) (i) $\frac{48}{100} = 0.48$ (ii) $\frac{70}{100} = 0.70$ (iii) $\frac{20}{100} = 0.20$

(iv) $1 - \frac{30}{100} = \frac{70}{100} = 0.70$

(c) (i) $p = 0.20$ (ii) 0.45

Question 6

Specific Objective(s): (b) 1, 2, 3, 4, 5, 6

The question examined solutions to systems of linear equations by matrix methods, as well as properties of matrices and determinants.

Although almost all the candidates attempted the question, generally, it was not well done. Very few candidates were able to obtain full marks in this question.

(a) (i) In a few cases, candidates were not able to express the system of equations in complete matrix form, that is, $AX = Y$.

(ii) Although candidates were able to identify the augmented matrix, in many instances this was not written using valid notation. Some candidates confused the augmented matrix with the adjugate matrix.

(iii) Most candidates were able to perform elementary row operations. There were, however, many arithmetic errors. There also seemed to be a problem in identifying when the matrix was in echelon form with candidates getting a simpler matrix to work with but not a matrix in the desired echelon form.

(iv) Most candidates were able to find the value of α from their reduced matrix.

(v) The majority of candidates were able to solve the system of equations up to a point. Many got as far as stating $y = -1$, $x + z = 11$. Only a few candidates were able to go further to choose x or z arbitrarily and state the final required answer.

(b) (i) This was generally well done, however, many candidates seemed not to be able to identify the 3 x 3 identity matrix, I. Some students misinterpreted $KI - A$ as $K(I - A)$.

(ii) Most candidates who correctly found $KI-A$ were able to at least make a valid attempt at finding the determinant. In many instances, there were errors in the simplification of the determinant and this led to the wrong cubic equation. Most candidates were able to use the factor theorem correctly to factorise the cubic. There were cases of candidates correctly factorising the valid cubic equations but losing marks by making errors such as $k^2 - 3 = 0 \Rightarrow k = \pm 3$ or by simply stating that $k^2 - 3 = 0 \Rightarrow k^2 = \sqrt{3}$.

Answer(s): (a) (i) $\begin{pmatrix} 1 & 1 & 1 \\ 3 & -2 & 3 \\ 2 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 35 \\ \alpha \end{pmatrix}$ (ii) $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 10 \\ 3 & -2 & 3 & 35 \\ 2 & 1 & 2 & \alpha \end{array}\right)$

(iii) $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 10 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & \alpha - 21 \end{array}\right)$ (iv) $\alpha = 21$

(v) $x = 11 - z,\ y = -1,\ z$ arbitrary

(b) (i) $kI - A = \begin{pmatrix} k & 1 & -1 \\ 1 & k & -1 \\ -1 & -1 & k-1 \end{pmatrix}$ (ii) $|kI - A| = 0 \Rightarrow k = 1\ k = \pm\sqrt{3}$

UNIT 2
PAPER 03/B (ALTERNATIVE TO INTERNAL ASSESSMENT)

SECTION A
(Module 1: Calculus II)

Question 1

Specific Objective(s): (c) 1, 2, 3

The question posed a mathematical modelling problem based on proportionality, exponentials and the solution of a differential equation.

Most of the candidates who took the paper were unable to form the differential equation required in Part (a).

While 10% of the candidates gained maximum marks, the majority earned less than 10% of the 20 marks.

Several candidates were unable to manipulate the exponential and logarithmic functions.

Answer(s): (a) $\frac{dA}{dt} = kA, \quad k < 0$ or $\frac{dA}{dt} = -kA, \quad k > 0$

SECTION B
(Module 2: Sequences, Series and Approximation)

Question 2

Specific Objective(s): (b) 4, 6, 8, 9

This question examined geometric series and mathematical modelling.

The general performance on this question was poor with the majority of candidates earning between 0 and 5 marks.

(a) Several of the candidates were unable to recall that the common ratio r of a convergent geometric series must satisfy the condition $|r| < 1$.

(b) (i) Some candidates were unable to express the answers to 2 significant figures.

(ii) Most candidates did not recognise that maximum output meant sum to infinity.

Answer(s): (a) $x < 6, \quad -1 < x < 1, \quad x > 6$

(b) (i) a) 1 300 000 to 2 sig. fig. b) 920 000 to 2 sig. fig.
(ii) 29 to 2 sig. fig.

SECTION C
(Module 3: Counting, Matrices and Modelling)

Question 3

Specific Objective(s): (a) 1, 2, 6, 7, 9, 10

The question covered arrangements of objects, probability, and solutions of system of equations. Mathematical modelling is also evident.

(a) Candidates were able to find the probability with no restrictions in (i), but had severe difficulties in obtaining the correct answer to Part (ii).

(b) Generally, this part of the question was well done with 95 per cent of the candidates obtaining the correct answers.

Answer(s): (a) (i) 10! (ii) 0.8

(b) (i)
$$2x + 2y + z = 5950$$
$$4x + y + z = 11\,450$$
$$5x + 3y + 2z = 14\,600$$

(ii)
$$\begin{pmatrix} 2 & 2 & 1 \\ 4 & 1 & 1 \\ 5 & 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5\,950 \\ 11\,450 \\ 14\,600 \end{pmatrix}$$

(iii)
$$x = 2800$$
$$y = 100$$
$$z = 150$$

PAPER 03

INTERNAL ASSESSMENT

Module Tests

The main features assessed are:

- The mapping of the items tested to the specific objectives in the syllabus;
- Coverage of content of each Module test;
- The appropriateness of the items tested for the CAPE level;
- The presentation of the sample (Module Tests and students' scripts);
- The quality of the teachers' solutions and mark schemes;
- The quality of the teachers' assessment – consistency of marking using the mark scheme.

In general, the question papers, solutions and detailed mark schemes met the CXC CAPE Internal Assessment requirements for both Unit 1 and Unit 2.

Too many of the Module tests comprised items from CAPE past examination papers. Untidy 'cut and paste' presentations with varying font size were common place. Teachers are encouraged to use the past CAPE examination papers ONLY as a guide and to include original and creative items in the Module tests.

The stipulated time for Module Tests ($1 - 1\frac{1}{2}$ hours) must be strictly adhered to as students may be at an undue disadvantage when Module Tests are too extensive or insufficient.

The specific objectives tested in a particular module test must be from that Module.

In most cases, the Internal Assessments that were based on collective testing of Modules were inadequate in terms of the coverage of the Unit.

Improvement is needed in the presentation of samples for moderation. (See Recommendations below).

The moderation process relies on the validity of teacher assessment. In a few instances, the students' scripts reflected evidence of rewriting of some solutions after scripts were formally assessed by the teacher. Also, there were a few cases where students' solutions were replicas of the teacher's solutions – some contained identical errors and full marks were awarded for incorrect solutions.

To enhance the quality of the design of Module tests, the validity and accuracy of teacher assessment and the validity of the moderation process, the following Internal Assessment guidelines are recommended.

Recommendations (Module Tests):

(i) Design a separate test for each Module.

(ii) In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered, and the same marking scheme used. ONE sample of FIVE students will form the sample for the centre.

(iii) **In 2008, the format of the Internal Assessment remains unchanged. [Multiple Choice Examinations will not be accepted].**

Please note Recommendations for Module Tests and Presentation of Sample.

Recommendations for Module Tests and Presentation of Samples

1. COVER PAGE TO ACCOMPANY EACH MODULE TEST

The following information is required at cover for each Module test.

- Name of School and Territory; Name of Teacher;
- Unit Number and Module Number;
- Date and duration ($1 - 1\,{}^{1}\!/_{2}$ hours) of Module Test;
- Clear Instructions to candidates;
- Total Marks allotted for Module Test;
- Sub-marks and total marks for each question must be clearly indicated.

2. COVERAGE OF SYLLABUS CONTENT

- The number of questions in each Module Test must be appropriate for the stipulated time of 1 - 1 $\frac{1}{2}$ hours;
- CAPE Past Examination papers should be used as a guide ONLY;
- Duplication of specific objectives and questions must be avoided;
- Specific objectives tested must be within the syllabus.

3. MARK SCHEME

- Detailed mark schemes must be submitted; holistic scoring is not recommended;
- FRACTIONAL MARKS MUST NOT BE AWARDED;
- The total mark for Module tests must be clearly stated on the teacher's solution sheet;
- The student's mark must be entered on the FRONT page of the student's script.

4. PRESENTATION OF SAMPLE

- Students' responses must be written on normal size paper, preferably 8 $\frac{1}{2}$" x 11";
- Question numbers are to be clearly written in the left margin;
- The total score for each question marked on students' scripts must be clearly written in the right margin;
- ONLY original students' scripts must be sent for moderation.
  Photocopied scripts will not be accepted;
- Module Tests must be typed using a legible font size, (or if handwritten must be neat and legible)
- The following are required for EACH Module test:

  – A question paper
  – Detailed solutions with detailed mark schemes
  – The scripts of the candidates comprising the sample
    (Students' scripts in the sample are to be organized by Modules)

- Marks recorded on PMaths 1 - 3 and PMath 2 - 3 must be rounded off to the NEAREST WHOLE NUMBER;
- In cases where there are five or more registered candidates, FIVE samples must be sent.