

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

**REPORT ON CANDIDATES' WORK IN THE
ADVANCED PROFICIENCY EXAMINATION
MAY/JUNE 2010**

APPLIED MATHEMATICS

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GENERAL COMMENTS

The revised Applied Mathematics syllabus was examined this year for the third time. This is a two-unit course comprising three papers. Paper 01, consisting of 45 multiple choice items, and Paper 02, consisting of 6 essay questions were examined externally, while Paper 03 was examined internally by class teachers and moderated by CXC. Contributions from Papers 01, 02 and 03 to each unit were 30 per cent, 50 per cent and 20 per cent, respectively.

Unit 1, Statistical Analysis, tested (1) Collecting and Describing Data; (2) Managing Uncertainty and (3) Analysing and Interpreting Data. Unit 2, Mathematical Applications, tested (1) Discrete Mathematics, (2) Probability and Probability Distribution and (3) Particle Mechanics.

For Unit 1, 456 candidates wrote Papers 01 and 02 and 11 wrote the Alternative to the Internal Assessment paper, Paper 03/2. For Unit 2, 197 candidates wrote Papers 01 and 02 and three candidates wrote Paper 03/2.

Approximately 80 per cent of the candidates registered for Unit 1, Statistical Analysis, and 83 per cent registered for Unit 2, Mathematical Applications, obtained acceptable grades, Grades I–V. The standard of work seen from most of the candidates in the examinations was satisfactory.

In Unit 1, candidates appeared to be well prepared in Describing and Collecting Data and in Managing Uncertainty, while Analysing and Interpreting Data continued to be a challenge for many.

In Unit 2, candidates appeared to be well prepared in Discrete Mathematics and Probability and Distributions. Particle Mechanics again seemed to be a challenge for many candidates. Generally, candidates are still having difficulty with algebraic manipulations.

DETAILED COMMENTS

UNIT 1

Paper 01–Multiple Choice

Performance on the 45 multiple-choice items on Paper 01 produced a mean of 56 out of 90 with scores ranging between 0 and 88.

Paper 02–Essay

Module 1

Question 1

This question tested candidates' ability to

- (a) distinguish between qualitative and quantitative data, and between discrete and continuous data
- (b) distinguish between a parameter and a statistic
- (c) distinguish between a sample and a population
- (d) identify simple random sampling, systematic sampling, stratified sampling, quota sampling and cluster sampling
- (e) determine class boundaries, class widths, and calculate frequency densities

(f) state some advantages of grouping data.

The question was attempted by all candidates, with about 75 per cent satisfactory responses. In Part (a), candidates performed exceptionally well.

In Part (b), about 60 per cent of the candidates did well. The other 40 per cent knew the definition of parameter or a statistic but did not know how to relate them to the context of the question given.

Part (c) was generally well done by most candidates; a few candidates had difficulty differentiating between a sample and a population.

Answers (i) sample (ii) population

In Part (d), about one-third of the candidates scored full marks. The other candidates showed some measure of confusion as to which technique to use. Responses included techniques such as convenience instead of quota, purposive, snowball or multistage instead of systematic.

Answers (i) cluster (ii) stratified (iii) systematic (iv) quota (v) simple random

Part (e) (i) and (ii) were well done by most of the candidates. A few candidates gave the boundaries as 24–29, while others gave no indication of how they got their answers.

Part (e) (iii) was poorly done by the majority of the candidates. Candidates did not understand the concept of frequency density.

In Part (e) (iv), many candidates knew that the data was grouped but still had difficulty estimating the number of children who took more than 60 minutes to complete the assignment. Very few candidates were able to correctly calculate the answer.

Part (v) was poorly done by most candidates. Many gave some notable responses such as ‘normally distributed’, ‘does not take exact values’ and ‘not accurate’.

Answers (i) 24.5, 29.5 (ii) 10 (iii) 3 (iv) 8 (v) loss of individual data values

Question 2

This question tested candidates’ ability to

- (a) (i) construct and use a stem and leaf diagram to display data
- (ii) give one disadvantage of using a stem and leaf diagram
- (b) determine the median and mode from ungrouped data
- (c) calculate the mean from ungrouped data
- (d) outline an advantage of using the mean from raw data
- (e) calculate the trimmed mean from given data
- (f) interpret and describe the shape of a distribution in terms of ‘skewness’
- (g) describe the shape of the distribution

This question was attempted by most of the candidates who gave satisfactory responses.

Part (a) (i) was exceptionally well done except for a few candidates who omitted the key. Part (a) (ii) was not well done. Only a few candidates were able to correctly state one advantage of using the stem and leaf diagram to display data.

In Part (b) (i), candidates performed exceptionally well except for a few who calculated the position of the median but were unable to give the final answer. For Part (b) (ii), the majority of the candidates were able to determine the modal age.

In Part (c), all the candidates knew the formula for calculating the mean but a few made errors in summing the given values. Part (d) seemed to have posed a great difficulty for most candidates although some knew the answer but had problems expressing the solutions correctly.

In Part (e), many candidates showed competency in calculating the 8 per cent trimmed mean which was $\frac{8}{100} \times 25 = 2$, but had difficulty calculating the mean using $\frac{\sum x}{n}$ which was $\frac{542}{21}$. Few candidates knew that two values should be subtracted from both sides of the data. Many candidates used n as 23 rather than 21.

For Part (f) (i), half of the candidates had difficulty determining the quartiles. They knew the formula but did not calculate the true value. Many gave the position of the term as the answer.

Part (f) (ii) was exceptionally well done. Candidates were able to calculate the semi-interquartile range of the ages based on their responses from Part (f) (i). A few candidates had the wrong formula and therefore used $\frac{Q_3 + Q_1}{2}$ instead of $\frac{Q_3 - Q_1}{2}$.

Part (g), was fairly well done. The majority of candidates correctly described the shape of the distribution as positively skewed.

Answers (a) (i) all data values are maintained (b) (i) 24 (b) (ii) 19
 (b) (iii) 27.08 (d) effected by extreme values or outliers (e) 25.84
 (f) (i) 20, 32 (f) (ii) 6 (g) positively skewed.

Module 2

Question 3

This question tested candidates' ability to

- (a) calculate:
- (i) the probability of an event A, where $P(A)$ represents the number of possible outcomes of A divided by the total number of outcomes
 - (ii) the probability of two events occurring as $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
 - (iii) the probability of an intersection of two events A and B i.e. $P(A \cap B)$
 - (iv) the conditional probability of two events A and B i.e. $P(A|B) = \frac{P(A \cap B)}{P(B)}$

- (b) use the property that $P(A') = 1 - P(A)$ where $P(A')$ is the probability that event A does not occur.
- (c) calculate, using an appropriate method, the different combinations of patties chosen by patrons at a restaurant.

Part (a) (i) was attempted by approximately 98 per cent of the candidates, with about 65 per cent giving a satisfactory response. Candidates inappropriately applied independent events in their solution.

Part (a) (ii) was attempted by most candidates, with approximately 75 per cent giving a satisfactory response. Some candidates interpreted $P(A|B)$ as $\frac{P(A)}{P(B)}$

For Part (a) (iii), the majority of candidates were unable to identify the region for $P(A' \cap B')$.

Part (b) (i) was generally very well done. However, a few candidates, in stating the probability as a fraction, used an incorrect denominator.

In Part (b) (ii), some of the candidates represented the data in a Venn diagram but incorrectly calculated the intersection. Other candidates incorrectly calculated $P(A \cap B)$ as $P(A) \times P(B)$ rather than using $P(A \cap B) = P(A) + P(B) - P(A \cup B)$.

For Part (b) (iii), approximately 50 per cent of the candidates did not recognize that this question was an event without replacement. Part (c) (i) was very well done. The 30 per cent who did (c) (ii) incorrectly did not cube the respective probabilities.

In Part (c) (iii), many candidates were unable to identify all six combinations of the three types of patties. For Part (c) (iv), some candidates did not recognize the use of the conditional probability. Some of them interpreted $P(\text{chicken and the same type})$ to be $P(\text{chicken}) \times P(\text{same type})$.

Answers (a) (i) 0.5 (ii) 0.694 (iii) 0.14 (b) (i) 8/15 (ii) 3/15 (iii) 3/7
 (c) (i) 0.091 (ii) 0.142 (iii) 0.189 (iv) 0.642

Question 4

This question tested candidates' ability to

- (a) (i) calculate the expected value of independent events
 (ii) use the Binomial to calculate probability of independent events
 (iii) interpret and calculate probability of independent events
- (b) (i) use the Binomial distribution to find n
 (ii) use the Binomial distribution to find expected value
 (iii) use the Binomial distribution to calculate probability
- (c) (i) decide when to use the Normal distribution as an approximation of a Binomial distribution
 (ii) calculate probabilities using the Normal distribution..

Many candidates scored fairly well on this question, with the most common score being 18 marks out of 25. However, many candidates did not follow instructions as it pertained to stating the answer to three significant figures. Several candidates rounded-off prematurely, and this affected the accuracy of their final answer.

Part (a) (i) was generally well done, with most candidates getting the correct solution of 16.5 days. Surprisingly many candidates did not know the number of days in the month of September (values ranging from 28 to 32 were used). Candidates who obtained no marks on this part of the question demonstrated a clear lack of understanding of discrete probability.

In Part (a) (ii), most candidates correctly identified the use of the Binomial. However, many candidates incorrectly substituted the values for p and q into the formula and therefore were unable to obtain the answer of 0.239.

In Part (a) (iii), many candidates could not interpret the probability of independent events and were unable to obtain the answer of 0.166. Part (b) (i) was very well done by those candidates who attempted it. Part (b) (ii) was generally also well done as most candidates were able to obtain the answer of 10.5.

Part (b) (iii) was generally well done by most candidates who knew that the use of the Binomial was required. However, not many of them obtained the answer of 0.206 as the values for p and q were incorrectly substituted.

In Part (c) (i), most candidates received partial credit as they did not state both conditions. A large sample size was interpreted by most candidates as $n > 30$; however, many of them did not realize that the value of p needed to be close to 0.5. Many candidates answered with $np > 5$ and $nq > 5$ as two separate conditions when these should have been stated together as one condition. Some candidates also gave $npq > 5$ which had to be recognized since it is stated as such in the syllabus.

For Part (c) (ii), many candidates used the Binomial distribution although an approximation with the Normal distribution was required. Many candidates who used the Binomial failed to include $P(x = 0)$ in the solution, and of those who used the Normal distribution, the continuity correction was incorrectly used.

Answers (a) (i) 16.5 (ii) 0.239 (iii) 0.166 (b) (ii) 10.5 (iii) 0.206 (c) (ii) 0.416

Module 3

Question 5

This question tested candidates' ability to

- (a) identify the distribution of the sample mean
- (b) calculate unbiased estimates of the population mean and standard deviation
- (c) interpret a confidence interval
- (d) construct and use a confidence interval.

In Part (a), many candidates were able to identify the distribution of the sample mean as the Normal distribution but had difficulty stating its parameters.

For Part (b) (i), most candidates failed to identify the sample mean as the middle of the given confidence interval. Many candidates set up the equations for both ends of the interval and attempted to solve them as simultaneous equations but did not complete them due to poor algebraic skills. As a result, many candidates did not obtain the answer of 0.575.

In Part (b) (ii), candidates were able to obtain the answer of 80. Most substituted whatever value was obtained in the previous part into one of the equations for the confidence interval and proceeded to solve for n . This part also indicated that some candidates have weak algebra skills. Most candidates recognized the need to round-off the answer for the discrete variable.

Part (c) was fairly well answered and most candidates obtained the answer of 2. Those getting this part incorrect attempted to construct the confidence interval rather than interpret the information that was given.

Part (d) (i) (a) was well done by the majority of the candidates who obtained the correct answer of 0.99 but failed to write the solution in ‘thousands of dollars’. The most common error was simply dividing the values given for ‘sum of all x ’ by ‘sum of all values of x ’ rather than using the given sample size, $\frac{\sum x^2}{\sum x}$ rather than $\frac{\sum x}{n}$.

Most candidates attempting Part d (i) (b) applied the formula incorrectly, however, for Part d (ii), most candidates knew the formula but were unable to obtain the correct z-value of 1.881.

Answers (b) (i) 0.575 (ii) 80

Question 6

This question tested candidates’ ability to

- (a) evaluate the t-test statistic
- (b) determine the appropriate number of degrees of freedom from a given data set
- (c) determine the t-values from the t-distribution table
- (d) apply a hypothesis test for a population mean using a small sample ($n < 30$) drawn from a Normal population with an unknown variance
- (e) determine the critical value from the chi squared (χ^2) tables and hence determine the critical region for the test
- (f) apply a χ^2 test for independence in a 2 x 3 contingency table.

In Part (a) (i), many candidates interchanged the null and alternative hypotheses. Some candidates incorrectly stated the null hypothesis as $H_0: \mu \geq 150$. For Part (a) (ii), most candidates lost a mark because they did not calculate the expected number of hardback cover textbooks to the nearest whole number. In Part (a) (iii), most candidates were unable to identify the correct rejection region, $\chi^2 > 5.991$. Other candidates gave the critical value rather than the rejection region. Most candidates who attempted Part (a) (iv) were able to state whether the null hypothesis was rejected or accepted, but could not interpret this with a valid conclusion.

For Part (b) (i), most candidates stated the conditions for the use of a t-test rather than the assumption. Part (b) (ii) was fairly well done; however, some candidates used the sample mean \bar{x} instead of the population mean μ and others did not state a mean. In Part (b) (iii), many candidates were unable to identify the correct rejection region. Some candidates did not state the answer as a region but as a value.

Most candidates who did Part (b) (iv) incorrectly used the test value as $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ instead of

$$t = \frac{\bar{x} - \mu}{\sigma/\sqrt{n-1}}.$$

Many of the candidates who attempted Part (b) (v) were able to identify the acceptance region of the null hypothesis, but did not state a clear conclusion as was required.

Answers (a) (ii) 7, 56 (iii) $\chi^2 > 5.991$ (iv) reject H_0 ; there is no association between the type of book and its cover.

(b) (i) normal distribution (ii) $H_0: \mu = 150, H_1: \mu > 150$ (iii) $t > 1.761$ (iv) accept $H_0: \mu = 150$, there is no evidence of an increase in the mean mass of the packages.

UNIT 2

Paper 01 – Multiple Choice

The performance on the forty-five multiple choice items on Paper 01 produced a mean of 62 out of 90 with scores ranging between 0 and 88.

Paper 02 – Essays

Question 1

This question tested candidates' ability to

- (a) derive and graph linear inequalities in two variables
- (b) determine the solution set that satisfies a set of linear inequalities in two variables
- (c) determine the feasible region of a linear programming problem
- (d) determine the optimal solution of a linear programming problem

All the candidates attempted this question but only about 60 per cent performed satisfactorily with scores between 18 and 25 marks.

Part (a) was poorly done. All the candidates who attempted this section knew that they should substitute values into the function $P = x + 2y$. However, most candidates did not use the significant points in the region to find the optimal solution. A few of the candidates misread the scales on the graph.

Part (b) was fairly well done. Most candidates knew how to find the row minimum. However, some candidates maximized instead of minimize. A few candidates utilized other methods such as solving the equations simultaneously rather than the required Hungarian method to find the solution. Candidates also used a version of the Hungarian method that required that the minimum uncovered element be doubled and added to the elements that were covered twice.

In Part (c), most candidates were able to list the three paths, only a few did not start at A and did not finish at A.

Answers

- (a) *Using $P = x + 2y$ and the points (5, 7), (17, 7) and (20, 4.4)
The maximum value of P is 31 and it occurs when $x = 17$ and $y = 7$.*

- (b)(i) *A is assigned to task 1
B is assigned to task 4
C is assigned to task 5
D is assigned to task 2
E is assigned to task 3*

- (b)(ii) *The total minimum time taken by the five persons = 50 + 50 + 50 + 50 + 49 = 249 minutes.
Any part that starts at A and ends at A. For example: ABCDA, ABEA, ABEDA, AECDA, ABCEA, etc.*

Question 2

This question tested candidates' ability to

- (a) represent a Boolean expression by a switching or logic circuit
- (b) use switching and logic circuits to model real-world situations
- (c) use the laws of Boolean algebra to simplify Boolean expressions
- (d) use truth tables to determine if propositions are equivalent
- (e) derive a Boolean expression from a given switching or logic circuit.

This question was attempted by all the candidates. However, only 70 per cent performed satisfactory.

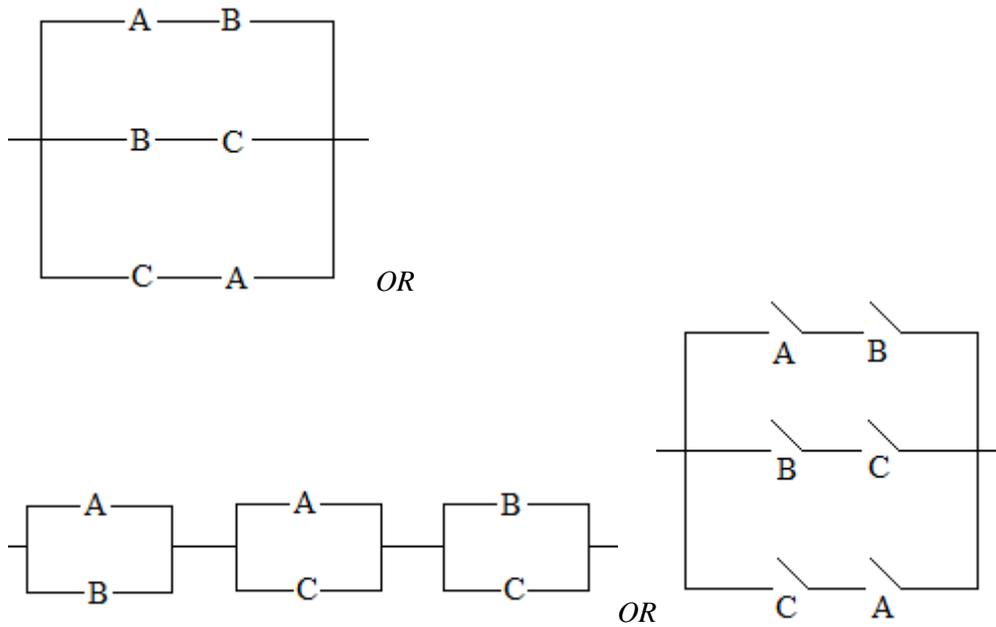
Part (a) was poorly done. Some candidates confused the switching and logic circuits. A few candidates used truth tables to assist them in drawing the switching circuit, but there were some who only did the truth tables, while some of them also used a combination of switching and logic circuits.

Part (b) was well done by most candidates. Only a small number of candidates did not complete the truth table. In Part (c), most candidates knew that they should use gates, but some of them did not combine the gates correctly.

About 90 per cent of the candidates attempted Part (d). Candidates knew how to calculate the float and the earliest start time but some of them did not know how to find the latest start times. Most candidates knew that they needed to use the float to determine the critical path. However, there were a few candidates whose float did not have anything to do with their critical path.

Answers

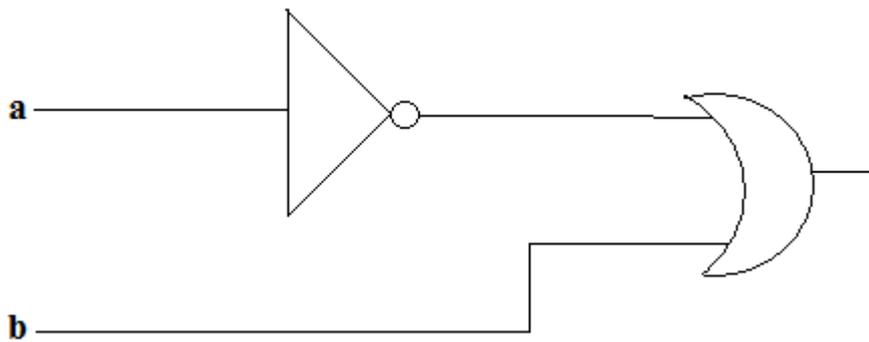
- (a) *Tom works hard if and only if he is successful.*



(b)

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$p \wedge \sim q$	$(\sim p \vee \sim q) \Rightarrow (p \wedge \sim q)$
T	T	F	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	T	F	F
F	F	T	T	T	F	F

(c)



(d) (i)

Activity	Earliest Start Time	Latest Start Time	Float
A	0	0	0
B	6	6	0
C	16	16	0
D	6	17	11
E	18	19	1
F	18	18	0
G	22	22	0

(ii) The critical path is: Start, A, B, C, F, G, Finish

Question 3

This question tested the candidates' ability to

- (a) use the cumulative distributive function $F(x) = P(X \leq x)$ to calculate
 - (i) the constant, k
 - (ii) the probability between two values
- (b) calculate the expected value, variance, median and other quartiles
- (c) calculate the probability density function, $f(x)$, from the cumulative distributive function, $F(x)$

The question was fairly well done.

In Part (a), most candidates used integration, instead of just substituting the value for $F(6) = 1$ to arrive at the correct solution.

For Part (a) (ii), most candidates related Part (ii) to Part (i) by using integration. They recognized that they were dealing with the boundary value and hence had to subtract.

Candidates who attempted Part (a) (iii) knew that the median value was 0.5, but did not use the correct formula to find its value. Similarly for Part (a) (iv), many candidates knew that the lower quartile value was 0.25, but did not use the correct formula to find its value.

In Part (b), most candidates recognized that they had to differentiate in order to get a probability density function. However, many of them differentiated the wrong function, and hence got the wrong graph.

For Part (b) (ii), most candidates knew the formulas for $E(X)$ and the $\text{Var}(X)$, but used incorrect functions. A few candidates did not subtract the $[E(X)]^2$ when finding $\text{Var}(X)$.

In Part (b) (iii), some candidates treated $\text{Var}(X)$ and $E(X)$ as being discrete and used the formula

$$E(X) = \sum xP(X = x) \text{ rather than } E(X) = \int_{x_1}^{x_2} xf(x)dx.$$

Answers

$$(a) (i) 1/3 \quad (ii) 1/2 \quad (iii) 4.5 \quad (iv) 3.75 \quad (b) (ii) 9/2 \quad (iii) 3/4$$

Question 4

This question tested candidates' ability to

- (i) solve problems involving probabilities of the Normal distribution using z-scores
- (ii) carry out a Chi-square (χ^2) goodness-of-fit test with appropriate number of degrees of freedom. (The situation in this question was modelled by the uniform distribution.)

The question was fairly well done by most candidates.

In Part (a) (i), most candidates were able to get the standardization of the normal distribution, but many could not follow a logical order of steps to get the correct answer. Also, some candidates used a premature approximation of the z value, but they were unable to follow through the solution.

Part (a) (ii) was well done by most candidates and they were able to get at least four out of the five marks.

In Part (b), a few candidates incorrectly used the table as a contingency table. Most candidates were able to get Part (b) (i) correct, with few candidates missing the key terms *independent* and *dependent* in both hypotheses.

The majority of the candidates got Part (b) (ii) correct, but a few candidates incorrectly interpreted the table.

In Part (b) (iii), the majority of candidates read the table incorrectly. Many of them calculated the degrees of freedom correctly, but some used the wrong formula for calculating the value of χ^2 . Most candidates had problems writing the region and wrote the critical value instead, that is, instead of writing $\chi^2 > 9.447$, they wrote $\chi^2 = 9.447$. Many candidates did not clearly state their conclusion. They were able to reject the null hypothesis, but could not give a concluding statement such as *time taken is not dependent on distance traveled*.

Answers (a) (i) 0.241 (ii) 0.204 (b) H_0 : the number of employees who work overtime is independent of the distance of their home from the workplace. H_1 : the number of employees who work overtime is dependent on the distance of their home from the workplace. (ii) 20 (iii) $\chi^2 = 11.30$

Question 5

This question tested candidates' ability to use equations of motion for a body moving in a straight line with constant acceleration and to find the velocity of a particle at a given distance from a starting point, given distances and times.

In Part (a), candidates recognized the need to apply an equation of motion for a body moving under the given conditions. However, many candidates encountered difficulty selecting the correct equation to enable them to solve the problem. Many candidates did not appreciate the significance of *constant acceleration* and stated that *distance = speed multiplied by time*.

In Part (b), the problems experienced by candidates were similar to those in Part (a).

Answers

(a) (i) acceleration: 2 ms^{-2} ; (ii) time = 9 secs.
(b) (i) acceleration: 0.25 ms^{-2} ; (ii) velocity = 5.3 ms^{-1} .

Question 6

This question tested the candidates' ability to

(a) calculate

(i) the tension in a string

(ii) the angle of inclination of a string to the vertical, given a particle hanging from a fixed point by an elastic string and drawn sideways by a horizontal pull on the particle, with the system in equilibrium.

6. candidates generally failed to link their findings to the purpose of their projects. In many instances, new information was found in the discussion of findings or the conclusion section which did not relate to the variables under consideration in the project.

UNIT 2

The following relate to internal assessment projects for Unit 2.

1. Generally, candidates demonstrated a high degree of mastery in the mathematical principles pertaining to the syllabus. In most cases, the mathematical analyses were relevant and carried out with few flaws.
2. There was evidence of originality and creativity.
3. Projects were appropriately applied to real-world problems and situations.
4. Over 90 per cent of the candidates were able to effectively communicate information in a logical way using correct grammar and mathematical language.

Some areas of concern were:

1. About 20 per cent of the candidates ignored the stipulated format for the presentation of the project.
2. The statement of the task was not explicit enough in about 30 per cent of the projects.
3. Some of the tables used were not clear; other tables were presented without headings and without reasons for their use.
4. Some candidates presented more data than was need for their analysis.

Recommendations

Candidates have to spend more time solving problems from the Mechanics Module as well as developing the ability to explain concepts.