CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE JANUARY 2009

MATHEMATICS

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MATHEMATICS

GENERAL PROFICIENCY EXAMINATIONS

JANUARY 2009

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. The Basic Proficiency examination is offered in May/June only.

There was a candidate entry of approximately 15 300 in January 2009. Forty-nine per cent of the candidates achieved Grades I-III. The mean score for the examination was 84.54 out of 180 marks.

DETAILED COMMENTS

Paper 01- Multiple Choice

Paper 01 consists of 60 multiple choice items. This year, twenty-eight candidates each earned the maximum available mark of 60. Sixty-eight per cent of the candidates scored 30 marks or more.

Paper 02 - Essay

Paper 02 consisted of two sections. Section I comprised eight compulsory questions totalling 90 marks. Section II comprised six optional questions: two each from Relation, Functions and Graphs; Trigonometry and Geometry and Vectors and Matrices. Candidates were required to answer any two questions from this section. Each question in this section was worth 15 marks.

This year, four candidates each earned the maximum available mark of 120 on Paper 02. There were five candidates who each scored 119 marks. Approximately twenty-eight per cent of the candidates earned at least half of the maximum mark on this paper.

Compulsory Section

Question 1

This question tested candidates' ability to:

- perform basic operations with fractions
- solve problems related to currency conversion
- calculate compound interest over two periods

The question was attempted by 99.7 per cent of the candidates, 7 per cent of whom earned the maximum available mark. The mean mark was 6.66 out of 11.

Candidates demonstrated good proficiency in inverting and multiplying when dividing one improper fraction by another and calculating the exchange rate from BDS$ to EC$.

Candidates showed weak performance in finding the lowest common multiple, using it correctly to subtract fractions and using the algorithm for subtracting fractions. Some incorrect procedures were:

$$\frac{7}{3} - \frac{5}{6} = \frac{2}{3} \quad \text{and} \quad \frac{(4+7) \quad - \quad (2+5)}{12}$$

Candidates also experienced difficulty calculating the compound interest for 2 years.

Solutions:

(a) $\frac{5}{2}$

(b) (i) EC $1.35 **(ii) BDS $320.00**

(c) $27 933.60

Recommendations

The use of the calculator is a necessary skill in learning mathematics. Teachers are encouraged to teach students how to use the calculator, as well as the requisite skills necessary for determining the accuracy of the calculation such as estimation and rounding.

Question 2

This question tested candidates' ability to:

- write the difference between two algebraic fractions as a single fraction
- evaluate the result of a binary operation on two integers
- factorize an expression of the form ax + bx + ay + by
- use algebraic symbols to represent information
- use linear equations to solve a worded problem

The question was attempted by 99 per cent of the candidates, 9 per cent of whom earned the maximum available mark. The mean mark was 6.07 out of 12.

Candidates demonstrated good proficiency in choosing the appropriate common denominator in part (a) and recognizing that $5 * 2 = 5^2 - 2$ in the binary operation. Candidates were also able to express most of the statements as algebraic expressions.

However, some of the weaker candidates had difficulty following the algorithm for subtracting fractions. Errors were also seen in completing the factorization, writing the expression for '3 cm shorter than the first piece' and using the trial and error strategy to solve for the value of x.

Solutions:

(a) $\frac{m}{3n}$

(b) 23

(c) (3 + x) (x – 2y)

(d) (i) x; x – 3; 2x **(ii) x + x – 3 + 2x** **(iii) x = 6**

Recommendations

In the teaching of fractions, teachers need to focus on the algorithms for adding and subtracting common fractions. Emphasis should also be placed on translating verbal statements into mathematical symbols and solving simple equations in one unknown by using approaches other than trial and error.

Question 3

This question tested candidates' ability to:

- construct and use Venn diagrams
- solve problems involving the use of Venn diagrams
- use Pythagoras' theorem to find the side of a right-angled triangle

The question was attempted by 99 per cent of the candidates, 11 per cent of whom earned the maximum available mark. The mean mark was 6.38 out of 12.

Candidates demonstrated strengths in reproducing the Venn diagram, correctly interpreting the worded information, illustrating it on the Venn diagram and finding the total of the subsets.

In part (b), many candidates correctly chose Pythagoras' theorem to find the length of the third side of a right angled triangle but had difficulty applying the theorem correctly. Choosing the appropriate trigonometric ratio for finding the length MK also proved challenging.

Solutions:

(a) (iii) x = 12

(b) (i) MK = 5m **(ii) JK = 7m**

Recommendations

Students need more practice in the practical application of Pythagoras theorem. Exposure to the development of the theorem by determining the areas of the squares whose lengths are the sides of a right-angled triangle may be beneficial.

Question 4

This question tested candidates' ability to:

- write the coordinates of two points located on the x and y axes respectively
- determine the gradient of a line segment
- find the equation of a line given its graphical representation
- find the equation of a line which passes through a given point and is perpendicular to a given line.

The question was attempted by 94 per cent of the candidates, 5 per cent of whom earned the maximum available mark. The mean mark was 3.30 out of 11.

Although most candidates were able to write the coordinates of P, the point on the x – axis, candidates had difficulty writing the coordinates of Q, the point of the line on the y-axis. However, many of the candidates were able to calculate the gradient of the line segment; use their value of m and the coordinates of a point to find the value of c in the equation $y = mx + c$ and recognize that for two perpendicular lines the product of their gradients is -1.

Candidates had difficulty recognizing that for a given value of x, the corresponding value of y could be found by substituting into the equation of the line.

Solutions:

(a) P (0, 3) Q (-2, 0)

(b) (i) $m = \frac{3}{2}$ **(ii)** $y = \frac{3}{2}x + 3$

(c) t = -9

(d) $y = \frac{-2}{3}x + 6$

Recommendations

Students should be encouraged to use graphical representations of linear equations to answer questions related to gradients, intercepts and equations of lines.

Question 5

This question tested candidates' ability to:

- calculate the volume of a right prism given the length of its edges
- calculate the surface area of the right prism
- solve worded problems based on the volume and dimensions of prisms

The question was attempted by 88 per cent of the candidates, 8 per cent of whom earned the maximum mark. The mean mark was 3.75 out of 10.

Candidates demonstrated proficiency in calculating the volume of the prism, the area of at least one face of the prism and dividing by 6 to find the volume of a small box.

However, many candidates did not recognize that the net of the prism had six faces and also had difficulty distinguishing between the volume and the area. In some cases, although the calculations were done accurately, the appropriate units of volume and area were not used.

Solutions:

(a) 7 200 cm^3

(b) 2 776 cm^2

(c) (i) 1 200 cm^3

(ii) 60 cm^2 = 6cm × 10cm or 15cm × 4cm or 20cm × 3cm

Recommendations

In teaching the concept of solids, teachers are urged to ensure that the students are familiar with the properties of the solids in relation to the nets of the solids, the dimensions, number of vertices, faces and edges.

Question 6

This question tested candidates' ability to:

- use a ruler and a pair of compasses to construct a rectangle
- identify and describe transformations given object and image
- find the centre of enlargement given the object and image
- determine the scale factor of an enlargement

The question was attempted by 92 per cent of the candidates, 2 per cent of whom earned the maximum available mark. The mean mark was 4.75 out of 12.

The majority of the candidates were able to construct or draw angles of 90^0, measure the sides of the rectangle accurately and complete the rectangle.

In part (b), candidates displayed weakness in determining the vector representing the translation. Many candidates also had difficulty locating the centre of enlargement and determining the scale factor of the enlargement.

Solutions:

(b) (i) $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$

(ii) b) G (-5,0); **c) scale factor of enlargement is 2**

Recommendations

Teachers are encouraged to include practical work and authentic activities when teaching transformations. In addition, transformational geometry should be taught with and without graph paper to ensure that the basic concepts and procedures are understood.

Question 7

This question tested candidates' ability to:

- complete the cumulative frequency column from a grouped frequency distribution
- draw a cumulative frequency graph
- use the cumulative frequency curve to solve problems
- compute simple probability

The question was attempted by 93 per cent of the candidates, 2 per cent of whom earned the maximum available mark. The mean mark was 5.81 out of 12.

Candidates displayed proficiency in completing the cumulative frequency column and using given scales to draw the cumulative frequency curve.

However, some candidates did not label the axes correctly. In addition, a number of candidates connected the points plotted with straight lines instead of drawing a smooth curve to represent the ogive. Candidates also had difficulty determining the number who passed when the pass mark was given.

Solutions:

(a)

Mark	**Cumulative Frequency**
31 – 40	**30**
41 – 50	**46**
51 – 60	**58**
61 – 70	**66**

(b)(ii) Assumption: No student obtained a mark less than 0.5 or 1

(c) 30 students

(d) $\frac{16}{70}$

Recommendations

Students should be given more practice in the drawing of non-linear graphs, in order to perfect the skill of drawing the ogive and other curves. Emphasis should also be placed on interpreting the graphs to obtain useful information.

Question 8

This question tested candidates' ability to:

- draw the next diagram in the sequence of diagrams
- recognize number patterns
- calculate unknown terms in a number sequence
- state the formula for the nth term in a sequence

The question was attempted by 94 per cent of the candidates, 7 per cent of whom earned the maximum available mark. The mean mark was 5.91 out of 10.

Candidates demonstrated strength in constructing the fourth diagram in the sequence and determining the missing number of dots and line segments by following the pattern in the table.

Candidates displayed weakness in determining the number of dots and lines segments for a pattern not represented in the table and stating the rule connecting the number of line segments to the number of dots.

Solutions:

	Dots	Pattern	Line Segments
(b) (i)	**62**	**$2 \times 62 - 4$**	**12**
(ii)	**92**	**$2 \times 92 - 4$**	**180**

(c) (i) 20 (ii) 42 (iii) $l = 2d - 4$

Recommendations

Students would benefit from exposure to problem solving involving the use of number patterns. These could be presented through projects where students work in groups or individually, exploring interesting real life situations which require the use of number patterns in making generalizations.

Question 9

This question tested candidates' ability to:

- change the subject of the formula
- complete the square
- find roots of a quadratic equation
- determine the domain interval over which a quadratic function is less
- than or equal to zero
- determine the minimum value of a quadratic function and state the
- value of x for which this minimum occurs

The question was attempted by 28 per cent of the candidates, 3 per cent of whom earned the maximum mark. The mean mark was 4.50 out of 15.

Candidates demonstrated strength in determining the roots of the quadratic equation and stating the minimum value of the quadratic function from the completed square. However, some candidates were unable to manipulate the equation to change the subject and complete the square. Very few candidates correctly stated the interval for which the function was negative or zero.

Solutions:

(a) $t = \frac{gp^2}{4} - r$

(b) (i) $\mathbf{f(x) = 2(x-1)^2 - 15}$ **(ii) x = -1.74, 3.74**

(iii) f(x) < 0 for -1.74 < x < 3.74]

(iv) Minimum value of f(x) is -15

(v) Minimum occurs when x = 1

Recommendations

The concept of changing the subject of a formula should be taught by placing emphasis on the inverse operation when transposing a quantity from one side of the equation to the other. Examples should include formulae with squares and square roots. Teachers must pay particular attention to the strategies used to teach the concept of completing the square. In addition, students should fully understand the importance of each component of the resulting expression.

Question 10

This question tested candidates' ability to:

- use function notation to determine the value of f(x) for a given value of x
- find the inverse of a function
- use the algebra of composite functions to prove an identity
- complete a distance/time graph from given information
- use a distance/time graph to solve problems

The question was attempted by 63 per cent of the candidates, 3 per cent of whom earned the maximum mark. The mean mark was 5.67 out of 15.

In part (a), candidates were able to calculate the value of the function and prove that the two given composite functions were equal. However, many were unable to determine the inverse of the function. While the majority of the candidates were able to determine values from the graph, errors were made in calculating average speed and the time taken to complete the journey. This was primarily as a result of candidates not using consistent units in the calculations.

Solutions:

(a) (i) $f(6) = 3$ **(ii) $f^{-1}(x) = x + 3$**

(b) (i) 20 minutes **(ii) 150 km/h** **(iii) 50 minutes**

Recommendations

Students should be exposed to a wide range of problems involving the use of travel graphs. Further, the importance of using the appropriate units should be emphasized.

Question 11

This question tested candidates' ability to:

- complete a table of values for $y = \frac{1}{2} \tan x$ for a given domain
- draw the graph of $y = \frac{1}{2} \tan x$ for a given domain
- use theorems in circle geometry to calculate the measure of angles

The question was attempted by 13 per cent of the candidates, 2 per cent of whom earned the maximum mark. The mean mark was 4.23 out of 15.

The performance in part (a) of this question was fair. The majority of the candidates were able to plot the given points on the recommended scale and determine the coordinates of the missing points. However, many of the candidates could not determine the estimate as required.

Generally, the candidates were unable to determine the measure of the unknown angles. In some cases, incorrect assumptions were made about the properties of the polygons in the circle and hence the answers and accompanying reasons were incorrect.

Solutions:

(a) (i)	**x**	**y**
	20^0	**0.18**
	40^0	**0.42**

(iii) For **y = 0.7,** **x = 55^0**

(b) (i) 90^0 **(ii)132^0** **(iii) 66^0** **(iv) 114^0**

Recommendations

Teachers should provide opportunities for more work on constructing and interpreting scales on graphs. Greater attention should be given to the interpretation of graphs than to the mechanics of drawing them. Theorems related to the geometry of the circle should be verified by the accurate construction of diagrams.

Question 12

This question tested candidates' ability to:

- find the length of the side of a triangle using the cosine formula
- find the measure of the size of an angle in a triangle using the sine formula
- find the area of a triangle given two sides and the included angle
- show lines of latitude and longitude on the circle representing the earth
- calculate the distance between two places on the earth measured along a circle of longitude
- determine the radius of a circle of latitude

The question was attempted by 21 per cent of the candidates, 2 per cent of whom earned the maximum mark. The mean mark was 2.89 out of 15.

The performance on the question was generally weak. In part (a), the majority of candidates correctly applied the sine rule to determine the measure of angle UVW. However, very few candidates correctly calculated the length UW or the area of triangle TUW. In part (b), candidates were able to correctly label the circles of latitude and longitude on the diagram but experienced difficulty calculating the required radius and distance.

Solutions:

(a) **(i)** **9.17 m** **(ii)** **32.4^0** **(iii)** **34.6 m^2**

(b) **(ii)a)** **5560 km** **b)** **5990 km**

Recommendations

Students should be exposed to a range of exercises involving the trigonometric ratios, where they will be required to select the appropriate ratio: sine, cosine or tangent. Practical and authentic activities should be incorporated, where possible, as well as accurate scale drawings to reinforce concepts.

Question 13

This question tested candidates' ability to:

- write coordinates of points as position vectors
- write the displacement of one point from another as a column vector
- state the condition for two vectors to be parallel
- determine the magnitude of a vector
- determine the values of two variables given two equal vectors
- use a vector method to establish that a given quadrilateral is a parallelogram

The question was attempted by 20 per cent of the candidates, 2 per cent of whom earned the maximum available mark. The mean mark was 3.78 out of 15.

Candidates demonstrated strengths in writing the co-ordinates of the given points as position vectors and adding vectors with numerical values. However, the majority of the candidates experienced challenges
routing vectors, proving that the vectors QR and OP are parallel, finding the magnitude of a column vector and proving that the given shape was a parallelogram.

Solutions:

(a) (i) $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ **(ii)** $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

(b) (i) $\begin{pmatrix} 9 \\ 6 \end{pmatrix}$ **(iii)** $\sqrt{74}$

(c) (i) $\begin{pmatrix} a+1 \\ b-3 \end{pmatrix}$ **(ii)** **a = 2; b = 5**

Recommendations

The concept of vectors should be taught using concrete examples. Students should be challenged to express a given route using a variety of paths and vectors.

Question 14

This question tested candidates' ability to:

- multiply 2 × 2 matrices
- multiply a matrix by a scalar
- calculate the determinant of a 2 × 2 matrix
- find the inverse of a non-singular 2 × 2 matrix
- use matrices to transform geometrical shapes
- use the matrix method to solve a system of linear equations

The question was attempted by 32 per cent of the candidates, 1 per cent of whom earned the maximum available mark. The mean mark was 5.13 out of 15.

In this question, candidates demonstrated competence in multiplying a matrix by a scalar, writing a pair of simultaneous equations in matrix form and inverting a given 2 × 2 matrix. The areas of weak performance included multiplying two matrices and stating the effect of a matrix transformation on an object.

Solutions:

(a) $\begin{pmatrix} 15 & 39 \\ 12 & 33 \end{pmatrix}$

(b) (i) **V produces an enlargement, scale factor 2, centre of enlargement (0,0)**

(ii) $\begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}$

(iii) **A' (-2,4)** **B' (-2,2)** **C' (-4,2)**

(c) **(i)** $\begin{pmatrix} 11 & 6 \\ 9 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6 \\ 7 \end{pmatrix}$

(ii) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 & -6 \\ -9 & 11 \end{pmatrix} \begin{pmatrix} 6 \\ 7 \end{pmatrix}$

Recommendations

Teachers should emphasize the conditions for multiplication of matrices, noting that not all matrices can be multiplied. The use of matrices in transformations needs to be reinforced.