

CARIBBEAN EXAMINATIONS COUNCIL

**REPORT ON CANDIDATES' WORK IN THE
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®**

MAY/JUNE 2012

APPLIED MATHEMATICS

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GENERAL COMMENTS

The revised Applied Mathematics syllabus was examined this year for the fourth time. This is a two-unit course comprising three papers. Paper 01, consisting of 45 multiple choice items, and Paper 02, consisting of six essay questions were examined externally, while Paper 03 was examined internally by class teachers and moderated by CXC. Contributions from Papers 01, 02 and 03 to each unit were 30 per cent, 50 per cent and 20 per cent, respectively.

Unit 1, Statistical Analysis, tested Collecting and Describing Data, Managing Uncertainty and Analysing and Interpreting Data.

Unit 2, Mathematical Applications, tested Discrete Mathematics, Probability and Probability Distributions and Particle Mechanics.

In 2012, the number of candidates writing the examinations for Units 1 and 2 were 672 and 237, respectively.

Approximately 80 per cent of the candidates registered for Unit 1, Statistical Analysis, and 83 per cent of the candidates registered for Unit 2, Mathematical Applications, obtained acceptable grades, I–V. The standard of work seen from most of the candidates in this examination was satisfactory.

In Unit 1, candidates appeared to be well prepared in Describing and Collecting Data and somewhat in Managing Uncertainty, while Analysing and Interpreting Data seemed to be a challenge for many candidates.

In Unit 2, candidates appeared to be well prepared in Discrete Mathematics and Probability and Distributions. Particle Mechanics again seemed to be a challenge for many candidates.

Generally, candidates are still having difficulty with algebraic manipulations.

Overall, for Paper 031, the School-Based Assessment (SBA), students demonstrated a high degree of mastery in the mathematical principles pertaining to the syllabus. In most cases, the mathematical analyses were relevant and carried out with few flaws.

There was evidence of originality and creativity and projects were appropriately applied to real-world problems and situations. Over 90 per cent of the students were able to effectively communicate information in a logical way using correct grammar and mathematical language.

DETAILED COMMENTS

UNIT 1

Paper 01 – Multiple Choice

Performance on the 45 multiple choice items on Paper 01 produced a mean score of 57.15, standard deviation of 18.15 and scores ranging from 14 to 90.

Paper 02 – Essay

Module 1: Collecting and Describing Data

Question 1

This tested the candidates' ability to

- distinguish between population and a sample, a census and a sample survey, a parameter and a statistic
- define a sample frame
- distinguish between random and non-random samples
- identify different sampling methods
- calculate the number of items in a sample
- construct a stem and leaf diagram
- calculate the inter-quartile range
- determine the median from a set of data.

Though this question was generally well done, many candidates could not identify cluster sampling. Most candidates were able to identify systematic sampling.

Candidates had difficulty explaining the advantages of using different methods of sampling.

Most candidates were able to calculate the number of workers in the sample using option B.

For Part (b), many candidates could not give a definition for a sampling frame. For Part (c), the stem and leaf diagram was generally well done, but then candidates could not state the advantage of using the stem and leaf diagram.

In determining the median, candidates were able to state the location of the median, but did not state the median.

Most candidates knew how to find the IQR, but many of them read the values for Q_1 and Q_3 incorrectly.

Answers

- (a) (i) A: Cluster sampling, non-random
 B: Stratified sampling, random
 C: Systematic sampling, random
- (ii) Not all workers will have a chance of being selected
- (iii) Possibility of bias
- (iv) 16 workers.
- (b) (i) Sampling frame is a set of elements from the population from which the sample is drawn
- (ii) Households may not have a telephone
- (ii) Only persons listening to the programme will call
- (c) (i) Stem Leaf
- | | | |
|---|-----------------|---------------|
| 4 | 0 2 7 | |
| 5 | 3 6 7 8 8 9 9 | |
| 6 | 0 1 2 3 4 6 7 9 | |
| 7 | 1 4 4 5 | |
| 8 | 1 2 2 | key: 7 4 = 74 |
- (ii) All data values are preserved
- (iii) Median = 62; IQR = 15

Question 2

This question tested candidates' ability to

- identify a population, census, sample, statistic, parameter
- determine the mode and median from a frequency distribution
- calculate the mean and the standard deviation from a frequency distribution
- use cumulative frequency curves.

Calculation of the standard deviation was a problem since many candidates could not correctly quote the formula for standard deviation.

Candidates also had problems identifying the normal distribution. Candidates used descriptions such as ‘symmetric to left and to the right’, ‘normally symmetric’, ‘left skewed normally’.

Whereas candidates could read values from the graph, using the graph to calculate values proved to be a challenge.

Answers

- (a) Population, census, sample, statistics, parameter
- (b) (i) Mode = 3, Median = 3
- (ii) Mean = 3; SD = 1.48
- (iii) The distribution is normal
- (c) 350; 35; 7.14 per cent IQR = 14; $x = 36.5$ minutes

Module 2

Question 3

This question tested candidates’ ability to

- calculate the probability of (i) the union of two events: $P(P \cup Q)$ and (ii) the conditional probability of two events $P(P|Q)$
- identify (i) independent and (ii) mutually exclusive events
- solve probability problems
- use a probability distribution table to obtain probabilities.

Parts (a) (i) a) was attempted by approximately 96 per cent of the candidates, with about 98 per cent giving a satisfactory response.

Part (a) (i) b) was attempted by approximately 96 per cent of candidates, with about 85 per cent giving a satisfactory response. Some candidates gave the incorrect definition for

$$P(R|Q) = \frac{P(R \cap Q)}{P(R)} \text{ or } \frac{P(R)}{P(Q)} \text{ or } \frac{P(Q)}{P(R)} .$$

Parts (a) (ii) a) and b) were attempted by approximately 96 per cent of candidates, with about 70 per cent giving a satisfactory response. Many candidates were unable to state their

answers in clear mathematical terminology (jargon) and provide correct reasons to support their answers.

Parts (b) (i) a) and b) were attempted by approximately 90 per cent of candidates, with less than 50 per cent providing satisfactory responses. Many candidates were unable to distinguish between Parts (a) and (b). This indicated that candidates lack proper understanding of the concept $P(F' \cap E')$ and $P(E \text{ only or } F \text{ only})$.

Part (b) (i) c) was attempted by approximately 90 per cent of candidates, with more than 80 per cent providing satisfactory responses. However, many candidates did not approximate correctly to three significant figures.

For Part (b) (ii), most candidates were unable to identify the concept of combined probability; as a result, less than 30 per cent of candidates who attempted this part provided satisfactory responses.

Parts (c) (i) and (ii) were generally well done, with approximately 96 per cent of candidates attempting and 98 per cent giving satisfactory responses. However, a few candidates were unable to identify and use the correct formula for the mean; they attempted to divide the mean by the total number of orders.

Answers

(a) (i) 0.88; 0.2 (ii) Not independent; Not mutually exclusive

(b) (i) 0.07; 0.65; 0.483 (ii) 0.21

(c) (i) 0.45 (ii) 6.48

Question 4

This question tested candidates' ability to

- identify a binomial distribution and state its parameters
- calculate probability using the binomial distribution
- use the normal distribution to calculate probabilities.

Most candidates attempted Part (a) (i); however, the parameters in some cases were either interchanged or incorrect. Some candidates were unable to state the distribution as binomial, using normal notation instead.

In Part (a) (ii) a), most candidates attempted to use the formula; however, values used were incorrectly substituted. In some cases, candidates incorrectly stated the formula

$$P(X = r) = C_r^n p^r q^{n-r} .$$

For Part (a) (ii) b), several candidates failed to use the property $P(X \geq 1) = 1 - P(X < 1)$ or $1 - P(X = 0)$. In many instances $1 - P(X = 1)$ or $1 - [P(X = 0) + P(X = 1)]$ was used.

The majority of candidates attempted Part (a) (iii) but did not do the entire question. They were able to correctly use the formula for $E[X]$.

For Part (b) (i), most candidates had a general understanding of standardizing; however, some used the wrong values at times. Candidates who demonstrated an understanding of this concept rounded off their values too early; this had an effect on their final answers. Some also used continuity correction which was unnecessary. The remaining set of candidates was unable to arrive at the correct answer because of standardizing using the variance rather than standard deviation or finding the root of 14.5 in a few cases. Also, some candidates did not know to subtract their value from 1. Another common mistake was misreading the values from the table or adding values incorrectly; for instance, for $\phi(1.379) = 0.9147 + 0.0014$ the expression $0.9147 + 0.14$ was frequently seen.

In Part (b) (ii), some candidates were able to formulate the statement $P(X > c) = 0.20$ and standardize; however, others read the table for $\phi(0.2)$ or even if they arrived at the correct conclusion $\phi^{-1}(0.8)$, they could not read the tables successfully. Candidates were able to derive the equation $c = (0.842 \times 14.5) + 630$, but in the other cases $c = 630 - (0.842 \times 14.5)$ was seen.

Most candidates failed to accurately complete Part (b) (iii). Again, continuity correction was commonly seen and many candidates were able to standardize but as in Part (b) (i), candidates rounded off too early so again table values were incorrect. Also, candidates failed to demonstrate an understanding of solving probabilities using the normal distribution. For instance, $P(-1.379 < Z < 1.379) = 2(1 - \phi(1.379))$ was frequently observed.

Very few candidates had difficulty understanding what was required for Part (b) (iv). The answer calculated in Part (b) (iii) was correctly used to solve this part of the question.

Answers

(a) Bin (12, 0.45); 0.0923; 0.999; 18

(b) 0.0839; 642.2; 0.8332; 54.

Module 3

Question 5

The question tested candidates' ability to

- state the mean, variance and distribution of the mean, \bar{x} of the sample
- calculate the probabilities that the sample mean is less than a given value
- calculate a 98 per cent confidence interval for the mean length
- calculate an unbiased estimate for the standard deviation
- state the condition necessary for a valid use of a t-test to test the hypothesis about the mean of x
- state the null hypothesis H_0 and an alternative hypothesis H_1
- apply a one-tailed test
- determine the critical value from tables for a given test and level of significance
- identify the critical region for a given test
- state a valid conclusion of the given test.

In Part (a) (i), many candidates were able to correctly identify the normal distribution and state the mean. However, they had difficulty identifying the variance.

In Part (a) (ii), many candidates used the incorrect value $\frac{\delta^2}{\sqrt{n}}$ rather than $\frac{\delta}{\sqrt{n}}$. However, most of them were able to read off the table values as well as apply $1-\phi(z)$ to calculate the probabilities.

In Part (b), many candidates used a one-tailed test as opposed to the two-tailed test. Many candidates also used the variance instead of the standard deviation. As a result, many candidates did not obtain the interval (75.73, 76.07).

In Part (c) (i), many candidates were able to write a correct formula. However, depending on the formula that they used, they did not use the $\frac{n}{n-1}$ factor. However, most candidates found the square root of their answer.

Part (c) (ii) was well answered and most candidates were able to identify two conditions necessary for the t-test; many candidates did not state that the distribution must be normal.

For Part (c) (iii) a), most candidates were able to set up the hypothesis correctly but neglected to test for μ , for example, $H_0 : x = 3.5$ and $H_1 : x < 3.5$. Many candidates used statements rather than symbols, but did not state that it was the mean battery life that was being tested. Several candidates used a two-tailed test.

In Part (c) (iii) b), many candidates used their unbiased estimation from Part (c) (i) and obtained an incorrect t_{calc} . Several candidates used $\frac{\delta^2}{\sqrt{8}}$ in the denominator. Several candidates obtained a positive value for t_{calc} because they used (3.5 - 3.4).

In Part (c) (iii), although many candidates were able to identify the correct degrees of freedom and read the correct table value of 1.895, they did not identify the critical region.

In Part (c) (iii), most candidates were able to state whether the null hypothesis was rejected or accepted, but could not interpret this with a valid conclusion.

Answers

$$(a) \bar{X} \approx N\left(420, \frac{12.7^2}{49}\right); 0.0491$$

$$(b) (75.73, 76.07)$$

(c) (i) 0.2; (ii) Small sample size, variance unknown, normal distribution (iii) Accept $H_0: \mu = 3.5$; the advertisement is not overstating μ

Question 6

This question tested candidates' ability to

- formulate a null hypothesis H_0 , and an alternative hypothesis H_1
- relate the level of significance of the probability of rejecting H_0 given that H_0 is true
- determine the critical values from tables for a given test and level of significance
- determine the degrees of freedom
- determine probabilities from χ^2 - tables
- apply a χ^2 test for independence in a contingency table
- draw scatter diagrams to represent bivariate data
- give a practical interpretation of the regression coefficient
- draw the regression line of y on x passing through (\bar{x}, \bar{y}) on a scatter diagram
- make estimations using the appropriate regression line.

This question was attempted by 95 per cent of the candidates. Eighty per cent of them performed exceptionally well on this question.

For Part (a) (i), most candidates gave appropriate responses for the hypothesis. Few of the candidates however misrepresented the null and the alternative hypothesis.

In Part (a) (ii), most candidates demonstrated the ability to calculate the degree of freedom using the correct formula. Some candidates included the totals in their rows and columns during their calculations.

In Part (a) (iii), candidates demonstrated their understanding of the critical region using either a graphical or calculated approach. Some candidates, after identifying the critical value, did not convert this to a critical region.

In Part (a) (iv), the majority of candidates was able to complete the table effectively. For Part (a) (v), most candidates were able to give an appropriate conclusion using their results from Part (a) (i) and (ii).

In Part (b) (i), the majority of candidates demonstrated their ability to plot six or more points correctly. For Part (b) (ii) a), only a minority of candidates were able to demonstrate their understanding of the x coefficient and its relationship to the rainfall at the two stations. In Part (b) (ii) b), most candidates were able to state the formula for mean and use it correctly.

In Part (c), candidates were able to plot the mean on their graph but a significantly large number of candidates were unable to draw the regression line correctly.

In Part (d), candidates demonstrated their knowledge to find y given x either graphically or by calculations.

Answers

(a) $\chi^2 \geq 5.991$, (v) reject H_0 ; opinion is dependent of sex.

(b) (ii) For every 1 cm increase of rainfall at Station A there will be a 0.8 cm increase at Station B

(iii) Mean rainfall at A = 4.3; Mean rainfall at B = 3.87

(c) $y = 3.98$

Paper 031 – School-Based Assessment (SBA)

Statistical Analysis

Approximately 80 per cent of students were able to score marks for project title and purpose. If they did lose marks, it was because they failed to clearly state the variables used in the study. Students are reminded to be concise in the statement of purpose. In some cases, the use of statistical theory was limited by students' choice of topic. Students also displayed a lack of creativity in topic selection.

A few students failed to secure the two marks allotted for data collection. In some cases the method selected was not appropriate and often when appropriate, students failed to adequately describe their choice. Most were able to describe the general method and select an appropriate method but failed to describe its application adequately.

Students were able to get the three marks allotted to data presentation and demonstrated adequate expertise in the use of statistical language, jargon and symbols. Marks were lost because tables and charts were not labelled and in some cases were ambiguous.

Most students demonstrated fair statistical knowledge. Minor inaccuracies of statistical concepts were identified and often overlooked by teachers. Teachers need to pay special attention to the calculation of averages as well as the expected values for the chi-squared distribution. In most cases, there was a logical flow of data and this was extended to the discussion.

Approximately half of the students failed to relate their findings to the purpose of the project, although all the calculations would have been completed. In many cases, this section was too wordy. Students need to be concise in their conclusion. The conclusion must also be clearly stated and must be related to the purpose of the project.

Students who refer to websites must make an effort to follow the accepted convention in a consistent manner. Students must also make an effort to select reliable websites. Most books listed were properly referenced. Students must endeavour to use multiple references.

Students' choice of font style and size made projects very difficult for reading. They are urged to follow conventional formatting used in academic papers. (Times New Roman, font size 12, double spacing, etc.)

Areas of Strengths and Weakness

Strengths

- Recalling of basic definitions, for example, a sample, a population
- Drawing a stem and leaf diagram
- Reading values from the normal distribution, the chi square and the t-tables
- Standardizing the normal distribution
- Recognizing when the binomial distribution is to be used.

Weaknesses

- Using independent events inappropriately
- Calculating unbiased estimate of the standard deviation
- Being unable to obtain a sample mean given the end points of a confidence interval
- Being unable to define a critical region given the critical value
- Interchanging the H_0 and the H_1 for the χ^2 test
- Determining when to use the t-test rather than the Z test
- Failing to use three significant figures when giving answers.

Paper 032 – Alternative to School-Based Assessment

Scripts for ten candidates were marked and the overall performance was satisfactory.

Question 1

This question tested candidates' ability to

- explain why sampling is necessary
- determine what constitutes a population
- identify sampling methods
- distinguish between qualitative and quantitative data
- construct a histogram from a grouped frequency distribution
- determine the mode and median from grouped frequency distribution
- calculate the mean and trimmed mean from a sample.

Marks for this question ranged from 7 to 17 with half of the candidates earning fewer than ten marks and the other half earning ten or more marks. Completing the histogram and calculating the mean were generally well done. Although candidates could easily identify the modal and median class, they had difficulty stating the actual mode and median. Most candidates had no idea what *trimmed mean* meant as they simply found ten per cent of the mean calculated in the previous section of the question.

Answers

1. (a) (i) Impossible to identify the entire population
 - (ii) The ice-cream lovers in the island
 - (iii) Cluster
 - (iv) Qualitative
- (b) (i) Individual data values are lost
 - (iii) Mode = 54.5 (iv) Median = 54.5
- (c) (i) Mean = 218.47 (ii) Trimmed mean = 191.72

Question 2

This question tested candidates' ability to

- list elements of a probability space
- identify elements of an event
- calculate the probability of event
- recognise the conditions for using the binomial distribution
- use the binomial distribution to calculate probability
- use the normal distributions as an approximation for the binomial distribution.

For this question, marks ranged from 0 to 20. Four candidates performed poorly, obtaining either zero or two marks and one candidate failed to respond to this question. The other five candidates performed well with scores ranging from 14 to the maximum possible marks obtainable, 20. Candidates who performed poorly obviously had no knowledge of this topic. Candidates who did well generally had difficulty with Part (b) (ii) – using an appropriate approximation for finding $P(x \geq 340)$.

Answers

- (a) (i) 0.049 (ii) 0.347 (iii) 0.189
- (b) (i) 0.240 (ii) 0.0032

Question 3

This question tested candidates' ability to

- calculate unbiased estimates for the mean and the variance
- formulate null and alternative hypotheses
- evaluate the t-test statistic
- apply the t-distribution in a hypothesis test of the mean and determine the validity of the t-test
- use the regression line to estimate values of y
- calculate and interpret the product moment correlation coefficient.

Marks awarded for this question ranged from 1 to 20 with one candidate not providing a response. Three candidates performed very well with scores of 18 or more, while all other candidates got awarded scores of ten or fewer than ten. From the three questions in the paper, this question had the poorest performance. The best performance was seen in Parts (a) (i) a) and (b) (i). Candidates failed to differentiate between working with a sample and working with a population. Although candidates had the formula and values for Part (b), these made errors in substitution and simplification.

Answers

(a) (i) Mean = 13.33 ; SD = 5.35

(ii) Accept H_0

(iii) Distribution must be normal

(b) (i) $y = 48.35$ (ii) $r = 0.51$; moderate, positive

The questions in this paper were of appropriate difficulty; adequate time was given to candidates to complete the examination and the marks were appropriately allotted to the various items. It is evident that some candidates were prepared for this paper while others were unprepared and therefore performed poorly.

UNIT 2

Paper 01 – Multiple Choice

Performance on the 45 multiple-choice items on Paper 01 produced a mean score of 58.42, standard deviation of 17.40 and scores ranging from 21 to 90.

Paper 02 – Essay

Question 1

This question tested candidates' ability to

- derive and graph linear inequalities in two variables
- determine whether a selected trial point satisfies a given inequality
- determine the solution set that satisfies a set of linear inequalities in two variables
- determine the feasible region of a linear programming
- identify the objective function and constraints of a linear programming problem
- determine a unique optimal solution (where it exists) of a linear programming problem
- formulate linear programming models in two variables from real-world data.

This question was attempted by the majority of candidates with about 60 per cent of them providing satisfactory responses and 20 per cent getting a mark over 19.

In Part (a), few candidates achieved full marks as they were unable to define the variables x and y as the number of backhoes of Type A and Type B respectively. Candidates were also unable to reproduce the correct linear programming model with some candidates forgetting to include the non-negative constraints, as well as to 'maximize' the objective function.

In Part (b), many candidates were unable to represent the equations of the straight lines on the Cartesian plane accurately. They were, however, able to shade the feasible region based on the lines that were drawn.

Part (c) posed problems for candidates as they read the coordinates of the vertices inaccurately, $(0, 15)$ instead of $(0, 16)$. Also one of the vertices $(16\frac{2}{3}, 8\frac{1}{3})$ did not contain non-negative integers. Candidates approximated this vertex to $(17, 8)$ which was found to lie outside the feasible region. Many candidates were, however, able to attain the correct number of each type of backhoe to give a maximum weight of the soil to be removed and the maximum weight of the soil.

Answers

- (a) (i) Maximize: $P = 40x + 60y$ where x is the number of backhoes of Type A and y is the number of backhoes of Type B
- (ii) $50x + 20y \leq 1000$; $2x + 8y \leq 128$; $x + y \leq 25$; $x \geq 0$; $y \geq 0$
- (c) (i) 12 backhoes of Type A and 13 backhoes of Type B
- (ii) The maximum weight of soil removed in one day = 1260 tonnes

Question 2

This question tested candidates' ability to

- formulate simple propositions
- determine if propositions are equivalent using truth tables
- use the laws of Boolean algebra to simplify Boolean expressions
- represent a Boolean expression by a switching or logic circuit.

Part (a) (i) was well done by the candidates. Most candidates obtained full marks. Part (a) (ii) was also well done by the majority of candidates.

For Part (b), many candidates found it difficult to determine the truth value. Candidates used truth tables to determine the answer. However, this method was not required. Candidates need to spend more time with this type of proof.

For Part (c) (i), 80 per cent of the candidates were able to assign propositional statements. However, only about 60 per cent were able to express the statement in symbolic form. Problems encountered were missing brackets and incorrectly placed brackets.

For Part (c) (ii), candidates encountered problems simplifying logic symbolic statements and did not change the OR to AND.

Candidates understood the concept of "NOT (NOT p)".

For Part (d) (i), the majority of candidates obtained the correct solution. Many candidates recognized the solution by carefully observing the diagram.

For Part (d) (ii), about 80 per cent of the candidates were able to construct the truth table from Part d (i) above. Simple mistakes were made in obtaining " $\sim B \vee C$ ". Hence, this affected their final result.

For Part (e), most candidates were able to draw the three logic gates. Weaker candidates did not realize that the brackets should have been worked first in order to obtain the final correct solution.

Answers

(a) (i)

p	q	r	$q \wedge r$	$p \Rightarrow (q \wedge r)$	$p \Rightarrow r$	$p \Rightarrow q$	$(p \Rightarrow r) \wedge (p \Rightarrow q)$
T	T	T	T	T	T	T	T
T	F	T	F	F	T	F	F
T	T	F	F	F	F	T	F
T	F	F	F	F	F	F	F
F	T	F	F	T	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

(ii)

p	q	$p \vee q$	$(p \vee q) \Rightarrow p$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	T

(b) $2+3=5 \Rightarrow \sim \{(2+2=4) \wedge (4+4=8)\}$

$$T \Rightarrow \sim \{T \wedge T\}$$

$$T \Rightarrow \sim T$$

$$T \Rightarrow F$$

Therefore, the statement is false

(c) (i) p : It is hot

q : It is sunny

$$\sim(\sim p \vee q)$$

$$(ii) \sim(\sim p \vee q) = \sim\sim p \wedge \sim q = p \wedge \sim q$$

(d) (i) $A \wedge (\sim B \vee C)$

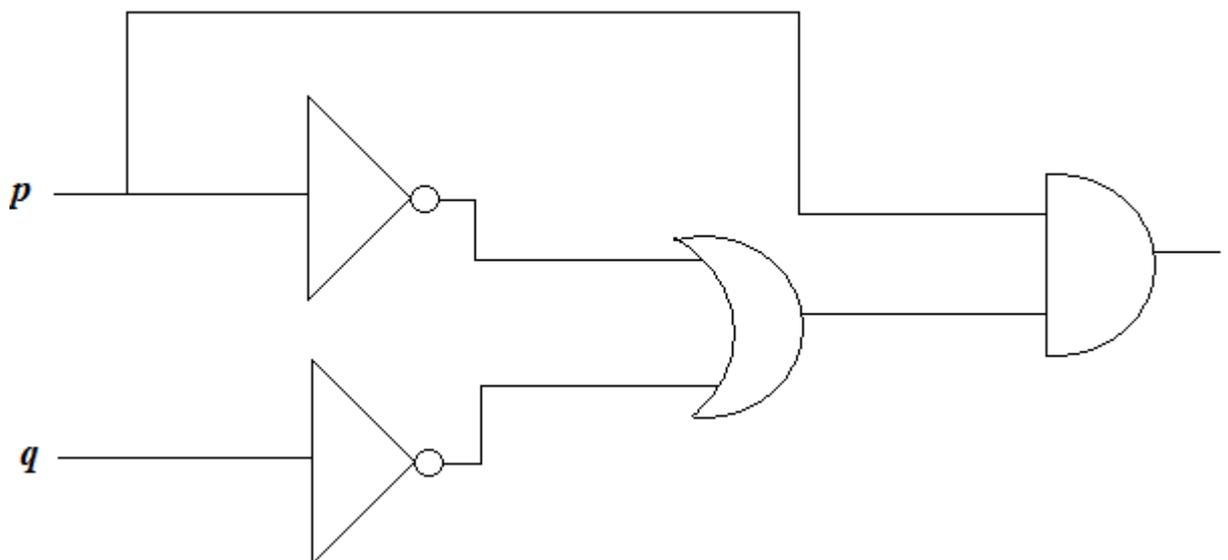
(ii)

<u>A</u>	<u>B</u>	<u>C</u>	<u>$\sim B$</u>	<u>$\sim B \vee C$</u>	<u>$A \wedge (\sim B \vee C)$</u>
1	1	1	0	1	1
1	1	0	0	0	0
1	0	1	1	1	1
1	0	0	1	1	1
0	1	1	0	1	0
0	0	1	1	1	0
0	0	0	1	1	0

The circuit is on when:

- A is on, B is off and C is off
- A is on, B is off and C is on
- A is on, B is on and C is on

(e)



Module 2

Question 3

This question tested candidates' ability to

- apply the properties of the probability density function f of a continuous random variable X
- calculate the expected value of a continuous function
- calculate $P(X \geq 1)$
- use the cumulative distribution function
- solve problems using the normal distribution with a continuity correction.

The question was generally well answered with over 65 per cent of candidates being awarded a mark of 16 or more (out of 25 possible marks). Of these candidates, a little less than half scored 20 marks or more.

Common errors made by several candidates were:

- Poor handling of algebraic manipulation of equations obtained after integration in Part (a) (i).
- Improperly interpreting the interval for Part (a) (i) by treating the variable as discrete rather than continuous
- Failing to use the given function as a cumulative distribution function although this was stated in Part (b),.
- In Part (c) (ii), candidates failed to take the inverse of ϕ when required to solve the equation $\frac{x - 16}{3} = \phi^{-1}(0.6732)$. Most candidates did not follow the instruction to give the answer exactly or to three significant figures.

Answers

(a) (i) $a = -\frac{3}{64}, b = \frac{1}{4}$

(ii) $P(X \geq 1) = \frac{57}{64}$

(b) (i) $\frac{26}{125}$ (ii) 3.97

- (c) (i) 0.0668 (ii) 17.3

Question 4

This question tested candidates' ability to

- formulate and use the probability function $f(x) = P(X = x)$
- apply the formula for probability using the Binomial distribution.

Nearly half of the candidates writing this paper were awarded scores higher than 15 out of the 25 possible marks. Close to 30 per cent of candidates did very well, obtaining scores of 20 or more. A small number of candidates did not respond to this question and 35 per cent obtained a score of ten or fewer than ten.

The question was set at an appropriate level of difficulty; however, some common errors were:

- Failing to take into account all the probabilities required by the question, especially $P(X = 0)$.
- Inappropriately making an approximation using the normal or Poisson distributions where the binomial was the appropriate distribution to use.
- Making computational errors when calculating probabilities in Part (a) (iii).
- Using an incomplete formula for calculating the variance.

The best responses were those given for Part (a) (ii).

Answers

- (a) (ii) $\{4, 5, 6, 7, 8\}$

(iii)

x	4	5	6	7	8
$P(X = x)$	$\frac{1}{21}$	$\frac{2}{7}$	$\frac{1}{3}$	$\frac{2}{7}$	$\frac{1}{21}$

- (iv) 6

- (v) 0.976

- (b) (i) ${}^{20}C_4 (0.04)^4 (0.96)^{16}$ (ii) 0.238

Module 3

Question 5

This question tested the candidates' ability to

- calculate the coefficient of friction between an object and a surface
- calculate the parallel force required to pull an object down an inclined plane at a constant speed
- calculate the total power output of the motion
- verify the horizontal range of a projectile and then determine the maximum range and the time of flight for a shell fired from a gun.

In Part (a) (i), candidates knew the formula $F = \mu R$, but applied it incorrectly.

Some candidates drew the 600 N force parallel to the horizontal instead of parallel to the plane. Candidates need to read the questions carefully.

Part (a) (ii) presented a challenge to candidates. Some of them used the 600 N from Part (a) (i). Candidates also had difficulty resolving forces parallel to the plane correctly.

Teachers need to remind students to draw force diagrams so they can get a visual representation of what they are doing.

Part (b) was poorly done. Candidates did not recall that in the equation $W = Fd$, the force needed to be found in the direction of the motion.

In Part (c), candidates were required to know when to use $v = u + at$ or $s = ut + \frac{1}{2}at^2$. If they set $v = 0$ in $v = u + at$, then they would get the time to reach the maximum height. Also, If they set $s = 0$ in $s = ut + \frac{1}{2}at^2$, then the non-zero value of t is the time of flight.

In conclusion, candidates need more independent practice in problem solving.

Answers

(a) (i) 0.517 (ii) 31.2 N

(c) (ii) 45°

(iii) a) 12250 m b) 49.5 s

Question 6

This tested candidates' ability to calculate the

- negative acceleration (deceleration) and substitute into an appropriate equation of motion to find the velocity
- velocity immediately after impact using the law of conservation of momentum
- kinetic energy
- constant force acting on the object
- braking force using $F = ma$ and $v^2 = u^2 + 2as$.

In Part (a) (i), candidates incorrectly implemented the conservation of momentum. Some did not negate the acceleration.

For Part (a) (ii), candidates incorrectly substituted the given velocity 6 ms^{-1} .

In Part (b), many candidates used the velocity from Part (a) to find the change in kinetic energy instead of finding the final velocity using the equation $v = u + at$ and using it to find the answer.

In Part (c), some candidates used the braking force as 3850 N instead of 3150 N. Some forgot to negate the acceleration. The majority of candidates got an answer of 487 m as a result of applying integration.

Answers

- (a) (i) Velocity = 3.464 ms^{-1} (ii) Velocity = 5.83 ms^{-1}
- (b) (i) Change in kinetic energy = 3360 J (ii) The force = 24 N
- (c) Minimum length of the runway = 595 m

Paper 031 – School-Based Assessment (SBA)Mathematical Applications

Some students failed to identify the relevant variables as well as mathematical concepts to be used in the statement of task and did not clearly state the method(s) being used, or provide an adequate description of its use.

In the section Mathematical Knowledge and Analysis, most students were able to successfully carry out simple mathematical processes and correctly interpret the resulting data.

The evaluation section was poorly done. Approximately 75 per cent of students failed to identify limitations and problems encountered during their study. Students were unable to articulate how to rectify the problems that were encountered during their study.

On the other hand, students were able to obtain full marks in the communication of information section. Many of them were able to articulate and present their findings in a logical manner, and in most cases, using proper grammar.

Overall, teachers did a fairly good job in guiding students in topic selection, and in the modelling of various experiments. However, attention must be paid to the assignment of marks in the rubric provided. *Teachers are reminded that fractional marks are not to be awarded to students.*

Allocation of Marks

- Approximately 84 per cent of students were able to secure marks in the 41–60 range.
- Approximately 60 per cent of all students sampled scored excellent grades in the SBA (51–60).
- Approximately five per cent of students scored under 31 marks out of a possible 60 marks.

Areas of Concern

- About 20 per cent of students ignored the stipulated format for the presentation of the project.
- The statement of the task was not explicit enough in about 30 per cent of the projects.
- Some students analysed the data before the data was even collected. As a result, some tables were not clear, and many tables were presented without headings and without reasons for their use.
- Some students presented more data than was needed for their analysis.

Areas of Strength and Weakness

Strengths

- Construction of the truth table.
- Construction of a logic gate.
- Identifying a feasible region.
- Producing a linear programming model.
- Working with resolving forces vertically and horizontally.

Weaknesses

- The majority of students did not write their answers to three significant figures.
- Few students used diagrams for the mechanics questions.
- Motion in two directions, for example, *forces perpendicular to each other*.
- Using the law of conservation of momentum too early to solve the problem.
- Too often students omitted to identify variables.
- Plotting straight lines.
- Not recognizing that they should use integer values: too many used decimal values
- Too often students did not identify logic assignments, for example, *p: it is hot*

Paper 032 – Alternative to School-Based Assessment

The seven candidates doing this paper performed poorly. Only one candidate performed well. Understanding of the basic concepts was weak.

Module 1

Question 1

This question tested candidates' ability to

- construct an activity network diagram
- calculate earliest start time, latest start time and float time
- identify the critical path in an activity network.

Candidates failed to demonstrate any understanding and synthesis of the graphical representation of the project flow network and how to compute the values (earliest start time, latest start time, float time) needed to determine the critical path and minimum completion time.

Module 2Question 2

This question tested candidates' ability to

- solve problems involving probabilities of the normal distribution
- calculate expected values
- use the formula for a geometric distribution.

Candidates demonstrated a lack of understanding of basic probability concepts. Candidates had an idea of the need to standardize but were unable to correctly carry out the desired operation.

Answers

(a) (i) 0.252 (ii) 0.161 (iii) 80

(b) 0.913

(c) 0.36

Module 3Question 3

This question tested candidates' ability to

- resolve forces, on particles, in mutually perpendicular directions
- calculate and use displacement, velocity, acceleration and time in simple equations
- apply Newton's laws of motion
- apply rates of change for velocity and acceleration.

Algebraic manipulation of the mechanics problem in Part (c), including the appropriate use of integration, was extremely weak. Generally, candidates failed to demonstrate a grasp of basic concepts in force, work and displacement. Most candidates used the incorrect function (sine) instead of the correct trigonometric function (cosine) when resolving the resulting force. Candidates mostly failed to identify the appropriate formula to use in computing for time in Part (b). Candidates did not recognize the use of differentiation of the velocity to generate the equation to find the time.

Answers

(a) $W = 5280 \text{ J}$

(b) $t = 35.7 \text{ s}$

(c) $k = 0.1; a = 10 e^{-0.2s}$.

Overall, the difficulty level of the questions was reasonable and appropriate for this level. The time provided for solutions was adequate. The conceptual grounding of the candidates (except one) attempting this paper was very weak.