CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

MAY/JUNE 2008

PURE MATHEMATICS
(Trinidad & Tobago)

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION
MAY/JUNE 2008

INTRODUCTION

This is the first year that the revised syllabus for Pure Mathematics is examined. The new format of Paper 01 is multiple choice (MC) and Papers 02 and 03 have retained the format with extended-response questions. Circumstances dictated that examination papers for the Trinidad and Tobago (T&T) candidates were not the same as those for the Rest of the Region (ROR), nevertheless, the Internal Assessment (IA) of candidates from Trinidad and Tobago was included in the overall IA for the entire region and a common report has been written for that aspect of the examination process. A copy of that report is appended under the heading PAPER 03.

Generally, the performance of candidates was very satisfactory with a number of excellent to very good grades. There still remained, however, too large a number of weak candidates who seemed unprepared for the examination. Approximately 3 500 scripts were marked.

GENERAL COMMENTS

The new topics in the revised Unit I syllabus are Cubic Equations, Indices and Logarithms, and L'Hopital's rule, with Complex Numbers moved to Unit 2. Of these new topics, candidates showed reasonable competence in Cubic Equations and Logarithms, but some seemed not to have been exposed to L'Hopital's rule. Among the old topics comprising Unit 1, candidates continue to experience difficulties with Indices, Mathematical Induction and Summation Notation (Σ). General skills at algebraic manipulation including substitution at all levels continue to pose challenges. A new area of difficulty has emerged, the topic of Trigonometric Identities. Strong performances were recorded in Differentiation, the Plotting of Graphs, Vectors, and Coordinate Geometry. This was encouraging. Some effort should be made in providing students with practice in connecting parts of the same question in order to facilitate efficient solutions.

DETAILED COMMENTS
UNIT 1
PAPER 01

Paper 01 comprised 45 multiple-choice items. Candidates performed satisfactorily. The mean score was 61.0 per cent and standard deviation was 8.6.

PAPER 02
SECTION A
(Module 1: Basic Algebra and Functions)

Question 1

Specific Objective(s): (d) 3, 8, 10; (b) 4; (f) 3, 5; g (2)

This question tested properties of the roots of quadratic equations, cubic equations, the modulus function and graphs.

(a) (i) There were several attempts at this part of the question. Many candidates wrote the condition for real roots without the 'greater than' sign in $b^2 - 4ac \geq 0$ and so did not obtain full marks. Others made errors in simplifying $(-2h)^2$.

(ii) This was one of the new topics and presented some challenges, but there were many encouraging attempts. Very few candidates used the fact that $5(5 - k)(5 + k) = 105$ to obtain the values of k. More practice is recommended.

(b) (i) (iii) This part of the question was very well done. Most candidates did not use the graph to find the values of x in Part (iii), but simply read the values from the table.

Answer(s): (a) (i) $h \geq 4$ **or** $h \leq -8$

(ii) $p = 71$, $k = \pm 2$

(b) (iii) $f(x) = g(x)$ when $x = 0$ **or** $x = 4$

Question 2

Specific Objective(s): (a) 5, 6, 8; (c) 1, 2, 3, 5

This question tested knowledge about indices, logarithms and the principle of mathematical induction.

(a) Several candidates attempted this part of the question, many of whom realized that each term could be expressed in the form 3^x, $x \varepsilon$ N. Due to errors in the algebraic manipulation, only about 50 per cent of those attempting the question obtained the correct answer.

(b) (i) Not many candidates knew how to derive this result although several of them knew how to use it as they demonstrated in Part (ii).

(ii) This was very well done by the many candidates who attempted it, although they found the underlying principle at (i) hard to derive.

(c) There were many good attempts at this question with several candidates obtaining at least 90 per cent of the marks. The step from n = k to n = k + 1, which is the main task in the principle of mathematical induction, still eludes many. More practice is recommended.

Answer(s): (a) $3^4 = 81$

(b) (ii) $y = 5$

SECTION B
(Module 2: Trigonometry and Plane Geometry)

Question 3

Specific Objective(s): (a) 9, 10, 12, 13; (c) 8, 9, 10

This question tested properties of vectors, trigonometric identities and solutions of trigonometric equations.

(a) This part of the question was quite well done although Part (iii) did pose a challenge to a few candidates.

(b) Several successful attempts were made in solving this part, however, some candidates had difficulty expanding cos 2A.

(c) The manipulation of the trigonometric identity troubled some candidates in this part of the question. It was also noted that not many candidates used the 'otherwise' route in Part (iii) to solve $\sin 3\theta = \sin \theta$.
More practice of this type of question and better use of the formula sheet are recommended.

Answer (s): (a) (i) $\lambda = -2$

(ii) $\lambda = 2$

(iii) $\lambda = 4 \pm 2\sqrt{3}$

(c) (iii) $\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

Question 4

Specific Objective(s): (b) 5, 7, 9

The question dealt with tangents to circles and basic properties of circles in the context of coordinate geometry.

(a) (i) Most candidates were able to find the coordinates of P, A and B. Approximately 90 per cent of attempts were successful.

(ii) Several candidates found it difficult to obtain the value of λ. Those who were successful substituted the coordinates of P but made simple errors in extracting the correct value of λ. Many obtained $\lambda = \frac{10}{3}$ instead of $\frac{-10}{3}$.

For those candidates who had a value of λ to carry forward, further marks were obtained from Parts (a) to (d).

(b) (i) Many candidates did not relate the result required to the trigonometric relationship $\sin^2 t + \cos^2 t = 1$ and hence missed out on the simplicity of the process in obtaining the required equation.

(ii) This part of the question was more of a challenge for the candidates. Few obtained full marks.

Answer(s): (a) (i) P $\equiv$ (1, 10), A $\equiv$ (2, 3), B $\equiv$ (6, 5)
(The coordinates of A, B may be interchanged)

(ii) a) $\lambda = -\frac{10}{3}$

b) $3x^2 + 3y^2 - 16x - 40y + 113 = 0$

c) $|PQ| = \frac{5\sqrt{5}}{3}$

d) $|PM| = 3\sqrt{5}$

SECTION C
(Module 3: Calculus I)

Question 5

Specific Objective(s): (a) 4, 7; (b) 8, 9(i), 10, 16; (c) 13, 14, 15

The question examined knowledge about limits, L'Hopital's rule for limits, differentiation of rational functions and maxima in mensuration.

(a) The topic of L'Hopital's rule for finding limits is a new topic in the revised syllabus and some candidates did not seem to be familiar with it. As a consequence, candidates were not penalized for using other methods to solve the particular problem posed.

(b) (i) This part of the question was generally well done. Several candidates obtained full marks for both a) and b).

(ii) Most candidates who attempted this part of the question obtained full marks. Several of them found $\frac{d^2y}{dx^2}$ by differentiating $\frac{dy}{dx}$ as a quotient. A small number of candidates used implicit differentiation to find $\frac{d^2y}{dx^2}$.

(c) (i) This part was very well done with the majority of candidates who attempted it gaining full marks.

(ii) Several candidates succeeded in doing this part correctly although some had difficulty in substituting h in the expression for V. A few did not find the second derivative in order to obtain the value of h for V a maximum.

Answer(s): (a) $\frac{4}{5}$

(b) (i) a) $\frac{dy}{dx} = \frac{1}{(1-4x)^2}$

(c) (ii) $h = 4$

Question 6

Specific Objective(s): (c) 1, 3, 4, 5, 6, 7, 8 (i), 9

The question tested knowledge and skill in differentiation and integration. Generally the question was well done, with approximately 70 per cent of candidates obtaining at least 16 of the 25 marks allocated.

(a) This part of the question was the most challenging for the candidates. Many seemed not to be familiar with integrating functions using 'substitution', which should be a regular procedure for problems whenever the integrand is not straightforward. Many candidates found difficulty manipulating $x\,dx$ to change from the variable x to the variable u. More practice is recommended.

(b) Too many candidates were unable to relate the gradient of the curve with the need for integrating to obtain the equation for the curve. Several candidates treated $x^2 - 4x + 3$ as the function $f(x)$ rather than $f'(x)$ and proceeded in the wrong direction.

(c) (i) Candidates found this part of the question easy, however, some struggled with the algebra involved.

(ii) There were some excellent responses to this part of the question. Common errors included:

a) Incorrect choice of limits
b) Attempting to combine the equations of the line and curve into a single function to integrate
c) Attempting to use approximation to find the area despite the stipulation to obtain the exact value

Answer(s): (a) $\frac{1}{3}\sqrt{3x^2+1}$ + a constant

(b) Equation of C is $\frac{x^3}{3} - 2x^2 + 3x - 1$

(c) (i) A ≡ (1, 3), B ≡ (0, 5), C ≡ (4, 0)

(ii) Exact value of area = 13 units^2

UNIT 1
PAPER 03/B (ALTERNATIVE TO INTERNAL ASSESSMENT)
SECTION A
(Module 1: Basic Algebra and Functions)

Question 1

Specific Objective(s): (a) 7; (c) 2, 3; (d) 7, 9; (f) 5(ii)

This question tested candidates' abilities in solving logarithmic equations, and their knowledge of the factor theorem, concept of a decreasing function, and the sigma notation relating to an arithmetic progression.

(a) (i) This part of the question was satisfactorily done.

(ii) This part of the question was not well done. Candidates found it difficult to express the given logarithmic equation in index form thus allowing for the solution of a simple linear equation.

(b) (i) A significant number of candidates were unable to use the intercepts of the curve to determine the constants required. Preparation for specific topics seem to be stereotype. Apparently, the use of the factor theorem is studied without any reference to the relationship of a curve and its intercepts.

(ii) Most of the candidates stated the range as seen on the graph but included the point where $x = -1$. Instances were seen where candidates attempted differentiation of f(x) to find the required range. More practice on graphs and how to use graphs to determine some features of a function should be done.

(iii) This part of the question was satisfactorily done.

Answer(s): (a) (i) p

(ii) $x = 9$

(b) (i) $h = 4,\ k = -1,\ m = -2$

(ii) $-1 < x \leq 0$

(c) 15 350

SECTION B
(Module 2: Trigonometry and Plane Geometry)

Question 2

Specific Objective(s): (b) 1, 2, 6, 7, 9.

This question tested candidates' abilities to determine a Cartesian curve from given parametric equations, finding a tangent and a normal to a Cartesian curve in linear form and in terms of its parameter, intersection of a line and a curve, and the distance between two points on a curve.

The majority of candidates performed poorly with approximately 10 per cent of them giving no responses.

Symbolic representation and application of the given data as required was a big challenge to most candidates. The algebraic skills demonstrated were very weak.

Such candidates require more preparation and practice to perform satisfactorily at these examinations.

Answer(s): (b) (ii) $y + t_1x - at_1^3 - 2at_1 = 0$

(iv) $2a(1 + t_1^2)$

SECTION C
(Module 2: Calculus1)

Question 3

Specific Objective(s): (a) 4, 5; (c) 2, 4, 5, 6; (b) 11.

This question tested the concept of limits and of definite integration, as well as the use of a simple model involving rate of change.

(a) (i) This part of the question was well done since candidates were given a useful hint which simplified the rational function.

(ii) Most candidates were able to follow through with the result from (i) to perform well on this part of the question.

(b) (i) This part of the question was satisfactorily done by most of the candidates.

(ii) Some candidates had difficulty separating the integral and using the result given for $\int_1^4 f(x)\,dx = 7$. In addition candidates could not use the fact that for a continuous function $\int_1^2 f(x)\,dx + \int_2^4 f(x)\,dx = \int_1^4 f(x)\,dx$. Candidates therefore could not obtain the correct answer. A number of candidates attempted integration of the problem in the form given with obvious difficulties.

(c) (i) The majority of candidates merely found $\frac{dV}{dh}$, apparently not aware that finding $\frac{dV}{dt}$ required multiplication of $\frac{dV}{dh}$ by $\frac{dh}{dt}$.

(ii) The candidates were required to find $\frac{dh}{dt}$ but many of them failed to do so since their result at (i) was incorrect. There were no correct responses to this part of the question.

Answer(s): (a) (i) $\frac{1}{6}$

(ii) $\frac{1}{48}$

(b) (i) $u = 2$

(ii) 4

(c) (i) $\frac{dV}{dt} = \frac{1}{3}\pi\left(48h - 3h^2\right)\times\frac{dh}{dt}$

$= \pi h(16 - h) \times \frac{dh}{dt}$

(ii) $\frac{25}{7\pi}$ cms^{-1}

GENERAL COMMENTS

UNIT 2

Topics satisfactorily covered were those relating to solution of Exponential Equations, Calculus of Composite Functions, (including Inverse Trigonometric Functions), First-order Differential Equations, Solution of Second-order Differential Equations, Series, Mathematical Induction, Permutations and Simple Probability, Approximations to Roots of Equations, Series, Complex Numbers (including De Moivre's theorem), and Matrix Algebra.

This examination tested the new topics which included Calculus of Inverse Trigonometrical Functions and the Second Derivative, the use of an Integrating Factor for First-order Differential Equations, Second-order Differential Equations, Maclaurin's Theorem for Series Expansions, Binomial Expansion Series, Reduction to Row-Echelon Form, and Row Reduction of an Augmented Matrix, Complex Numbers with application of Demoivre's Theorem for integral n.

The majority of candidates continue to display weaknesses in tasks requiring algebraic manipulation or involving substitution. It is imperative that more emphases be placed on these areas of weaknesses. Extensive practice in the use of substitution and algebraic manipulation is demanded if candidates are to be well prepared to show improved performances in these areas. Candidates continue to demonstrate a lack of appreciation for questions which allow for "hence or otherwise". They fail to see existing links from previous parts of the questions and never seem disposed to using "otherwise" thus employing any other suitable method for solving the particular problem.

UNIT 2
PAPER 01

Paper 01 comprised 45 multiple-choice items. The candidates performed satisfactorily. The mean score on this paper was 68.4 per cent and the standard deviation was 8.6.

UNIT 2
PAPER 02
SECTION A
(Module 1: Calculus II)

Question 1

Specific Objective(s): (a) 7, 9; (b) 1, 2, 3, 5, 6, 7.

This question tested differentiation of various functions, exponential, logarithmic, and inverse trigonometrical, as well as parametric equations, and distinguishing a point of inflexion.

This question was generally well done by the majority of candidates. The average marks obtained were within the range 17 – 20 from a maximum of 25 marks.

(a) Most of the candidates gained full marks for this part of the question. A small number of candidates found it difficult to solve the quadratic equation obtained in terms of e^x. Some candidates attempted to take the natural log of each term with the obvious difficulties experienced.

(b) This part of the question was well done. All the candidates used second differentiation to prove the result. No candidate attempted to use implicit differentiation.

(c) (i) Generally, responses to this part of the question were good. Some candidates had difficulties with the Multiple Composite Functions. An application of the product rule over three terms is not a regular feature and more practice would be needed in this regard. $\frac{d}{dx}\sin^{-1}(2x)$ was not well done. Most candidates used the result of $\frac{d}{dx}\sin^{-1}(x)$ without paying attention to the composite $2x$. With the testing of this new topic it was not unexpected that lack of adequate practice would be evident.

(ii) a) This part of the question was well done. A few candidates attempted to set t in terms of x and y before differentiation. Clearly, they had an idea but failed to develop it successfully.

b) Very few candidates obtained full marks for this part of the question. The majority of candidates set $\frac{dy}{dx} = 0$ to find the point of inflexion. Those candidates who attempted to find $\frac{d^2y}{dx^2}$ merely found $\frac{d}{dx}\left(\frac{dy}{dx}\right)$ and not multiplying by $\frac{dt}{dx}$. In fact distinguishing a point of inflexion appeared to be new to most of the candidates. A lot of practice is required in this regard.

Answer(s): (a) $x = \ln 7, x = 0$

(c) (i) $\ln x\left(\frac{2x}{\sqrt{1-4x^2}} + \sin^{-1} 2x\right) + \sin^{-1} 2x$

Question 2

Specific Objective(s): (c) 1, 3, 6, 8, 11, 12 (ii)

This question required candidates to evaluate an indefinite integral using integration by parts, solving a first-order differential equation using an integrating factor, the general
solution of a second-order differential equation with the principal integral being a trigonometric function, resolving partial fractions, and integration involving $\int \frac{f'(x)}{f(x)} dx$.

The majority of candidates attempted this question. The average range of marks obtained was 15 – 20 with a satisfactory number of candidates earning marks in the range 21 – 25 out of a maximum of 25 marks.

(a) (i) Approximately 30 per cent of the candidates who attempted this question obtained full marks. The substitution $u = \ln x$ was widely used. The practice of not stating the constant of integration continues to be a source of concern. Emphasis must be placed on this aspect for candidates to appreciate the importance of this constant and to earn full marks.

(ii) Some candidates had problems finding the correct integrating factor. They also failed to write the equation in the form $I\frac{dy}{dx} + Iy = I \ln x$. However, the majority of candidates seemed to have grasped the concept of using an integrating factor.

(b) (i) Most candidates successfully found the first and second derivatives of $m \cos x + n \sin x$. However, too many of these candidates made simple errors in calculating the values of m and n.

(ii) Candidates had no difficulties finding the complementary function correctly. Due to errors made in (i) some marks were lost overall.

(c) (i) The majority of candidates performed well in this part of the question.

(ii) Generally most of the candidates who were successful in Part (i) were able to integrate correctly. A very small number of candidates mistakenly found . . . $\int \frac{3x}{(x^2 + 1)} dx$ as arctan x. Candidates are guilty of omitting the constant of integration.

Answer(s): (a) (i) $\frac{1}{2}\{\ln(x)\}^2 + C$

(ii) $xy = \frac{1}{2}\{\ln(x)\}^2 + C$

(b) (i) $m = 2,\ n = 1$

(ii) $y = Ae^{3x} + Be^{x} + 2\cos x + \sin x$

(c) (i) $\dfrac{2}{x-1} - \dfrac{3x}{x^2+1}$

(ii) $2\ln|x-1| - \dfrac{3}{2}\ln|x^2+1| + C$

SECTION B
(Module 2: Sequences, Series and Approximations)

Question 3

Specific Objectives: (b) 1, 3, 6, (e) 1, 2, 4

This question tested the candidates' abilities with respect to arithmetic progressions, the principle of mathematical induction, the intermediate value theorem for the existence of a real root, and the Newton-Raphson method.

The majority of candidates attempted this question. The range of marks obtained for this question was between 10 and 20 out of a maximum of 25 marks.

(a) (i) a) Most candidates used the approach of evaluating the sums for S and T by using formulae stated in the Formulae Booklet. However, they did not understand the concept tested and failed to gain marks for this part of the question.

b) Candidates, having failed to answer (a) correctly, proceeded to find the sum of the arithmetic series, S, using the formula stated in the Formulae Booklet. This type of candidate needs adequate practice to be proficient with algebraic manipulation and the deductions made from these manipulations.

(ii) Most candidates demonstrated a sound understanding of the principle of mathematical induction. However, some candidates are still unclear of the inductive process and failed at the step where the assumption that P_k is true is used to show that P_{k+1} is true for n = some k. More work on the principle of mathematical induction is required. Some candidates simply substituted $k + 1$ for k in the statement for P_k. Candidates also failed to express $(k + 2)(2k + 3)$ in the form $[(k + 1) + 1][2(k + 1) + 1]$ to show that the statement P_{k+1} is of the form P_k for $n = k + 1$. It is clear, however, that candidates are becoming more adept at applying the principle of mathematical induction.

(iii) This part of the question required the use of the substitution of the formulae for $\sum_{r=1}^{n} r^2$ *and* $\sum_{r=1}^{n} r$, and simplifying to get the answer. A number of candidates mistakenly applied the principle of mathematical induction to prove the result. Those candidates who used substitution of the formulae obtained full marks.

(b) (i) Most candidates found f (0) and f (1), concluding that since there was a sign change that condition was sufficient for the existence of a real root in the interval (0, 1). Candidates were not aware that the function must be continuous in that interval to use the intermediate value theorem. Many candidates failed to obtain full marks due to this omission.

(ii) The majority of candidates were not able to use the concept of differentiation to show that the function was continuously increasing, hence the existence of only one real root. Some candidates very logically showed by way of two graphs that $f(x) = x^3$ and $g(x) = 3 - 6x - 3x^2$ had only one point of intersection. A few candidates used the roots of a cubic equation to show that the cubic equation had one real root and two complex roots.

(iii) More than 60 per cent of the candidates were able to obtain maximum marks for this part of the question. Very few candidates showed weaknesses in applying the Newton-Raphson method.

Answer(s): (b) (iii) 0.41

Question 4

Specific Objective(s): (a) 1, 2; (c) 1, 3; (e) 3, 4

This question tested the use of sequences defined by recurrence relations, simple algebraic proofs, and the binomial theorem. The average range of marks for this question was between 10 and 15 from a maximum of 25 marks. Generally the responses to this question were unsatisfactory.

(a) (i) Because of arithmetical errors, a few candidates did not earn full marks for this part of the question.

(ii) A number of candidates showed weaknesses in basic algebraic simplification and correct numerical answers.

(iii) a) b) Most candidates were unable to answer this part of the question logically. Simple algebraic proofs continue to be problematic and much more is required in this regard.

(b) The majority of candidates did well on this topic. Generally, most of them used an inspection method to determine the term independent of x. In fact, few candidates used the binomial expansion to determine the term required. Emphasis must be placed on the binomial theorem.

(c) A significant number of candidates found this part of the question difficult. Many of them simply used the calculator to find the difference. The use of the binomial expansion series for approximations seemed unfamiliar to most of the candidates. Practice in this regard is necessary.

Answer(s): (a) (i) $\frac{20}{11}, \frac{31}{16}$

(ii) $\frac{2(a_n-2)}{4+a_n}$

(b) $\frac{15!}{6!\,9!}\left(6^6\right)$

(c) 10.28620

SECTION C
(Module 3: Counting, Matrices and Complex Numbers)

Question 5

Specific Objective(s): (a) 1, 2, 7, 8, 9; (b) 2; (c) 1, 2, 4, 5, 11

This question tested selections using permutations, classical probability, complex numbers including De Moivre's theorem, and matrix algebra. The overall performance by candidates was satisfactory. The average range of marks was between 10 and 20 from a maximum 25 marks.

(a) (i) a) Most candidates responded well to this part of the question. A few candidates found distinguishing between combinations and permutations rather challenging.

b) The majority of candidates gave satisfactory responses to this part of the question.

(ii) This part of the question was well done.

(b) Candidates demonstrated a good understanding of probability theory, including the use of Venn diagrams and laws of probability.

(c) (i) A majority of candidates substituted 3 + 4i into the equation but had difficulties comparing coefficients since many of them made arithmetic errors in the expansions. Very few candidates used the principles of complex conjugate and the sum and product of roots of a quadratic equation. Not many candidates were able to obtain full marks.

(ii) The majority of candidates demonstrated an understanding of De Moivre's theorem. However, many of them made errors in the expansion, particularly the terms involving i^2. A number of candidates were unaware that they had to consider the real part of the expansion for cos 3θ. Very rarely did candidates define the complex number cos θ + i sin θ as z, cos θ - i sin θ as $\frac{1}{z}$ and used the principle of $\left(z + \frac{1}{z}\right) = 2\cos\theta$. More practice in this topic will improve candidates' understanding and performance.

Answer(s): (a) (i) a) $6^4 = 1296$

b) 360

(ii) $\frac{1}{3}$

(b) $\frac{13}{28}$

(c) (i) $h = -6, k = 25$

Question 6

Specific Objective(s): (b) 1, 2, 7, 8.

This question tested matrix algebra including solution of a variable for a singular matrix, multiplication of conformable matrices, finding the inverse of a non-singular matrix, and solution of a system of equations using matrix algebra. Approximately 20 per cent of the candidates gained full marks on this question. The average range of marks for this question was between 10 and 25 from a maximum of 25 marks.

(a) The majority of candidates performed well in this part of the question. Some candidates made errors in the cubic expansion for x and subsequently found it difficult to factorize the cubic equation correctly.

(b) (i) This part of the question was well done.

(ii) a) This part of the question was well done. Very few candidates made some arithmetic errors in multiplication.

b) A number of candidates failed to deduce the inverse of A from the result at a). Many of them went on to calculate A^{-1} as $\frac{1}{|A|}$ adj A. This showed a weakness in understanding the concepts involved. Much practice in this topic will improve performance.

(iii) This part of the question was well done by candidates who deduced or otherwise found A^{-1} correctly. However, some candidates failed to obtain full marks because they included the number of coaches in their answers.

Answer(s): (a) $x = 1, 2, -3$

(b) (i) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 34 \\ 49 \\ 71 \end{pmatrix}$

(ii) a) $AB = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ b) $\frac{1}{2}B = \begin{pmatrix} -2 & 0 & 1 \\ 0 & 3 & -2 \\ 1 & -2 & 1 \end{pmatrix}$

(iii) 24

UNIT 2
SECTION A
PAPER 03/B (ALTERNATIVE TO INTERNAL ASSESSMENT)
(Module 1: Calculus II)

Question 1

Specific Objective(s): (a) 5, 6, 7, 8; (c) 5

This question examined an exponential and a logarithmic expression, integration by parts of a trigonometric function, and an exponential model. Overall this question was poorly done.

(a) Most of the candidates who attempted this part of the question did not demonstrate an understanding of exponential functions and of natural logarithms. This part of the question was poorly done.

(b) Candidates could not separate $\cos^3 x$ as $\cos x\,(1 - \sin^2 x)$ in order to substitute $\cos x\, dx$ for du. Many of the candidates only found $du = \cos x\, dx$. No candidate got marks beyond this point.

(c) (i) Many candidates failed to use the fact of $t = 0$ to find the answer to this part of the question.

(ii) Candidates were required to find the value of the constant k before proceeding to find the answer to this part of the question. However, substitution and subsequent solution proved beyond the ability of most of the candidates.

Answer(s): (a) $y = \frac{1}{2}\left(e^{x} + e^{-x}\right)$

(b) $\sin x - \frac{1}{3}\sin^{3} x + C$

(d) (i) 70° C

(ii) 7.5 minutes

SECTION B
(Module 2: Sequences, Series and Approximations)

Question 2

Specific Objective(s): (b) 3, 11, 12

This question tested the sum of a convergent series using the method of differences, and a model involving a geometric progression. The overall response was poor. Candidates seemed generally unprepared.

(a) (i) Candidates found it difficult to express the general term of the series. They appeared unfamiliar with patterns and sequences.

(ii) Follow through from (i) was not possible since the majority of candidates did not get the correct partial fractions to work with.

(iii) Most of the candidates did not respond to this part of the question. The few who did could not determine the nature of the series since answers to (i) and (ii) were either non-existent or wrong.

(b) (i) (ii) Overall they were very few and very poor responses to this part of the question. Analysis of the problem proved to be challenging for the candidates.

Answer(s): (a) (i) $\frac{1}{(2r-1)(2r+1)}$

(ii) $\frac{1}{2} - \frac{1}{2(2n+1)}$

(iii) $\frac{1}{2}$

(b) (i) $\$\left(100 + \frac{1}{10}r^{2}\right)$ (ii) \$6 205

SECTION C
(Module 3: Counting, Matrices and Complex Numbers)

Question 3

Specific Objective(s): (b) 7, 8

This question tested complex numbers and matrix algebra. Candidates showed a fair understanding of these topics. However, some candidates had difficulty finding the inverse of the invertible matrix.

(a) (i) This part of the question was well done by the majority of candidates.

(ii) Very few candidates made arithmetical errors in this part of the question. Generally, this part of the question was well done.

(b) (i) A significant number of candidates did not demonstrate a clear understanding of the process required to find the inverse of an invertible matrix. No candidate attempted the row reduction of an augmented matrix of the identity matrix.

(ii) There were no correct responses to this part of the question.

Answer(s): (a) (i) $\frac{-13}{2} - \frac{9}{2}i$

(ii) $5\sqrt{\left(\frac{5}{2}\right)}$

(b) (i) $\frac{1}{5}\begin{pmatrix} 8 & 7 & -6 \\ -2 & 2 & -1 \\ -5 & -5 & 5 \end{pmatrix}$

(ii) $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$

PAPER 03 – INTERNAL ASSESSMENT

The Internal Assessment comprises three Module tests.

The main features assessed are the:

- Mapping of the items on the Module tests to the specific objectives in the syllabus for the relevant Unit
- Content coverage of each Module test
- Appropriateness of the items tested for the CAPE level
- Presentation of the sample (Module tests and students' scripts)
- Quality of the teachers' solutions and mark schemes
- Quality of the teachers' assessments – consistency of marking using the mark schemes
- Inclusion of mathematical modelling in at least one Module test for each Unit

GENERAL COMMENTS

1. Too many of the Module tests comprised items from CAPE past examination papers.

2. Untidy 'cut and paste' presentations with varying font size were common place.

3. This year there was a general improvement in the creativity of the items, especially with regards to mathematical modelling. Teachers are reminded that the CAPE past examination papers should be used ONLY as a guide.

4. The stipulated time for Module tests (1-$1^1/_2$hours) must be strictly adhered to as students may be at an undue disadvantage when Module tests are too extensive or insufficient. The following guide can be used: 1 minute per mark. About 75 per cent of the syllabus should be tested and mathematical modelling **MUST** be included.

5. Multiple-choice Questions will **NOT** be accepted in the Module tests.

6. Cases were noted where teachers were unfamiliar with recent syllabus changes, for example, complex numbers, 3-dimensional vectors, dividing a line segment internally or externally.

7. The moderation process relies on the validity of teacher assessment. There were a few cases where students' solutions were replicas of the teachers' solutions – some containing identical errors and full marks were awarded for incorrect solutions. There were also instances where the marks on the students' scripts did not correspond to the marks on the Moderation sheet.

8. Teachers **MUST** present evidence of having marked each individual question on the students' scripts before a total is calculated at the top of the script. The corresponding whole number score out of 20 should be placed at the front of the students' scripts.

9. To enhance the quality of the design of the Module tests, the validity of teacher assessment and the validity of the moderation process, the Internal Assessment guidelines are listed below for emphasis.

Module Tests

(i) Design a separate test for each Module. The Module test MUST focus on objectives from that module.

(ii) In cases where several groups in a school are registered, the assessments should be coordinated, common tests should be administered, and a common marking scheme used. One sample of FIVE students will form the sample for the centre.

(iii) In 2009, the format of the Internal Assessment remains unchanged.

[Multiple Choice Examinations will NOT be accepted].

GUIDELINES FOR MODULE TESTS AND PRESENTATION OF SAMPLES

1. COVER PAGE TO ACCOMPANY EACH MODULE TEST

The following information is required on the cover of each Module test.

- Name of School and Territory, Name of Teacher, Centre Number.
- Unit Number and Module Number.
- Date and duration (1-1$^1/_2$ hours) of Module Test.
- Clear instruction to candidates.
- Total marks allotted for the Module Test.
- Sub – marks and total marks for each question MUST be clearly indicated.

2. COVERAGE OF THE SYLLABUS CONTENT

- The number of questions in each Module test must be appropriate for the stipulated time of 1-1$^1/_2$hours.
- **CAPE past examination papers should be used as a guide ONLY**.
- Duplication of specific objectives and questions must be avoided.
- Specific objectives tested must be from the relevant Unit of the syllabus.

3. MARK SCHEME

- Detailed mark schemes MUST be submitted, holistic scoring is not recommended that is, **one mark per skill should be allocated.**
- FRACTIONAL DECIMAL MARKS MUST NOT BE AWARDED.
- The total marks for Module tests MUST be clearly stated on the teacher's solution sheets.
- A student's marks MUST be entered on the FRONT page of the student's script.
- Hand written mark schemes MUST be NEAT and LEGIBLE. The marks MUST be presented in the right hand side of the page.
- Diagrams MUST be neatly drawn with geometrical/mathematical instruments.

4. PRESENTATION OF THE SAMPLE

- Student's responses MUST be written on normal sized paper, preferably 8 $^1/_2$ ×11.
- Question numbers are to be written clearly in the left margin.
- The total marks for each question on students' scripts MUST be clearly written in the right margin.
- ONLY original students' scripts MUST be sent for moderation. **Photocopied scripts will not be accepted.**
- Typed Module tests MUST be in a legible font size (for example, size 12). Hand written tests MUST be NEAT and LEGIBLE.
- The following are required for each Module test:

  - A question paper.
  - Detailed solutions with detailed mark schemes.
  - The scripts (for each Module) of the candidates comprising the sample. The scripts MUST be collated by Modules.

- Marks recorded on the PMath 1 – 3 and PMath 2 – 3 forms must be rounded off to the nearest whole number.
- The guidelines at the bottom of these form should be observed. (see page 57 of the syllabus, no.6).
- In cases where there are five or more candidates, FIVE samples MUST be sent.
- In cases where there are five or less registered candidates, ALL samples MUST be sent.