C A R I B B E A N E X A M I N A T I O N S C O U N C I L

REPORT ON CANDIDATES' WORK IN THE SECONDARY EDUCATION CERTIFICATE EXAMINATIONS

JUNE 2008

MATHEMATICS (T & T)

MATHEMATICS

GENERAL PROFICIENCY – TRINIDAD AND TOBAGO

MAY/JUNE 2008

The General Proficiency Mathematics examination is offered in January and May/June each year, while the Basic Proficiency examination is offered in May/June only.

In May/June 2008 approximately 20 000 candidates registered for the General Proficiency examination. At the General Proficiency level, approximately 47 per cent of the candidates achieved Grades I – III.

DETAILED COMMENTS

General Proficiency

No candidate scored the maximum mark on the overall examination. Forty per cent of the candidates scored at least half the available marks.

Paper 01 – Multiple Choice

Paper 01 consisted of 60 multiple-choice items. Approximately 57 per cent of the candidates scored at least half the total marks for this paper.

Paper 02 – Essay

Paper 02 consisted of two sections. Section I comprised eight compulsory questions totaling 90 marks. Section II comprised six optional questions: two each in Relations, Functions and Graphs; Trigonometry and Geometry; Vectors and Matrices. Candidates were required to choose any two questions. Each question in this section was worth 15 marks.

This year, fourteen candidates earned the maximum available mark on Paper 02. Approximately 33 per cent of the candidates earned at least half the maximum mark on this paper.

Question 1

This question tested candidates' ability to:

- Perform basic operations on mixed numbers
- Solve problems associated with income tax
- Calculate a percentage of a derived quantity
- Write as a percentage the ratio of two quantities

The question was attempted by 100 per cent of the candidates, 15 per cent of whom scored the maximum available mark. The mean score was 7.42 out of 12.

Responses were satisfactory. Candidates demonstrated competence in:

- Choosing an appropriate strategy for subtracting common fractions
- Converting a mixed number into an improper fraction
- Inverting and multiplying when dividing improper fractions
- Calculating annual salary given monthly salary
- Calculating 22 per cent of a quantity
- Expressing one quantity as a percentage of another

Some candidates employed incorrect algorithms for performing operations on mixed numbers.

e.g. $1\frac{5}{8}$ was equated to $\frac{40}{8}$

$2\frac{1}{5}-1\frac{1}{3}$ was simplified to $\frac{13-5}{15}=\frac{8}{15}$

Many of the candidates disregarded the fact that the family had 3 children and so computed the total taxable allowance on the assumption that there was only one child. Some candidates did not recognize the difference between taxable allowance and income tax payable. They therefore, calculated 22 % of the taxable allowances.

Recommendations

It would be useful to expose students to more real-life situations based on income tax.

Solutions:

(a) 8/15

(b) (i) $90000.00 (ii) $22500.00 (iii) $14850.00
(iv) 16.5 %

Question 2

This question tested candidates' ability to:

- Square a binomial
- Change the subject of the formula
- Factorize expressions using common factors and the difference of two squares
- Solve a pair of linear simultaneous equations

The question was attempted by 97 per cent of the candidates, 8 per cent of whom scored the maximum available mark. The mean score was 4.2 out of 12.

A high proportion of candidates scored zero. The areas of good performance were:

- Factorizing the expression $3mn - 6n^2$
- Choosing strategy for solving simultaneous equations

Students in general experienced difficulty in:

- Squaring the binomial
- Changing the subject of the formula
- Factorizing the difference of two squares
- Applying strategy for solving simultaneous equations

Recommendations

Topics which are covered in the junior school need to be reviewed systematically.

Solutions:

(a) $\mathbf{9a^2 - 6a + 1}$

(b) $p = \frac{5-3q}{q}$

(c) **(i)** $\mathbf{3n\,(\,m - 2n\,)}$ **(ii)** $\mathbf{(\,5p - q\,)(\,5p + q\,)}$

(d) $\mathbf{x = 5\ ;\ y = -2}$

Question 3

This question tested candidates' ability to:

- Describe a set
- Identify the cardinal number of a set
- Determine the complement of a given set
- Determine the elements in the intersection of a set
- Use a ruler and protractor to draw angles and measure line segments

The question was attempted by 98 per cent of the candidates, 2 per cent of whom scored the maximum available mark. The mean score was 6.01 out of 12.

Students were able to:

- List the members of the intersection
- Measure lines and angles with some degree of accuracy

Students in general:

- Failed to demonstrate an understanding of the complement of a set
- Were unable to find the cardinal number for a set
- Were unable to recognize relations among numbers, hence, the terms ‘multiples’ and ‘factors’ were used interchangeably
- Experienced difficulty in describing the range of a set
- Appeared to have measured from ‘1’ rather than ‘0’ on their rulers
- Attempted to construct the quadrilateral rather than draw it

Recommendations

Teachers should pay attention to instructional terms such as draw, sketch, construct, describe, define.

Solutions:

(a) (i) a) { 12, 24 } **b) { 4, 8, 16, 20 }**

(ii) 9

(iii) a) even numbers less than or equal to 30

b) multiples of 6 less than or equal to 30

Question 4

This question tested candidates’ ability to:

- Calculate the area of a right-angled triangle
- Calculate the length of a prism given its volume and cross-section
- Use Pythagoras theorem to find the length of a line segment
- Identify the number of faces, edges and vertices in a given prism
- Use, correctly, the S.I. units of measure for area and length

The question was attempted by 92 per cent of the candidates, 14 per cent of whom scored the maximum available mark. The mean score was 4.64 out of 10.

Candidates were generally able to calculate the area of the triangle. However, many candidates either omitted the units of measure for area (cm^2) or wrote the incorrect S.I. units of measure (cm). Many of them chose an appropriate strategy for finding the length of AC (use of Pythagoras or trigonometric ratio). However, they encountered difficulty following through with the computation.

Candidates found difficulty in calculating the length of the prism given its volume and area. Many misinterpreted the length of the prism to be the height of its cross-section. Several candidates interchanged the number of edges and vertices.

Recommendations

Teachers may wish to make greater use of concrete objects in teaching the properties of solids.

Solutions:

(a) 8 cm^2

(b) 9 cm

(c) 5.7 cm

(d) 5 faces, 9 edges, 6 vertices

Question 5

This question tested candidates' ability to:

- State the relation between an object and its image as a combination of transformations
- Locate the image of a set of points when the transformations (reflection and enlargement) are performed
- Determine the ratio of the area of the object and its image under an enlargement

The question was attempted by 89 per cent of the candidates, 3 per cent of whom scored the maximum available mark. The mean score was 5.38 out of 12.

The responses to this question were generally satisfactory. Candidates knew how to reflect the original triangle in a line. Most of them were able to plot the points of the triangle representing the enlargement.

The majority of the candidates were able to identify the transformation as an enlargement. However, they were unable to describe the transformation fully, particularly the negative aspect of the scale factor and the centre of enlargement (the origin). Students were unfamiliar with the use of the square of the scale factor to find the ratio of the areas of the triangle and its enlarged image.

Solutions:

(**c) (ii) Enlargement; centre (0, 0); scale factor -2**

(d) 4

Question 6

This question tested candidates' ability to:

- Represent given statistical data graphically, using specified graph and scale
- Use a graph to make estimates and to draw inferences from it

The question was attempted by 88 per cent of the candidates, 11 per cent of whom scored the maximum available mark. The mean score was 5.9 out of 11.

Most candidates were able to identify the five-year periods where there were increases in the population. Generally they were able to show on the graph how to estimate the population of the country in 1989. Most candidates were able also to draw correctly, the scales on both axes. However, a large number of them

- Interchanged the axes
- Used the values from the table as intervals on the population axis
- Drew bar charts or histograms
- Joined the points without using a straight edge

Most candidates were unable to explain in a concise manner (using terms such as gradient, slope) two facts about the population.

Recommendations

Teachers may wish to expose their students to the full range of graphs as specified by the syllabus. Interpretation of information represented graphically needs to be reinforced.

Solutions:

(b) 1.75 to 1.85 million
(c) (i) 1990 – 1995 (ii) 1995 - 2000

Question 7

This question tested candidates' ability to:

- Calculate the coordinates of the end point of a line segment given the midpoint and the other end point
- Determine the image on to which a particular function maps a member of the domain

- Find the inverse of a linear function

- Find the domain value for which the composite function is equal to a real number

The question was attempted by 77 per cent of the candidates, 8.62 percent of whom scored the maximum available mark. The mean score was 4.24 out of 11.

Candidates were generally able to employ a suitable strategy to find the values of j and k, some used $\frac{1+j}{2}=4$ and $\frac{8+k}{2}=5$ while others plotted the coordinates and extended the line segment.

Candidates were able to substitute correctly for $f(0)$ and $g(2)$ but multiplication by zero proved problematic, for example, 5×0 was commonly written as 5 and $\frac{4}{2+1}$ was equated to 2.

Responses to finding the inverse function were generally satisfactory. However, after candidates interchanged the domain and range, transposition of terms proved difficult for them.

Candidates experienced difficulty with the composite function. They simply multiplied the functions. In some cases they substituted 1 for x rather than equating the composite function to 1. Many candidates could not proceed beyond $5(\frac{4}{x+1})-2$ to solve for x.

Solutions:

(a) j = 7 ; k = 2

(b) (i) -2 (ii) 4/3 (iii) $f^{-1}(x)=\frac{x+2}{5}$ **(iv)** $\frac{17}{5}$

Question 8

This question tested candidates' ability to:

- Continue a sequence of diagrams given three terms in the sequence

- Follow number patterns in order to make generalizations

The question was attempted by 87 per cent of the candidates, 10 per cent of whom scored the maximum available mark. The mean score was 4.67 out of 10.

Most candidates were able to

- Sketch the fourth pattern in the sequence although in some cases they did not distinguish between grey and white triangles

- Deduce the pattern for the total number of triangular shapes and the number of grey triangular shapes. However, they had difficulty in generalizing the number of white triangular shapes

Recommendations

Teachers might choose to expose students to a wide range of problem solving experiences involving number patters and generalizations. Investigative methods should prove to be helpful.

Solutions:

(b) (i) 36, 21, 15 (ii) 400, 190

(c) n^2 , $\frac{n(n+1)}{2}$

Question 9

This question tested candidates' ability to:

- Solve a pair of simultaneous equations – one linear and one quadratic
- Establish an identity
- Determine the minimum value of a function
- Sketch the graph of a quadratic function to show roots of the function and the values of x at which the function is a minimum

The question was attempted by 25 per cent of the candidates, 12 per cent of whom scored the maximum available mark. The mean score was 5.32 out of 15.

Most candidates were able to:

- Eliminate one of the variables and to solve for x and y
- Solve the quadratic equation and to sketch the curve showing the minimum value and the intercept on the x-axis

Candidates generally had difficulty:

- Completing the square
- Identifying the x coordinate of the minimum point

Recommendations

Teachers might consider teaching the completion of the square as a means to an end which may include curve sketching, solving quadratic equations, maximizing and minimizing equations.

Solutions:

(a) x = 1, y = -14; x = 4, y = -8

(b) (i) $(x-\frac{3}{2})^2-1\,4\frac{1}{4}$ **(ii) – 14** $^{\frac{1}{4}}$ **(iii) 5.3; -2**.3

Question 10

This question tested candidates' ability to:

- Draw and use the graph of the function $y = a x^{-2}$
- Draw a tangent to a curve at a point
- Estimate the gradient of a curve at a point

The question was attempted by 46 per cent of the candidates, 10 per cent of whom scored the maximum available mark. The mean score was 9.26 out of 15.

The majority of the candidates who attempted this question were able to do part (a) quite well. They were also able to plot points on the Cartesian plane and connect the points to form a smooth curve.

An area of difficulty was in part (a) when calculating the y-values for the corresponding x-values. Candidates were unable to convert the fractional values into decimals (they simply divided by the smaller number).

Drawing axes and using the given scales also posed some problems, candidates simply used the values of y at intervals on the y-axis.

Part (d) posed the greatest difficulty. In many cases candidates were unable to draw a tangent at the given point and calculate the gradient of that tangent. Some candidates used points on the curve itself, to calculate the gradient. Also, when calculating the gradient of the tangent, candidates neglected the negative sign and when they quoted the formula $\frac{y_2 - y_1}{x_2 - x_1}$, they interchanged x and y values. In some cases they inverted the formula to obtain $\Delta x/\Delta y$.

Solutions:

(a) 5; 0.8; 0.4

(c) (i) y = 1 (ii) x = 2.4

(d) - 1.5

Question 11

This question tested candidates' ability to:

- Sketch a diagram given bearings and distances
- Calculate unknown angles
- Calculate lengths of two sides of a figure using trigonometry

- Read bearings

The question was attempted by 29 per cent of the candidates, 5 per cent of whom scored the maximum available mark. The mean score was 4.37 out of 15.

Stating the size of angle QRS and reading the bearings at Q and S were well done. Even if the angle QSR was inaccurately calculated, candidates were able to obtain the bearing of Q.

Many candidates were unable to read angles RQS or QST. Obtaining QT was difficult for some candidates. The cosine rule was necessary to obtain it.

Recommendations

- More attention must be given to:

- Reading and understanding written instructions. (some candidates sketched four diagrams instead of one)

- Locating the position of a point , given bearings

- Reading bearings from a given sketch

- Assisting candidates in recognizing when to use the sine rule and the cosine rule. (They should know that the trigonometric ratios sine, cosine, and tangent can only be used with right angled triangles.)

- Reading and referring to an angle using three vertices (some candidates found angle QSR instead of angle QST.

Solutions:

(c) (i) 57^0 (ii) 95^0 (iii) 152^0

(d) (i) 7 km (ii) 9 km

(e) 298^0

Question 12

This question tested candidates' ability to:

- To solve problems related to the properties of a circle

- Know and use the properties of an isosceles triangle

- Calculate the distance between two points on the earth along lines of longitude or latitude

The question was attempted by 13 per cent of the candidates, 2 per cent of whom scored the maximum available mark. The mean score was 4.06 out of 15.

The majority of the candidates who attempted this question were able to successfully copy the diagram and correctly indicate the 60 degree north latitude and the 25 degree west longitude. Another area of good performance was in part (a) where the students were able to recognize that triangle KLN was isosceles and to use its properties.

A common error made by candidates was incorrectly assuming that the line NL was a diameter of the circle and hence angles NKL and NML were both 90 degrees. Students demonstrated lack of understanding of the concept of the alternate segment in a circle. Although candidates recognized that NKLM was a cyclic quadrilateral, they wrongly stated that the opposite angles were equal as opposed to being supplementary.

In part (b) candidates failed to correctly state the formula for finding arc length. They showed great deficiencies in calculating angle θ as being 65 degrees and in calculating the radius of the circle of latitude. Most times the angle for the centre (65 degrees) was used instead of 60 degrees.

Teachers should ensure that students review the basics of angles, sectors and circles before attempting questions related to earth geometry.

Solutions:

(a) (i) 24^0 (ii) 132^0 (iii) 48^0

(b) (iii) 3611 km

Question 13

This question tested candidates' ability to:

- Write position vectors of given points
- Add vectors
- Find the magnitude of a vector
- Prove three points are collinear

The question was attempted by 21 per cent of the candidates, 5 per cent of whom scored the maximum available mark. The mean score was 3.47 out of 15.

Candidates were able to state the position vectors with little difficulty.

However, many were unable to find the value of p. Some did not know the magnitude formula for a vector while others who knew the formula had difficulty solving the resulting quadratic equation.

Recommendations

Teachers must encourage students to:

- Use appropriate mathematical vocabulary relevant to the topic when teaching

- Use and interpret vector notation, for example, the difference between a point and its position vector
- Use a vector method to establish a proof

Solutions:

(i) $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$ **(ii)** $\begin{pmatrix} -5 \\ 4 \end{pmatrix}$ **(iii)** $\begin{pmatrix} 7 \\ p \end{pmatrix}$

(b) **(i)** $\begin{pmatrix} 4 \\ =1 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 8 \\ -3+p \end{pmatrix}$

(c) $\boldsymbol{p}$ **= 9, -3**

Question 14

This question tested candidates' ability to:

- Identify an 2 × 2 singular matrix
- Perform multiplication of matrices
- Use matrices to solve problems in algebra
- Use matrices to solve problems in transformation geometry

The question was attempted by 23 per cent of the candidates, 7 per cent of whom scored the maximum available mark. The mean score was 4.04 out of 15.

In part (a), most candidates associated the singularity of a matrix with its determinant. Part (b) proved to be a great challenge to candidates as they were unable to perform the correct matrix multiplication. Many candidates left out the addition sign between 10 and b and 5a and 4b. There were many instances where arbitrary guesses were made with no working shown, for instance candidates wrote a = 1, b = 2.

In part (c) many candidates simply stated the matrix that represented the transformation mapping E onto E' as $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. In other cases, they showed a good working knowledge of the choice of strategy to be used in (c) (i) but failed to correctly apply this strategy, for instance, they wrote 5y = 5. There were a few cases where candidates used a graphical approach to get values for x and y.

Part(c)(ii) was well done, but an area of difficulty was sometimes seen in transposing points from coordinate form to column matrix form in order to perform the necessary calculations.

Candidates showed weakness in obtaining the coordinates of H. They were unable to perform this inverse translation. Examples of errors were adding H' to T or adding H to T, forming equations and equating to H'

Candidates showed a lack of understanding of the concept of a combined transformation. Some common errors were performing T first then S or attempting to combine S and T first.

Recommendations

Teachers should help students to follow instructions given in a logical order and practise using matrices to solve problems.

Solutions:

(b) $a = 3$; $b = -2$

(c) **(i)** $x = 1$; $y = -1$ **(ii)** $p = 5$; $q = 2$ **(iii)** P' **(9, 6).**