C A R I B B E A N   E X A M I N A T I O N S   C O U N C I L

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

MAY/JUNE 2013

PURE MATHEMATICS

GENERAL COMMENTS

In 2013, approximately 4,800 and 2,750 candidates wrote the Unit 1 and 2 examinations respectively. Overall, the performance of candidates in both units was consistent with performance in 2012. In Unit 1, 72 per cent of the candidates achieved acceptable grades compared with 70 per cent in 2012; while in Unit 2, 81 per cent of the candidates achieved acceptable grades compared with 83 per cent in 2012. Candidates continue to experience challenges with algebraic manipulation, reasoning skills and analytic approaches to problem solving.

DETAILED COMMENTS

UNIT 1

Paper 01 – Multiple Choice

The paper comprised 45 items, 15 items per module. Most candidates performed satisfactorily. Marks on this paper ranged from a minimum of 6 to a maximum of 45. The mean mark for the paper was 64.58 per cent.

Paper 02 – Structured Questions

The paper consisted of six compulsory questions, two questions per module. The maximum score was 149 out of 150. The mean score was 52.26.

Section A

Module 1: Basic Algebra and Functions

<u>Question 1</u>

Specific Objectives: (a) 2, 4; (b) 1, 3, 5; (c) 2, 3, 4; (d) 3, 8

The topics tested in this question included the use of truth tables, binary operations, proof by mathematical induction and the factor theorem. Overall, candidates demonstrated competence in this question with approximately 90 per cent of them attempting it and obtaining at least 16 marks. A number of candidates were also able to obtain the maximum score.

In answering Part (a), candidates used a variety of styles to represent the inputs and outputs such as ‘1 and 0’, ‘True and False’, ‘x and √’. The majority of candidates attempted Part (a) (i) and was successful. In Part (a) (ii), some candidates misinterpreted $\sim(p \wedge q)$ and used it as if it were $\sim p \wedge \sim q$.

Part (b) was misinterpreted by many candidates. They substituted $x = 2, -2$ instead of $y = 2$ in the given function. Other candidates treated $y \oplus x$ as $y + x$, solved for y by replacing x and hence substituted y as -2. They also had difficulty factorizing and solving the quadratic equation.

For Part (c), a majority of candidates were able to achieve the first four marks allocated. However, most candidates did not apply the induction steps correctly. Some were more familiar with questions involving the sigma notation and incorporated the sigma notation in their solution. A few candidates did not use the smallest natural number, 1, to begin the proof by induction and instead used 0 and 2. Other candidates were able to recognize the k + 1th term but they were unable to simplify the term 5^{k+1} because they did not realize that it could be expressed as $5^k \times 5^1$. The conclusion also posed a challenge to many candidates. Candidates should be reminded that the conclusion should relate to the hypothesis. The general format for the conclusion could be as follows: Since $P(1)$ is true and $P(k) \rightarrow P(k+1)$, the proposition $P(k)$ is true for all positive integers 'k'.

Most candidates obtained full marks in Part (d). Some candidates substituted the value of 'p' into the function to prove that $(x+1)$ is a factor as opposed to using the factor to find 'p'. In general, long division was used to show that the remainder is zero under division by $(x+1)$. This method could also have been used for Part (d) (ii) to obtain the quadratic equation, followed by factorizing the quadratic equation to obtain the other two factors and then equating each factor to zero to solve the cubic function. In some cases, candidates were able to factorize the cubic function correctly but were unable to identify the roots.

Solutions

(a) (i) (ii)

p	q	$p \rightarrow q$	$p \wedge q$	$\sim(p \wedge q)$
T	T	T	T	F
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

(b) $x = 8$ or $x = 1$

(d) (ii) $f(x) = (x - 8)(x - 2)(x + 1)$ (iii) $x = 8,\ x = 2,\ x = -2$

Question 2

Specific Objectives: (e) 2, 3, 4; (f) 2, 3; (g)

This question tested the concept of a one-to-one function on the domain of real numbers; the inverse of a linear and an exponential function; the inverse of the composite of a linear and exponential function; quadratic inequalities and the modulus function.

Part (a) failed to attract answers from the majority of candidates. Among the methods used for the proof of a one-to-one function was (i) the graphical approach, (ii) a deductive approach, (iii) differentiation and (iv) proof by induction. Candidates who attempted to use the graphical test used a vertical line test instead of the horizontal line test, not indicating the domain clearly on the graph used and in some cases graphed the quadratic function without indicating the specific domain over which the given function is one-to-one. Candidates who attempted to show that $\mathrm{f}(a) = \mathrm{f}(b) \Rightarrow a = b$ could not show the correct algebraic simplification and subsequent deductive proof. Those candidates who attempted differentiation to show the required result simply could not proceed beyond merely differentiating $x^2 - x$. Proof by induction was beyond the ability of those candidates who attempted to use this approach.

In Part (b), the results of $\mathrm{f}^{-1}(x)$ and $\mathrm{fg}(x)$ were easily shown. However, a significant number of candidates failed to find the expression for $g^{-1}(x)$. Common errors included the inability to take $\log_e$ for the change of variable and in cases where $\log_e$ was taken candidates incorrectly cancelled the logs on each side of the equation. In some cases candidates used $\log_e (x - 2)$ as $\log_e x - \log_e 2$. Evidence was seen where some candidates mistakenly interpreted $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$ as $\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{f}(x)$ and $\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{g}(x)$.

In Part (c) (i), candidates used the quadratic graph to find the correct range of values of x. Candidates who used the results $(x+2)(3x-2) \le 0$ incorrectly reasoned that

$$x + 2 \le 0 \Rightarrow x \le -2 \text{ and } 3x - 2 \le 0 \Rightarrow x \le \frac{2}{3}.$$

Some candidates used methods including squaring both sides of the equation and thus finding values of x for which the resulting quadratic equation was equal to zero. However, they failed to test the values of x found and were not able to obtain the mark given for showing or stating that $x = -\frac{7}{4}$ was inadmissible. Candidates who used the concept of $x + 2 = 3x + 5$ and $-(x + 2) = 3x + 5$ could not reason the correct value of x for which the equation was true.

Solutions

(b) (i) a) $f^{-1}(x) = \frac{x-2}{3}$ $g^{-1}(x) = \frac{1}{2}\ln x$ b) $fg(x) = 3e^{2x} + 2$

(c) (i) $-2 \le x \le \frac{2}{3}$ (ii) $x = -\frac{3}{2}$ only

Section B

Module 2: Trigonometry, Geometry and Vectors

Question 3

Specific Objectives: (a) 2, 3, 4, 5, 6

This question tested trigonometric identities of multiple angles; solving trigonometric equations involving multiple angles; expressing a cos x + b sin x in the form r cos (x + α); determining maximum and minimum values of trigonometric expressions; and proof of simple trigonometric equations.

In Part (a) (i), most candidates performed satisfactorily. Challenges encountered included the inability to use the identity given to obtain a quadratic equation in terms of tan θ. Candidates who obtained the correct equation $\tan\theta\left(1-\tan^2\theta\right)=0$ wrongly divided the equation by $\tan\theta$ thus losing the roots of the equation $\tan\theta = 0$. Candidates also deduced $\tan^2\theta = 1 \Rightarrow \tan\theta = 1$. This error resulted in candidates being unable to get the solutions for $\tan\theta = -1$.

Part (b) (i) was generally well done. Some errors included solving $\alpha = \tan^{-1}\left(\frac{3}{4}\right)$ and $\alpha = \tan^{-1}\left(-\frac{4}{3}\right)$. Part (b) (ii) a) was satisfactorily done. Errors made by candidates included stating that the maximum value of $5\cos\left(\theta+\alpha\right)$ is 1.

However, Part (b) (ii) b) was not satisfactorily done. Most candidates seemed unable to deduce the maximum value of a reciprocal function, particularly when another term is added to the denominator.

Responses to Parts (b) (iii) a) and (b) were very poor. Candidates did not demonstrate knowledge of the fact that the sum of the interior angles, A, B and C of the triangle given, was π radians. Evidence of candidates expanding sin (B + C) and being unable to link that expansion with sin A was observed. Some candidates attempted to substitute numerical values for angles A, B and C with no success.

Solutions

(a) (ii) $\theta = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}, 2\pi$

(b) (i) $5\cos\left(\theta + 0.927^c\right)$ (ii) a) $\mathrm{f}(\alpha)_{\max} = 5$ (ii) b) $\left(\frac{1}{8 + \mathrm{f}(\theta)}\right)_{\min} = \frac{1}{13}$

Question 4

Specific Objectives: (b) 1, 2, 5, 6; (c) 4, 5, 6, 8, 9

This question tested geometry of the circle; parametric equations to a Cartesian equation; three dimensional vectors; and equations of lines in the Cartesian of a plane.

Part (a) (i) was generally well done. Some candidates expressed the equation of the circle using completion of the square to deduce the coordinates of the centre and the radius. A number of candidates used the equation $x^2 + y^2 + 2\mathrm{f}x + 2\mathrm{g}y + c = 0$ to deduce that the coordinates of the centre were (−f, −g) and the radius $\sqrt{\left(\mathrm{f}^2 + \mathrm{g}^2 - c\right)}$. Some errors were made when expressing the coordinates of the centre as (f, g).

In Part (b) (ii) a), many candidates deduced that the normal to the circle lies along the diameter while other candidates worked though the equation $y - y_P = \frac{y_P - y_C}{x_P - x_C}\left(x - x_C\right)$.

In Part (b) (ii) (b), some candidates used the gradient of the normal to find the equation of the tangent. Those candidates who used the gradient of the tangent, $-\left(\frac{x_\mathrm{P} - x_\mathrm{C}}{y_\mathrm{P} - y_\mathrm{C}}\right)$ could not interpret the meaning of the resulting gradient $-\frac{3}{0}$. It was not unusual to see candidates stating the gradient as 0. The fact of the tangent being parallel to the y-axis was not understood by a significant number of candidates. Further, a number of candidates drew a graph of the circle and indicated the tangent parallel to the y-axis but were unable to state the equation as $x = 6$.

In Part (b), various methods were used by candidates. Apart from expressing $t = \frac{y + 4}{2}$ and substituting for $x = \left(\frac{y+4}{2}\right)^2 + \frac{y+4}{2}$, some candidates substituted the parametric equations given into the equation required to be shown. Common errors resulted from poor algebraic simplification. For example $\left(\frac{y+4}{2}\right)^2 = \frac{y^2 + 16}{4}$.

Generally Part (c) (i) was done satisfactorily. Some candidates made the basic error that the vector $\overline{\mathbf{AB}} = \overline{\mathbf{OA}} + \overline{\mathbf{OB}}$ and $\overline{\mathbf{BC}} = \overline{\mathbf{OB}} + \overline{\mathbf{B}C}$. Part (c) (ii) appeared challenging to many candidates. It was generally understood that the dot product was required. However, most candidates used the $\overline{\mathbf{OA}}$ and $\overline{\mathbf{OB}}$ with the vector $\mathbf{r} = -16\mathbf{j} - 8\mathbf{k}$ to attempt to show the perpendicular property without success. A small number of candidates understood that they were required to show perpendicularity between the vectors $\overline{\mathbf{AB}}$ and $\mathbf{r} = -16\mathbf{j} - 8\mathbf{k}$ and between the vectors $\overline{\mathbf{BC}}$ and $\mathbf{r} = -16\mathbf{j} - 8\mathbf{k}$. It was interesting to observe some candidates using the vector cross product to show the perpendicular vector $\mathbf{r} = -16\mathbf{j} - 8\mathbf{k}$.

In Part (c) (iii), candidates quoted the vector equation of the plane, but were not able to use the correct resulting point and the normal vector to the plane to complete the equation $\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$. Those candidates who used the correct values were unable to express the Cartesian equation of the plane as required.

Solutions

(a) (ii) a) normal$_{(6,\,2)}$: $y = 2$ b) tangent$_{(6,\,2)}$: $x = 6$

(c) (i) $\overline{\mathbf{AB}} = -2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$ $\overline{\mathbf{BC}} = -2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ (iii) $2y + z = 0$

Section C

Module 3: Calculus I

<u>Question 5</u>

Specific Objectives: (a) 3, 7, 9, 10; (b) 3–8, 12

This question tested limits; continuity; differentiation using the quotient rule; parametric differentiation; and finding the area enclosed by two curves using integration.

In Part (a) (i), the majority of candidates gave satisfactory performances although only a small number of candidates presented their statements in acceptable mathematical language. However, Part (a) (ii) was poorly done since a number of candidates made guesses on the question of continuity and were unable to justify their responses mathematically.

In Part (b), most candidates showed a good understanding of the quotient rule for differentiation. However, there were cases in which candidates used the product rule by expressing the denominator as a multiplying term. Some candidates committed errors in their differentiation of the term $(x^2 + 2)^3$. They mainly differentiated the cubic term and neglected the differential of x^2. Weaknesses in algebraic manipulation prohibited many candidates from obtaining the required simplified result.

Almost half of the candidates who responded to Part (c) demonstrated a lack of knowledge of differentiation of parametric terms. Some candidates opted to convert the equation to Cartesian form and to proceed with the differentiation. However, the term in y^2 made it difficult for them to successfully complete the correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. Only a small number of candidates were able to obtain the correct result.

Generally, the responses to Part (d) were satisfactory. Most candidates were familiar with the concept tested in this part of the question. Some errors included candidates subtracting the area enclosed by the curve $y = 4x$ from the area enclosed by the curve $y = x^2 + 3$ incorrectly. Very few candidates used the integral of $4x - (x^3 + 3)$ but preferred to find the area using the difference of two areas.

Solutions

(a) (i) lim = 4 (ii) f (x) is not continuous since f (x) is not defined at $x = 2$

(c) $\frac{2}{3}\cot\theta$ (d) (i) P (1, 4) Q (3, 12) (ii) $\frac{4}{3}$ units^2

<u>Question 6</u>

Specific Objectives: (c) 3, 4, 6, 8, 9 (b)

This question tested indefinite integration using substitution; the theorem of the integral of sums being equal to the sum of integrals, determining maxima using differentiation; and determining the constants of integration given initial conditions.

Part (a) (i) tested integration using substitution. The substitution $x = 1 - u$ was given and the majority of candidates demonstrated a good understanding of having to express dx in terms of du. However, in proceeding to complete the substitution of $x\,(1 - x)^2$ in terms of u, the majority of candidates stated the expression as $x\,u^2$. Hence, they could not continue integration in this form since the variable x was not expressed as $1 - u$. Many candidates who made the correct substitution and successfully integrated in terms of u failed to express their answer in terms of x.

Part (a) (ii) was generally well done. Some errors seen included:

$\int 4\sin 5t\,\mathrm{d}t = -4\cos 5t$, $4\cos 5t$ and $20\cos 5t$.

Part (b) was generally well done. However, in Part (b) (i), a significant number of candidates were unable to find the correct formula for the area of a simple plane figure. Candidates who used the correct formula for the area were not able to simplify the expression thus allowing for easy differentiation. As a result, a majority of the candidates failed to obtain the correct differential to proceed to find the value of x. However, candidates demonstrated the knowledge that it was necessary to solve $\frac{dA}{dx} = 0$.

Part (c) (i) required candidates to find the first and second differentials of y, explicitly given in terms of x. The terms to be differentiated involved a product of x and a trigonometric term in x. The majority of candidates failed to apply the product rule in differentiating $-x \sin x$. The common results shown were $-x \cos x$, $x \cos x$ and $\sin x$. This error was compounded by adopting the same approach for the second differential. Consequently, candidates were unable to show the required answer. Overall, candidates performed poorly on this part of the question.

Part (c) (ii) required that candidates find the values of two constants of integration for an explicit function of y in terms of x given the boundary conditions. This required substitution of the values of y for given values of x. A significant number of candidates failed to recognize this simple procedure and appeared to think integration was required. Those candidates who recognized the methods required for solving this part of the question made algebraic and arithmetic errors in their substitutions.

Solutions

(a) (i) $-\frac{1}{12}(1-x)^3(1+3x)+C$ (ii) $5 \sin t - \frac{4}{5} \cos 5t + C$

(b) (ii) 84 metres approx.

(c) (ii) $y = -x \sin x - 2 \cos x + \frac{1}{\pi}x + 3$

Paper 032 – Alternative to School-Based Assessment

Section A

Module 1: Basic Algebra and Functions

Question 1

Specific Objectives: (a) 1, 3; (b) 2, 4; (c) 1, 2; (d) 1, 2, 4, 6; (f) 3

This question tested the converse, inverse and contrapositive of a conditional statement; solving logarithmic and exponential equations; and graphing a modulus and a linear function of x.

For Part (a), candidates generally demonstrated a lack of knowledge in these topics. A small number of candidates attempted to use truth tables to show the required results but performed poorly.

In Part (b), candidates demonstrated understanding of expressing a sum of two logs as a single log. However, they failed to express 3 as $\log_2 2^3$ which would have allowed them to proceed with the solution of the correct quadratic equation $(x+3)(x+2)=8$.

Very few candidates attempted Part (c). Those who did substitute A = 0 could not proceed to express the resulting equation $3e^{4t} - 7e^{2t} - 6 = 0$ in a convenient quadratic form to solve for t.

In Part (d), the majority of candidates graphed the line f (x) = 2x + 3 correctly but did not graph g (x) = $|2x+3|$ correctly.

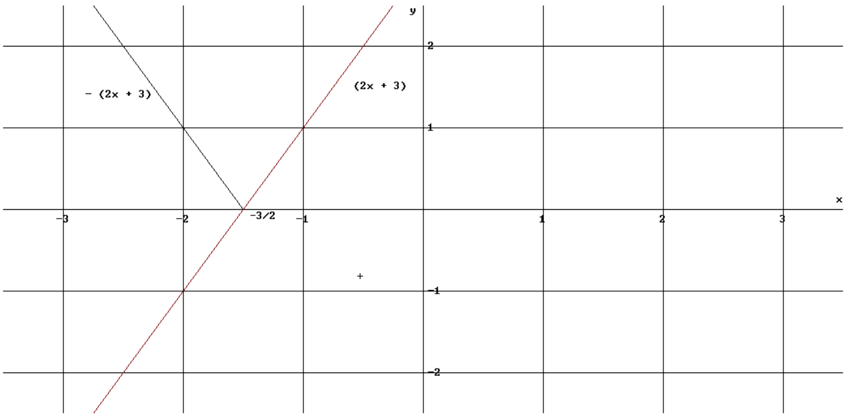
Solutions

(a) (i) $(q \vee \sim p) \rightarrow (p \wedge q)$

(b) x = 0.37 (2 d.p.)

(c) $t = \frac{1}{2} \ln 3$

(d)



Section B

Module 2: Trigonometry, Geometry and Vectors

Question 2

Specific Objectives: (a) 1, 3, 4, 7, 8; (b) 3, 4, 6; (c) 1, 7, 10

This question tested the exact value of cos 3A given exact values of sin A and cos B; the solution of a trigonometric equation involving a mix of cos 2θ and sin θ in a given range; and coordinate geometry involving the intersection of two circles.

In general, the responses to Part (a) were unsatisfactory. Candidates attempted to convert $\sin A = \frac{4}{5}$ and $\cos B = -\frac{3}{5}$ into degrees and substitute for $\cos 3A = 3 \cos 3$ (their value of A). It was clear that candidates did not have the required knowledge of this topic.

The responses to Parts (b) and (c) were also unsatisfactory. Candidates could not correctly express $\cos 2\theta$ in terms of $\sin \theta$ and the arithmetic errors made by candidates resulted in incorrect values for x and y in Part (c). However, most candidates demonstrated an understanding of using the equation two curves to find point(s) of intersection.

Solutions

(a) $-\frac{117}{125}$ (b) 6.031 radians (c) $(1.46, -1)$ and $(-5.46, -1)$

Module 3: Calculus I

Question 3

Specific Objectives: (a) 2, 4, 5; (b) 1, 3, 9–12; (c) 2, 5, 7, 8 (b)

This question tested limits, differentiation from first principles; finding minimum and maximum stationary points; and integration to find the volume of revolution about the x-axis. Overall, the majority of candidates gave no response to this question. A few unsuccessful attempts were made for Part (c) with some candidates managing to find $\frac{\mathrm{d}}{\mathrm{d}x} \mathrm{f}(x)$.

Solutions

(a) (i) a) 1 (b) 2 (ii) not continuous since it is not defined at $x = 2$

(b) $-\frac{1}{2\sqrt{2}\sqrt{(x)^3}}$

(c) $\frac{1}{3}$ and $-\frac{3}{2}$

(d) $\frac{100}{3}$ units^3

UNIT 2

Paper 01 – Multiple Choice

The paper comprised 45 items, 15 items per module. Most candidates performed satisfactorily. Marks on this paper ranged from a minimum of 6 to a maximum of 44. The mean mark for the paper was 69.18 per cent.

Paper 02 – Structured Questions

The paper consisted of six compulsory questions, two questions per module. The maximum score was 150 out of 150. The mean score was 47.52.

Section A

Module 1: Complex Numbers and Calculus II

Question 1

Objectives: (a) 7, 8, 12, 13; (b) 1–5, 8

In Part (a), the majority of candidates completed the differentiation satisfactorily. Some candidates who completed the differentiation were unable to differentiate the natural log of the term $\ln(x^2 y)$ correctly. Some common results for differentiating this term include $\frac{1}{x^2 y}$ and $\frac{2x\frac{dy}{dx}}{x^2 y}$. A few candidates applied the natural log laws to separate the terms before differentiating and many were successful with the differentiation using this approach.

In general, the candidates showed a lack of understanding of partial derivatives in Part (b). Many inserted additional terms. Common responses included:

$\frac{\partial f}{\partial y} = 3z^2 - e^{4x}\, cos4z - 6y - y$ and $\frac{\partial f}{\partial z} = 6yz + 4sin4ze^{4x} - 4cos4ze^{4x}$.

Part (c) was generally well done, with candidates gaining the majority of marks. Almost all candidates were able to identify that only the real part of the complex number was needed for the solution.

In Part (d), many candidates recognized that $\tan^{-1}$ was needed to find the argument. However, most obtained:

$$\arg(z) = \tan^{-1}(-1)$$
$$= -\frac{\pi}{4}$$

and completely ignored the use of the Argand Diagram. In addition, a majority of candidates obtained $|z| = \sqrt{2}$ but left out the exponent, 7, when doing their final calculation and hence they did not write the modulus as $\left(\sqrt{2}\right)^7$.

Solutions

(a) Undefined but marks awarded for finding the derivative and explaining why the gradient could not be found.

(b) $\frac{\partial z}{\partial y} = \frac{3(z^2 - 2y)}{2(3yz - 2e^{4x} Sin4z)}$

(d) (i) $z = \sqrt{2^7} e^{i7\left(\frac{3\pi}{4}\right)}$

<u>Question 2</u>

Objectives: (c) 1–3, 6, 8, 9, 11

Most candidates attempted Part (a) (i) using integration by parts. However, some candidates either differentiated or integrated the parts incorrectly while others had difficulty manipulating the signs. Some candidates used the identity $cos2\theta = 1 - 2sin^2\theta$ to simplify the integral to $\int sinx\, dx - 2\int sin^3 dx$, but most of them could not manipulate $\int sin^3 dx$.

In Part (a) (ii), the majority of candidates knew how to substitute the values into their answer from Part (a) (i) above and easily obtained full marks.

In Part (b), it was evident that many candidates did not understand what was meant by *four intervals*. Several candidates interpreted four intervals as four ordinates instead of four trapezia. Hence, they used n = 3. In other cases, candidates used n = 5 instead of n = 4. This resulted in a variety of incorrect responses. Further, most candidates did know what to do when the curve went below the x-axis. They did not take the modulus of the part of the curve that was below the x–axis, that is, $|f(-0.75)| = |-0.5625| = 0.5625$. Consequently, their responses to the problem were often smaller than the expected area between the curve and the x-axis.

The two common problems which arose in Part (c) were candidates not recognizing that partial fractions was not required and not competently dealing with irreducible quadratic factors and repeated roots. The vast majority therefore missed out on the opportunity to solve the problem easily by working on the right-hand side. Many candidates did not get the basic form of the expansion correct. Some did not recognize that $(x^2 + 4)$ is a factor of $(x^2 + 4)^2$. They made their denominator $(x^2 + 4)(x^2 + 4)^2$ which made the question more complicated and left them unable to complete the solution.

In Part (c) (ii), candidates experienced several difficulties. Many did not know how to use the substitution $x = 2tan\theta$. Some candidates determined $\frac{1}{2}\int \frac{1}{sec^2\theta} d\theta$, but could not go any further. Those who were able to manipulate the trigonometric function and integrate it to obtain $-\frac{1}{8}\sin 2\vartheta + \frac{3}{4}\vartheta + C$ experienced great difficulty changing the variable in $\sin 2\vartheta$ back to x. They did not realize that they could have used the identity $\sin 2\theta = 2\sin\theta\cos\theta$ and a right-angled triangle to get their final answer in terms of x.

Solutions

(a) (i) $-\frac{2cos^3x}{3} + cos\, x + c$ or $-\frac{1}{6}\cos 3x + \frac{1}{2}\cos x + c$ (ii) $-\frac{1}{3}$

(b) 4.22 square units

(c) (ii) $\frac{3}{4}tan^{-1}\left(\frac{x}{2}\right) - \frac{1}{2}\left(\frac{x}{x^2+4}\right) + c$

Section B

Module 2: Sequences, Series and Approximation

<u>Question 3</u>

Specific Objective(s): (a) 2–4; (b) 3, 6, 7, 9

This question tested the concepts of mathematical induction, telescoping and the Taylor series.

In Part (a), it was very clear that most candidates lacked understanding of the process of induction since they were unable to deduce what was to be proved. Many candidates attempted to prove the recurrence relation via mathematical induction, but they ignored the inequality. For those who recognized the inequality as the induction hypothesis and proved the base case, many found it difficult to carry out the inductive procedure.

Part (b) was generally well done. In Part (i) (b), most candidates applied the method of differences in the summation of a series although a few candidates opted to use mathematical induction to prove the equality with limited success. In Part (b) (ii), some candidates saw the connection to Part (b) (i) (b) and were able to complete the solution competently. However, in many cases the correct limit notation was not used.

Overall, candidates performed below expectations in Part (c) (i). Several candidates started correctly but experienced difficulty completing the series. Some candidates did not differentiate sine and cosine functions correctly while others confused Taylor with Maclaurin's series. In Part (c) (ii), candidates saw the connection with the solution of Part (c) (i) and attempted to substitute $\frac{\pi}{16}$ into their solution. However, many equated $\left(x - \frac{\pi}{4}\right)$ to $\frac{\pi}{16}$ and used $x = \frac{5\pi}{16}$ instead. Others failed to arrive at the correct solution because of faulty algebraic manipulation and arithmetic errors.

Solutions

(c) (i) $\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2 + \frac{\sqrt{2}}{12}\left(x - \frac{\pi}{4}\right)^3$

(ii) 0.977

<u>Question 4</u>

Specific Objective(s): (c) 1–4; (d) 1, 2, 6

This question examined the use of the binomial theorem to approximate a surd and to compute a coefficient of a term in an expansion; the Intermediate Value Theorem in testing for the existence of a root in an equation; and the 'interval bisection' method in finding successive approximations to a root in an equation.

Candidates generally found Part (a) (i) manageable and showed good understanding of the binomial expansion. However, a significant number of candidates simplified $\sqrt[4]{(1+x)} + \sqrt[4]{(1-x)}$ incorrectly as $(1+x)^2 + (1-x)^2$. Some candidates also had difficulty obtaining the correct binomial coefficients due to arithmetical errors, particularly the signs of coefficients.

In Part (a) (ii), most candidates substituted $x = \frac{1}{16}$ into the binomial expansion rather than first substituting $x = \frac{1}{16}$ into $\sqrt[4]{(1+x)} + \sqrt[4]{(1-x)}$ in order to determine how to modify the expansion to give the desired result. Simplifying surds also continues to be an area of difficulty for candidates. For example, several candidates wrote $\sqrt[4]{\frac{17}{16}} = \frac{\sqrt[4]{17}}{\sqrt[4]{16}} = \frac{\sqrt[4]{17}}{4}$, which is incorrect.

Part (b) examined candidates' ability to extract the coefficient of x^5 from a binomial expansion. Most candidates opted to expand both expressions first then multiply them. Some deviated from this and only wrote down the terms that had the desired power of x. On the other hand, certain candidates used more advanced approaches. They were able to apply the binomial expansion to the product of the two binomials and obtain the correct coefficient of x^5. In Part (b) (ii), most candidates appeared to be unfamiliar with interval bisection and continued to calculate midpoints using $b_n = 3$ or found an approximation using the Newton–Raphson method. Most candidates who demonstrated some knowledge of the topic produced at least two successive iterations. However, one commonly observed problem was the incorrect application of the stopping criterion. Interval bisection is a geometrical approach to finding a root; therefore, the use of diagrams in the teaching this topic should be encouraged in order to strengthen the responses in this area.

Solutions

(a) (i) $2 - \frac{3}{16}x^2$ (ii) 3.9986

(c) (i) $f(2) < 0;\ f(3) > 0\ hence$ continuity (ii) 2.92

Section C

Module 3: Counting, Matrices and Differential Equations

<u>Question 5</u>

Objectives: (a) 2, 4, 6, 16, 17, (b) 4–6.

In Part (a) (i), although the majority of the candidates was able to provide the required solution, some candidates did not know how to construct a tree diagram. In many cases, candidates used the actual letter of the word BRIDGE to show the outcomes rather than classifying the outcomes into vowel and consonant before recording the outcome on the tree. They were then unable to determine the required probability. In some cases although candidates correctly determined the number of outcomes, they were unable to calculate the associated probabilities.

In Part (b), a few candidates did not understand the use of the word *system* or what was meant by a system of equations having a "unique" solution. This question did not restrict candidates to using the row reduction approach and as a result, some candidates used the determinant method to arrive at their conclusion.

Performance on Part (c) (i) was unsatisfactory. Most candidates did not seem to understand how to move from the conditional probabilities given to the total probability required. Several candidates gave values greater than 1 which indicated that they lacked understanding of the basic concept of a probability. Some also used the values of 45 per cent, 30 per cent and 25 per cent and not the fractional form in their calculations. Consequently, they obtained incorrect responses.

Solutions

(b) (i) The system is not consistent since we have $0x + 0y + 0z = 9$. *(The justification will depend on how the reduction is executed.)*

(ii) The solution is unique. The matrix can be reduced to $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 4/3 \\ 0 & 1 & 0 & 25/6 \\ 0 & 0 & 1 & 9/2 \end{array}\right)$.

Since only the leading diagonal elements are non-zero, the solution is unique.

Alternatively, $|A| = 12$ which is not 0. The system of equations can therefore be solved.

(c) (i) 0.7975 (ii) 0.2665

<u>Question 6</u>

Objectives: (c) 2, 3 (c)

The first part of this question appeared to be the most difficult for candidates who were unable to determine the correct integrating factor and thus failed to arrive at the correct integral. In Part (a) (ii), several candidates used degrees instead of radians in their solutions. They were, therefore, unable to use the cancellation to assist in finding the answer. The given value of $y = \frac{15\sqrt{2}\pi}{32}$ was also not interpreted correctly by some candidates resulting in incorrect constants of integration.

In Part (b), most candidates appeared to know the appropriate form of the complementary function given the auxiliary equation. However, several candidates did not use the quadratic formula correctly to evaluate and many were unable to evaluate $\sqrt{-16}$. Most candidates who attempted Part (b) (iii) were able to find the particular solution correctly although some provided the general solution. Candidates were awarded the marks for either of these solutions.

Solutions

(a) (i) $y = x^2 \cos x + C \cos x$ (ii) $C = \frac{7\pi^2}{8}$

(b) (i) (a) $\lambda = -1 \pm 2i$ (b) $u_p(t) = C_1 e^{-t} \sin 2t + C_2 e^{-t} \cos 2t$

(iii) $y(t) = 0.981e^{-t}\sin 2t + 0.255e^{-t}\cos 2t - \frac{16}{17}\cos 2t + \frac{4}{17}\sin 2t$

Paper 032 – Alternative to School-Based Assessment

Section A

Module 1: Complex Numbers and Calculus II

Question 1

Objectives: (a) 4–6, 9; (b) 3, 4, 8; (c) 4, 5, 7, 8, 10

In Part (a), it was evident that candidates were unable to interpret the partial derivatives required. Similarly, in Part (b), most candidates were unable to choose appropriate expressions for the integration by parts. Consequently, they were unable to solve the problem. Those who were able to begin the integration by parts ended up with expressions containing incorrect signs. More exposure to the reduction formula is recommended.

In Part (c), most candidates were able to generate the simultaneous equations needed to solve for the square root. However, they experienced difficulty recognizing that the resulting equation was a quadratic equation in x^2.

Solutions

(a) 0.16% approximately

(c) $z^{1/2} = 1.453 + 0.344i$ and $z^{1/2} = -1.453 - 0.344i$

Section B

Module 2: Sequences, Series and Approximation

Question 2

Specific Objective(s): (b) 1, 2, 4, 6, 8; (c) 4

This question examined the use of binomial theorem to approximate a decimal number; Maclaurin and geometric series.

In Part (a) (i), a significant number of candidates did not express $\frac{1}{2}x$ in brackets. As a result, they raised only x to the various powers instead of the entire expression $\frac{1}{2}x$. In Part (a) (ii), some candidates substituted $x = 1.377$ into their expansion rather than stating that $1 + \frac{1}{2}x = 1.377$ to find x as 0.754.

In Part (b), quite a few candidates simply copied the Maclaurin expansion from the formula booklet instead of deriving it as the question required and were unable to find the derivatives of the logarithmetic function given. Of the few who recognized the general term, some did not use the sigma notation and others could not derive the appropriate sign for the terms of the sum.

In Part (c) (ii), most candidates were unable to show that $S_2 < 4$. Some found it difficult to work with the algebraic expressions as the terms of the series while others could not manage the reasoning required to complete the proof.

Solutions

(a) (ii) 3.60

(b) (i) $x - \frac{1}{2}x^2 + \frac{1}{3}x^3$ (ii) $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{x^k}{k}$

(c) (i) $-2 < x < 2$

Section C

Module 3: Counting, Matrices and Differential Equations

Question 3

Objectives: (a) 2, 3, 6; (b) 2, 7, 8

In general, candidates knew how to find the determinant of the matrix in Part (a) and demonstrated the ability to multiply matrices. However, many did not write out the 3 × 3 zero matrix. Instead, they simply wrote 0.

Most candidates did not recognize the link between the equation in Part (a) (ii) and the inverse of the matrix and instead attempted to use row reduction to find the inverse. Similarly, row reduction was used to solve the simultaneous equations. This approach required much more work and many computational errors were made.

In Part (b), the majority of candidates were unable to recognize that the problem involved permutations and used the $\binom{n}{r}$ format, instead of ${}^6P_3$. Further, the concept of probability was not understood.

Solutions

(a) (i) 18 (ii) b) $A^{-1} = \frac{1}{18}C^T = \frac{1}{18}\begin{bmatrix} 8 & 4 & 2 \\ 7 & -1 & -5 \\ -3 & 3 & -3 \end{bmatrix}$

(ii) (c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

(b) (i) 120 (ii) 0.8