

**CARIBBEAN EXAMINATIONS COUNCIL**

**REPORT ON CANDIDATES' WORK IN THE  
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®**

**MAY/JUNE 2013**

**APPLIED MATHEMATICS**

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## GENERAL COMMENTS

The revised Applied Mathematics syllabus was examined in 2013 for the fifth time. This is a two-unit course comprising three papers. Paper 01, consisting of 45 multiple choice questions, and Paper 02, consisting of six essay questions were examined externally, while Paper 03 was examined internally by class teachers and moderated by CXC. Contributions from Papers 01, 02 and 031 to each unit were 30 per cent, 50 per cent and 20 per cent respectively.

Unit 1, Statistical Analysis, tested (1) Collecting and Describing Data; (2) Managing Uncertainty and (3) Analysing and Interpreting Data.

Unit 2, Mathematical Applications, tested (1) Discrete Mathematics; (2) Probability and Probability Distributions; and (3) Particle Mechanics.

For Unit 1, 652 candidates wrote the 2013 paper and 12 wrote the Alternative to the Internal Assessment paper, Paper 032. For Unit 2, 313 candidates wrote the 2013 paper and 10 candidates wrote the Alternative to the Internal Assessment Paper, Paper 032.

Generally, candidates are still having difficulty with (i) algebraic manipulations and (ii) problem solving in the mechanics module.

## DETAILED COMMENTS

### UNIT 1

#### Paper 01 Multiple Choice

Performance on the 45 multiple choice questions on this paper produced a mean of approximately 62.5 out of 90, standard deviation of 17.9, with scores ranging from 18 to 90.

#### Paper 02 Essay

#### Module 1: Collecting and Describing Data

##### Question 1

This question tested candidates' ability to

- identify whether variables were qualitative or quantitative
- distinguish between population, sample, census or sample survey
- use random sampling numbers provided in the statistical tables to obtain a random sample of a given size.

Parts (a), (b) and (c) were done exceptionally well by most candidates. However, it was common for candidates to also classify qualitative data as either discrete or continuous. Although the solutions were provided in the exam script, spelling errors were common. Part (d) was fairly well done by most candidates. However, several candidates were unable to express themselves clearly. This part required that candidates provide two reasons why it may be necessary to conduct a sample survey rather than use a census.

Similarly, for Part (e) (i), explanations provided by candidates were not clear. Most did not provide a complete explanation of how to use the random table. Although a two-digit random number was provided several explanations included a four-digit table or using the random table to assign numbers which would then be drawn from a hat to select the sample of 15.

Approximately half of the candidates scored full marks for Part (e) (iii). Many provided some parts of the process and generally failed to provide an appropriate range for assigning numbers or not indicating that the start position is a random point.

For Part (e) (i), any response that indicated that non-overlapping groups are required. Alternatives could include that the groups were not well defined, members of each group were not unique or that the groups were not mutually exclusive.

### Question 2

This question tested candidates' ability to

- determine the mode class, mean and standard deviation from a frequency distribution
- determine the median and interquartile range from a stem and leaf diagram.

For Part (a) (i), the majority of candidates performed exceptionally well. Some candidates obtained at least one mark for identifying the modal class but failed to identify the boundaries. Part (a) (ii) was poorly answered. Most candidates did not recognize that 67 was the midpoint of the interval and therefore needed to estimate the frequency by taking one-half of the total frequency for the interval. Many failed to add the frequency of the higher interval. The most common response was to take the sum of both intervals. Most candidates obtained at least one mark for identifying the total number of students (20). The correct response to this part was:  $\frac{\frac{10}{5}+2}{20} \times 100 = 35 \text{ per cent.}$

Part (a) (iii) was answered correctly by approximately 60 per cent of the candidates. The most common error was taking the sum of the mid points and dividing by the number of intervals. In some cases, errors in calculations were observed. The correct response to this part was 65.65 inches.

Part (a) (iv) was poorly done as most candidates did not apply the formula correctly. The correct response was 2.593 inches.

Part (b) (i) was completed at a satisfactory level. A correct response would have been that all data values are retained. Acceptable alternatives included stating that the mode or range, are clearly visible or that the exact value of the mean and median can be calculated. Part (b) (ii) a) was correctly answered by nearly all candidates.

Because of the way the data values were distributed some candidates seemed to obtain the correct response but were in fact reading the wrong position of 13 rather than the required position of 13.5.

In Part (b) (ii), the majority of candidates knew they needed to subtract the two quartiles. However, most had problems calculating the exact values of both quartiles. Some candidates identified the position using  $\frac{n+1}{2}$  and  $\frac{3}{4}(n+1)$  which did not provide correct quartiles due to a small sample size.

A few candidates found the semi-interquartile - range. Part (b) (ii) (d) required that candidates construct a box-and-whiskers diagram. It was evident that some candidates were not knowledgeable about this type of diagram they proceeded to construct dot plots or bar charts. Although the graph paper provided included the scale, some candidates developed an alternative scale while others used a different graph paper.

For Part (b) (ii) e), most candidates obtained the two marks allocated. The majority of candidates was able to identify that the distribution was skewed but they had problems determining whether the distribution was negatively or positively skewed.

### Question 3

This question tested candidates' ability to

- calculate the probability of the intersection of two events, P and Q
- construct a Venn diagram
- solve probability problems.

Part (a) (i) was attempted by 98 per cent of the candidates with about 60 per cent giving a satisfactory response. Some candidates did not use the conditional probability but multiplied  $P R$  by  $P T$ , for example  $P R \cdot P T$ .

Approximately 97 per cent of the candidates attempted Part (a) (ii) and responses were satisfactory.

Some candidates either identified the regions on the diagram without calculating the respective probabilities or they calculated the probabilities without identifying them on the Venn diagram.

Part (a) (iii) a) was attempted by approximately 95 per cent of the candidates with about 55 per cent of them successfully completing the question. Some candidates did not take into account that Q and T were independent.

Part (b) was also attempted by 98 per cent of the candidates, with only 40 per cent giving satisfactory responses.

### Question 4

This question tested candidates' ability to

- calculate the expected value, variance and probability from a probability distribution
- carry out calculations based on a probability density function.
- identify a binomial distribution and use it to calculate probabilities.

Parts (a) (i) and (ii) were generally well done. For Part (a) (ii) b), the most common mistake was that candidates neglected to square the mean before subtracting.

Part (b) (i) was attempted by approximately 90 per cent of the candidates with about 55 per cent giving satisfactory responses. Some responses failed to acknowledge that the p.d.f. started at 1, hence the length of the rectangle was taken as  $k$  and not  $k - 1$ . Some candidates used the integration method with some success.

For Part (b) (ii), most candidates gave a satisfactory response. Many candidates were able to arrive at the correct answer by simply subtracting  $\frac{1}{5}$  from the total area of 1.

For Part (c) (i), some candidates failed to use the binomial distribution. Part (c) (ii) was attempted by almost every candidate with about 97 per cent giving satisfactory responses. Some candidates used the probability that was calculated from Part (c) (i) as the value of  $p$ .

In Part (c) (iii), most candidates correctly used the normal approximation to the binomial; however, many either neglected or incorrectly applied the continuity correction. Some candidates used the variance instead of the standard deviation when standardizing. Almost every candidate was able to correctly read off a value from the table, but a few continued to subtract this value from 1.

### Question 5

This question required candidates to

- state the distribution of the sample mean
- calculate the probability that the sample mean is greater than a given value
- calculate a 94 per cent confidence interval for the mean
- formulate a null hypothesis and an alternative hypothesis
- carry out a particular test at the 5 per cent level of significance

For Part (a) (i), most candidates were able to obtain two marks by identifying the distribution as normal and the mean as 35. However, many candidates failed to identify the variance correctly which should have been  $\frac{100}{8}$ . Many candidates did not use the appropriate symbolic form in their statement.

In Part (a) (ii), most candidates were awarded five marks for correct calculation of 0.148. Some candidates were able to calculate the  $z$ -value but were unable to read the correct value from the table. Approximately 40 per cent of candidates did not use  $\sqrt{8}$  in the formula and therefore lost a mark.

For Part (b) (i), the majority of candidates was able to calculate the correct confidence interval of 3.894, 4.506. Many candidates used the limits for a 95 per cent confidence interval.

In Part (b) (ii), although many candidates were able to give the correct solution as 38 packages, others failed to recognize that the data given was discrete and apply this to the calculated value.

Part (c) (i) was well done with candidates providing acceptable responses, which included:

- normal distribution and small sample ( $n < 30$ ) or unknown variance.

Part (c) (ii) was answered well by most candidates. However, the appropriate symbols were not always used. Although the question stated the use of symbols, some candidates provided the answer in words. The correct responses were  $H_0: \mu = 10$  and  $H_1: \mu < 10$ .

Part (c) (iii) posed challenges for most candidates. Several did not use the appropriate degree of freedom ( $8 - 1 = 7$ ). Although they had the hypothesis correctly stated in the preceding sub-part, they proceeded to use a two-tailed test. Many candidates failed to state the conclusion in words relevant to the question and only stated it as 'accept' or 'reject'.

### Question 6

This question tested the candidates' ability to utilize regression analysis and chi-square analysis in solving problems.

In Part (a) (i), many candidates did not correctly interpret the value of 0.09 in the equation. Most candidates made reference to the gradient and  $y$ -intercept, but were unable to relate the information to the question and offer meaningful interpretations.

Part (a) (ii) and (iv) were well done.

For Part (b) (i), the majority of candidates was able to provide a favourable response. Parts b (ii) to (iv) were done fairly well.

For Part (b) (iii) b), most candidates were able to state or show the correct critical region. However, some candidates calculated an incorrect critical value for the chi-squared test; while others read the value from the table under the 5 per cent column, instead of the 95 per cent column.

In Part 6 (b) (iii) c) there were many variations of values due to rounding errors. In some cases, an incorrect formula or the incorrect usage of the formula was also applied.

### **Paper 031 – School-Based Assessment (SBA)**

Approximately 80 per cent of candidates were able to score full marks for project title and purpose. If they did lose marks, it was because they failed to clearly state the variables used in the study. Candidates are reminded to be concise in the statement of purpose. In some cases the use of statistical theory was limited by the candidates' choice of topic. Candidates also displayed a lack of creativity in topic selection.

Some candidates failed to secure the two marks allotted for data collection. In some cases the method selected was not appropriate and often when appropriate, the candidate failed to adequately describe their choice. Most were able to describe the general method and select an appropriate method but failed to describe its application adequately.

Data presentation and the use of statistical language, jargon and symbols were for the most part satisfactory but candidates lost marks due to tables and charts being unlabeled and in some cases ambiguous.

Most candidates demonstrated a fair statistical knowledge. Minor inaccuracies of statistical concepts were identified and often overlooked by teachers. Teachers need to pay more attention to the calculation of expected values for the Chi-squared distribution. In most cases there was a logical flow of data and this was extended to the discussion.

In some cases, candidates failed to relate their findings to the purpose of the project, although all the calculations would have been completed. In many cases, this section was too wordy.

Candidates need to be concise in their conclusion. The conclusion must also be clearly stated and must be related to the purpose of the project.

Candidates must endeavor to use multiple references, but at the same time, when using websites as a reference they must make an effort to follow the accepted convention in a consistent manner. Candidates must also make an effort to select reliable websites.

Candidates' choice of font style and size made it very difficult for reading. Candidates are urged to follow conventional formatting used in academic papers. (Times New Roman, font size 12, double spacing, etc.)

Many of candidates sampled were able to performed, fairly well in the Unit 1 Internal Assessment.

### **Paper 3/2 – Alternative to the School Based Assessment**

#### Question 1

This question tested candidates' ability to

- State whether given statements involve the use of interviews or observations
- Construct frequency distribution from raw data
- Construct and use histograms to analyse data
- Outline the relative advantages and disadvantages of using frequency distribution in data analysis
- Determine or calculate mode for grouped data.

Parts (a) (i) and (a) (ii) were fairly well done. Most candidates understood how to distinguish between the usages of interviews and observations.

Parts (b) (i) and (b) (ii) were well done by most candidates.

For Part (c), most candidates demonstrated a high level of competence in distinguishing between the usage of bar charts and histograms.

Part (d) (i) was generally well done. Part (d) (ii) posed the most difficulty for some candidates since they did not use any of the two methods which involved using the histogram or the formula for the mode which is  $L + \frac{\Delta_1}{\Delta_1 + \Delta_2} C$ .

### Answers

- (a) (i) Interview            ii) Observation  
 (b) (i) Be more specific about family. How many people live in your immediate household?  
 (ii) Define young, middle age, old using ranges  $< 35$ ,  $35 \leq 50$ ,  $> 50$   
 (c) (i) Bar chart            ii) Histogram  
 (d) (i)

Range	Frequency
10 - 19	7
20 - 29	9
30 - 39	13
40 - 49	10
50 - 59	7
60 - 69	4

- (d) (ii) Individual data values are lost  
 (d) (iii) Reducing the amount of data  
           Easier to see at a glance  
           Clearer idea of distribution of the data  
 (d) (v) 35

### Question 2

This question tested candidates' ability to

- identify and use the concept of probability
- calculate simple probability without replacement using
  - $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
  - Mutually exclusive events  $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$
- state the assumptions made in modelling data by a binomial distribution
- identify and use the binomial distribution as a model of data where appropriate
- use the notation  $X \sim B(n, p)$  where  $n$  is the number of independent trials and  $p$  is the probability of a successful outcome in each trial.
- calculate the probabilities  $P(X = a)$ ,  $P(X > a)$ ,  $P(X < a)$  or any combination of these where  $X \sim B(n, p)$
- use the normal distribution as a model of data as appropriate
- determine probabilities from tabulated values of the standard normal distribution  $Z \sim N(0, 1)$
- solve problems involving probabilities of the normal distribution using  $z$ -scores.

All candidates answered Part (a) (i) correctly while for Part (a) (ii), some candidates were unable to write down the required probabilities needed to arrive at the correct solution.

For Part (b) (i), most candidates were able to give three assumptions made in modelling data by the binomial distribution. However some statements were not mathematically precise.

The majority of candidates was able to answer Part (b) (ii) question correctly.

Most candidates who attempted Part (c) were able to convert the values to z-scores, use the correct formula for  $\Phi$  and read the table values correctly. However, some candidates were unable to simplify the  $\Phi$  formula correctly while others gave the incorrect table values.

### Answers

(a) (i)  $\frac{1}{4}$

(a) (ii) a)  $\frac{1}{22}$

(a) (ii) b)  $\frac{21}{44}$

(b) i) Assumptions for binomial:

- n- distinct trials
- independent trials
- 2 outcomes for each trial
- Probability of success the same for each trial

(ii) a) 0.0318

(ii) b) 13.5 or 14

c) 0.7745 or 0.775

### Question 3

This question tested candidates' ability to

- calculate the confidence intervals for a population mean or proportion using a large sample ( $n > 30$ ) drawn from a population of known or unknown variance
- evaluate a  $t$ -test statistic
- determine the appropriate number of degrees freedom for a given data set
- apply a hypothesis test for a population mean using a small sample ( $n < 30$ ) drawn from a normal population of unknown variance.
- formulate a null hypothesis and an alternate hypothesis  $H$
- apply a one-tailed or two-tailed test appropriately
- determine the critical values from tables for a given test and level of significance.
- identify the critical or rejection region for a given test and level of significance
- evaluate from sample data the test statistic for testing a population mean or proportion

Candidates' scores ranged from 0-19 for this question, with less than 40 per cent of the candidates getting over 12 out of a total of 20 marks.

For Part (a) (i), some candidates used the formula correctly; however; marks were lost for using the incorrect confidence factor and not finding the square root of the sample size. In Part (a) (ii), most candidates used  $E(x) = np$  correctly to find the expected value.

Part (b) (i) was well done. However, some candidates failed to state the hypotheses correctly and also used the incorrect test, that is, they used a one-tailed test instead of two-tailed test. In Part (b) (ii), some candidates did not know the criteria for using a t-test and therefore failed to use the t-test in this question.

In Part (b) (iii), most candidates had the correct degree of freedom but failed to write the critical value as a region  $t > 2.365$ ,  $t < -2.365$ .

Parts (b) (iv) and (v) were well done by most candidates.

### Answers

- (a) (i) 95 per cent confidence interval for  $U = (23.2, 24.8)$   
 (ii) Expected value (54)
- (b) (i) Hypothesis for the tailed test =  $H_0: \mu = 14$ ,  $H_1: \mu \neq 14$   
 (ii) Correct test to use was the  $t$ -test  
 Reasons = Sample size is small and the variance of the population is unknown  
 (iii) Critical Region =  $-2.365 < t < 2.365$   
 (iv) Calculated Value  $T_{cal} = 1.91$  or  $1.79$   
 (v) Decision – Accept  $H_0$  since  $T_{cal}$  is in the acceptance region; that is  
 $1.79 < 2.365$  or  $1.91 < 2.365$

## UNIT 2

### Paper 01 – Multiple Choice

Performance on the 45 multiple-choice items on Paper 01 produced a mean of approximately 65.3 out of 90, standard deviation of 17.93 and scores ranging from 22 to 90.

### Paper 02 - Essay

#### Question 1

This question tested candidates' ability to

- establish the truth value of the negation of simple propositions and compound propositions that involve conjunctions, disjunctions and negations;
- use truth tables to determine whether a proposition is a tautology or contradiction and if propositions are equivalent;
- use the laws of Boolean algebra to simplify Boolean expressions
- represent a switching circuit or logic circuit. Using a Boolean expression

This question was generally well done, with the majority of candidates obtaining 17 marks and over. Parts (a) and (b) were attempted by all candidates with most candidates getting full marks. De Morgan's law was used incorrectly in Part (c) with candidates obtaining  $\sim p \vee \sim q$  instead of  $\sim p \wedge \sim q$ .

In Part (d), candidates forgot to assign statements to variables, that is, let  $p$  represent it is hot and  $q$  represent it is sunny. Candidates who wrote equivalent propositions instead of statements lost a mark.

For Part (e) (i), most candidates obtained full marks, whilst Part (e) (ii) was poorly done. Most candidates were unable to simplify the expression using the Absorption law. Also, candidates were unable to differentiate between a switching circuit and logic gates. They must be reminded to identify the laws of Boolean algebra being used. Part (f) was generally well done. There were some candidates, however, who simplified the statements using De Morgan's law and then drew the circuit using logic gates.

### Answers

(a)

$p$	$q$	$p \wedge q$	$p \vee q$	$\sim(p \vee q)$	$(p \wedge q) \wedge \sim(p \vee q)$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

(b)  $(p \wedge q) \wedge \sim(p \vee q)$  is a contradiction since the final column of the truth table contains all "F"s.

(c)  $\sim(p \vee q) \equiv \sim p \wedge \sim q$

(d) It is not raining or it is not sunny.

(e) (ii)  $p \wedge q \wedge (p \vee r) \wedge (q \vee (r \wedge p) \vee s) \equiv p \wedge q$ . The corresponding switching

circuit is 

Range of Marks	Responses
NR	2
0 4	7
5 7	10
8 11	33
12 14	41
15 16	23
17 18	48
19 25	86

## Question 2

This question tested candidates' ability to:

- determine the degree of a vertex
- calculate the earliest, latest starting time and float time
- identify the critical path in an activity
- solve a minimization assignment problem by the Hungarian algorithm.

This question was well done with at least 60 per cent of the candidates obtaining a score between 19 and 25.

For Part (a) (i), many candidates were able to determine the earliest start time of S and X. A wide range of answers were given for the minimum completion time of the project in Part (a) (ii). Part (a) (iii) was well done as candidates were able to correctly work out the latest starting times for X and Q. In Part (iv), many candidates recognized that there was more than one critical path for the activity network. The majority of candidates were able to accurately calculate the float time of T in Part (v). A diagram or table indicating EST, LST and Float times would have assisted candidates in answering this question.

For Part (b), a variety of responses were received such as 45<sup>0</sup>, 60<sup>0</sup>, 360<sup>0</sup>. This indicated that some candidates did not understand the meaning of *degree of a vertex* and also that the degree of a vertex has no units. Many of them did not realize that a loop has a degree of two.

Part (c) was generally well done. However, candidates failed to perform the final shading to indicate the end of the algorithm. Some candidates repeatedly found row and column minimums indicating that they did not know when to terminate the algorithm. Candidates used a variety of algorithms to get the answer with most obtaining the correct matching and the correct total minimum cost.

### Answers:

- (i) EST of S and X are 4 and 7 respectively.
- (ii) 13 days
- (iii) LST of X and Q are 7 and 0 respectively.
- (iv) Start-Q-T-X-Finish, Start-R-T-X-Finish, Start-Q-S-X-Finish
- (v) 0 (5-5)
- (b) Degree of vertices A and B are 4 and 2 respectively
- (c) (i)  $W_1 - S_2, W_2 - S_1, W_3 - S_3, W_4 - S_4$
- (ii) \$ 16.00

Range of Marks	Responses
NR	1
0 4	2
5 7	3
8 11	14
12 14	19
15 16	21
17 18	26
19 25	127

Question 3

The question tested candidates' ability to

- calculate and use the expected values and variance of linear combinations of independent random variables
- calculate  $P(X = x) = q^{x-1}p$ , where  $X = \{1, 2, 3, \dots\}$
- use formula for  $E(X)$  and  $Var(X)$  where  $X$  follows a discrete uniform, binomial geometric or Poisson distribution
- solve problems involving probabilities of the normal distribution using z-scores.

For Parts (a) (i) and (ii), the majority of candidates scored full marks. They displayed a good understanding of the concepts and showed a high level of competency in solving the given problem.

In Part (a) (iii), candidates were asked to calculate the variance. Some candidates did not square the coefficient of the variance or use the '+' sign in calculating variance, resulting in loss of marks.

For Part (b) (i), some candidates used the incorrect inequality sign for example. Candidates were asked to calculate  $P(X \leq 3)$ , and seemed not to know what to do. Common errors were  $1 - P(X \leq 3)$  and  $P(X = 1)$ ,  $P(X = 2)$ . Some candidates did not recognize that zero should be included in the latter if they are using the alternative method that is  $1 - P(X \leq 2)$  which include  $\{0, 1, 2\}$ .

In Part (b) (ii), candidates demonstrated mastery of content as full marks were awarded to the majority of candidates. In Part (b) (iii), candidates were asked to calculate  $P(X = 5)$ . Some candidates used the incorrect formula for example,  $pq^r$  and  $pq^{r-1}$ , instead of  $q^4p$  which is the correct formula.

In Part (c), very few candidates used the continuity correction method instead of the normal distribution. Candidates need to be taught when to use the continuity of correction method the Poisson method/normal distribution for approximation.

**Answers**

- (a)(i) 12                      (ii) -11                      (iii) 48
- (b)(i)  $\frac{4}{9}$                       (ii) 3                      (iii)  $\frac{16}{243}$  or 0.658
- (c)(i) 0.309 to 3dp                      (ii) 0.964 to 3 dp

Question 4

This question tested candidates' ability to

- formulate and use the probability function  $f(x) = P(X = x)$
- apply the formula  $P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$   $x = 0, 1, 2, 3, \dots$
- use the Poisson distribution as an approximation to the binomial distribution
- apply the properties of the probability density function,  $f$ , of a continuous random variable:
  - i)  $F(x) \geq 0$
  - ii)  $\int_{-\infty}^{+\infty} f(x) dx = 1$

Parts (a) (i) and (ii) were well done by the majority of candidates. Some candidates used the incorrect inequality sign. Part (a) (iii) was poorly done by most candidates as many of them were unable to use the binomial formula correctly.

In Part (b), approximately 25 per cent of the candidates misinterpreted the question and used the binomial formula instead of the Poisson approximation.

Part (c) was well done by the majority of the candidates.

### Answers

(a) (i) 0.184 (3 s.f)      (ii) 0.567 (3 s.f)      (iii) 0.285 (3 s.f)

(b) 0.677 (3 s.f)

(c) (i)  $k = \frac{1}{3}$       (ii)  $t = \frac{9}{4}$

### Question 5

This tested the candidate's ability to

- resolve forces, on particles, in mutually perpendicular situations
- use the appropriate relationship  $F = \mu R$  or  $F \leq \mu R$  for two bodies in limiting equilibrium;
- solve problems involving concurrent forces in equilibrium;
- apply Newton's Law of Motion
- apply wherever appropriate the following rates of change:

$$v = \frac{dx}{dt},$$

$$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx}$$

- Formulate and solve first order differential equation as models of linear motion of a particle when the applied force is proportional to its displacement or its velocity.

This question was poorly done. Those who attempted it scored less than 12 marks.

In Part (a) (i), some candidates had difficulty interpreting the words and creating a diagram. The weight of the rod was incorrectly put at the end, the string was incorrectly drawn and the point R was placed incorrectly. Candidates also forgot the force at the hinge P. They did not draw the coplanar force diagram meeting at one point or in the form of a closed triangle (as if it was in equilibrium). Candidates drew two forces with the correct angle between the two. In Part (a) (ii), only two candidates used Lami's theorem and a few used the moments method correctly.

In Part (b), most candidates drew the diagram correctly; however, the forces were added incorrectly and the resolution of the weight was also done incorrectly. Candidates did not use the resultant force to be  $ma$  and some even stated it was equal to zero. Some candidates attempted to use the work-energy theorem but did so incorrectly, stating potential energy plus initial kinetic energy minus final kinetic energy was equal to the work done. Also, those who stated that the work done equals change in kinetic energy eliminated the negative sign because they did not consider the fact that the final force was less than the initial force which implied deceleration.

Part (c) was generally well done. Most candidates separated the variables and integrated, with a few reciprocating both sides and integrating with respect to  $x$ . A few candidates used definite integrals which eliminated the substitution to find the integration constant. Those who got this question wrong either integrated incorrectly or did not find the constant of integration.

Part (c) (ii), was also well done with the majority of candidates substituting well. Some candidates made the mistake of using the initial velocity as opposed to the final velocity and some of them did not recognize that  $\frac{dx}{dt}$  was the velocity. They used  $v = u + at$  with  $t = \frac{1}{k} \ln \frac{10}{10-kx}$ .

**Answers:**

(a) (ii)  $T = 20\sqrt{2} N$

Range of Marks	Responses
NR	4
0 4	68
5 7	50
8 11	57
12 14	25
15 16	15
17 18	17
19 25	12

### Question 6

This question tested candidates' knowledge of

- acceleration and tension relating to two particles connected by a light inextensible string which passes over a pulley
- projectiles.

Most candidates attempted this question. However, it was poorly done. Less than 40 per cent of the candidates scored 12 marks or more out of a maximum of 25 marks.

Part (a) (i) was generally well done. For Part (a) (ii), most candidates substituted correctly in the formula and therefore gained marks. In Part (a) (iii), some candidates used the incorrect formula,  $v = \frac{d}{t}$  or  $d = vt$  is used for constant velocity, instead of  $s = ut + \frac{1}{2}at^2$ . However, the substitution for their answer from Part (a) (i) was done correctly for the most part.

For Part (b), candidates knew the equation impulse = change in momentum. However, they failed to realize that it was a vector quantity, hence when the ball bounced from the wall the sign would have changed.

Part (c) was poorly done as candidates had problems using the equations of motion to determine the equation of trajectory.

In Part (d), most candidates used the correct formula but it was noted that candidates had problems converting the formula to have only one variable or ratio that is,  $y = x \tan \alpha - \frac{1}{2} \frac{gx^2}{v^2 \cos^2 \alpha}$ . Also, to form the equation into a quadratic and solve it was problematic for most candidates.

**Answers**

(a) (i)  $\frac{2}{7} g ms^{-2}$       (ii)  $\frac{450}{7} N$  or 64.3 N      (iii) 51.4 m

(b) 18Ns

(c)  $y = x \tan \alpha - \frac{1}{2} \frac{gx^2}{v^2 \cos^2 \alpha}$

(d)  $\alpha = 77^\circ$  or  $71^\circ$

Range of Marks	Responses
NR	11
0 4	40
5 7	28
8 11	33
12 14	8
15 16	11
17 18	11
19 25	29

**Paper 031 – School-Based Assessment (SBA)**Mathematical Applications

Approximately 40 per cent of the candidates failed to identify the relevant variables as well as mathematical concepts to be used in the statement of task. Additionally, some candidates did not clearly state the method(s) being used, nor provided an adequate description of its use.

The section Mathematical Knowledge and Analysis, was fairly well done as candidates were able to successfully carry out simple mathematical processes and correctly interpret the resulting data.

The evaluation section was however poorly done. Candidates failed to identify limitations and problems encountered during their study. Some candidates were unable to articulate how to rectify the problems that were encountered during their study.

However, many candidates were able to obtain full marks in the communication of information section. Candidates were able to articulate and present their findings in a logical manner, and in most cases, using proper grammar.

Overall, it was evident that students were well guided in the selection of a topic, and in the modeling of various experiments.

**General comments**

1. Generally, candidates demonstrated a high degree of mastery in the mathematical principles pertaining to the syllabus. In most cases, the mathematical analyses were relevant and carried out with few flaws.
2. There was evidence of originality and creativity.
3. Projects were appropriately applied to real world problems and situations.
4. Over ninety percent of the candidates were able to effectively communicate information in a logical way using correct grammar and mathematical language.

### Areas of Concern

1. Some candidates ignored the stipulated format for the presentation of the project.
2. The statement of the task was not explicit enough in some of the projects.
3. Some candidates analysed the data before the data was even collected. The result of this was tables that were not clear, and even some tables that were presented without heading and without reasons for their use.
4. Some candidates presented more data than was need for their analysis.

### Areas of Strength and Weakness

#### Strengths

1. Originality and creativity.
2. Appropriately applied to real world problems and situations.
3. Effectively communicate information in a logical way using correct grammar and mathematical language.

#### Weaknesses

1. Vague titles.
2. Variables not mentioned or defined.
3. Table headings not stated.
4. Some discussions and conclusion not clearly related to the purpose.
5. References – did not use a recognised format.

## Paper 032 Alternative to the School Based Assessment

### Module 1

#### Question 1

This question tested candidates' ability to

- derive and graph linear inequalities in two variables
- determine whether a selected trial point satisfied a given inequality
- determine the solution set that satisfies a set of linear inequalities
- determine the feasible region of a linear programming problem
- formulate, in symbols, compound propositions
- graph terminology: *path*

For Part (a), most of the candidates were able to derive and graph the linear inequalities in two variables. Fewer candidates were able to identify the feasible region. The majority of candidates was unable to use  $\bar{c} = \bar{x} + 2\bar{y}$  to calculate the minimum value.

In Part (b), most candidates were able to recall the converse, contrapositive and the inverse.

Part (c), was particularly well done as most candidates understood the question and were able to give the correct solution.

**Answers**

- (a) (i)  $\{(0,12),(3,0)\}\{(3,0)(0,21)\}\{(0,3)(9,0)\}$  (ii)  $C = 6.9$  at  $(2.7, 2.1)$   
 (b) (i)  $Q \implies P$  (ii)  $\sim Q \implies \sim P$  (iii)  $\sim P \implies \sim Q$   
 (c) AC, ABC, ADC

**Question 2**

This question tested the candidates' ability to:

- calculate the number of ordered arrangements of  $n$  objects taken  $r$  at a time, with or without restrictions
- calculate probabilities of events (which may be combined by unions of intersections) using appropriate counting techniques
- carry out a chi-square ( $\chi^2$ ) goodness of fit test, with appropriate number of degrees of freedom

For Part (a), the majority of candidates was able to answer the question correctly. Part (b) was well done as the majority of candidates were able to score full marks.

For Part (c), the majority of the candidates was able to use binomial formula correctly to calculate the expected value.

**Answers**

- a)  $\frac{7!}{2!}$   
 b)  $\frac{1}{21}$   
 c) i) 0.6

(ii) 52.2, 36.8, 9.8, 1.1, 0.1

$H_0$ : data fit binomial distribution with  $n=4$ ,  $p=0.15$

$H_1$ : data do not fit binomial distribution with  $n=4$ ,  $p=0.15$

$$\chi^2_{\text{test}} = 0.669$$

since  $\chi^2_{\text{test}} < 3.841$  accept  $H_0$

**Question 3**

This question tested candidates' ability to

- use vectors to represent forces
- calculate the resultant of two or more coplanar forces
- resolve forces on particles on incline planes
- calculate the work done by a constant force
- solve problems involving kinetic energy and gravitational potential energy
- apply the principle of conservation of energy
- solve problems involving power
- apply the work energy principle in solving problems

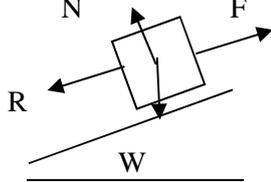
Parts (a) (i) and (a) (ii) were well done by most candidates. For Part (b), most of the candidates were able to recall the converse, contrapositive and the inverse.

Part (c) was answered very poorly, all candidates were unable to solve this question involving kinetic energy and gravitational potential energy. However, two candidates obtained full marks using another method which involved  $v^2 = u^2 + 2as$  and  $F = ma$ .

**Answers:**

a) i)  $6i+12j$       ii)  $\sqrt{180}$

b) i)      N      F      ii) 194N



c) 295N

**Recommendations**

- For the Hungarian algorithm, teachers should teach the different methods for solving the optimization problem.
- Teachers should teach students how to interpret the obtained results such as the regression coefficients.
- Students need to practise more worded problems in the Mechanics section. They do need to draw diagrams to assist them in problem solving.
- It would be good practice for students to use the APA style for writing references in the School-Based Assessment (SBA).