CARIBBEAN EXAMINATIONS COUNCIL

# REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

## MAY/JUNE 2014

## APPLIED MATHEMATICS

## GENERAL COMMENTS

Applied Mathematics syllabus is a two-unit course comprising three papers. Paper 01, consisting of 45 multiple-choice questions and Paper 02, consisting of six essay questions were examined externally, while Paper 03 was examined internally by class teachers and moderated by CXC. Contributions to candidates' final score from Papers 01, 02 and 03 to each unit were 30 per cent, 50 per cent and 20 per cent, respectively.

Unit 1, Statistical Analysis, was tested in three modules: (1) Collecting and Describing Data, (2) Managing Uncertainty and (3) Analysing and Interpreting Data.

Unit 2, Mathematical Applications, was tested in three modules: (1) Discrete Mathematics, (2) Probability and Distributions and (3) Particle Mechanics.

Both units were tested in two papers: Paper 01, which consisted of 15 multiple-choice items from each module, and Paper 02, which had two questions from each module. Candidates were also required to complete a project for school-based assessment (SBA) on each unit. Candidates who did not do the SBA wrote an alternative Paper 032, consisting of three questions, one from each module. Each unit was tested at three cognitive levels - Conceptual knowledge, Algorithmic knowledge, and Reasoning. Each question in the Paper 02s was worth 25 marks, while each in the Paper 032s was worth 20 marks.

For Unit 1, 691 candidates wrote the 2014 examination and 14 wrote Paper 032, the Alternative to the School-Based Assessment (SBA). For Unit 2, 321 candidates wrote the 2014 examination and 10 candidates wrote the Paper 032, the SBA.

## DETAILED COMMENTS

### UNIT 1

### Paper 01 – Multiple Choice

Performance on the 45 multiple-choice questions on this paper had a mean of approximately 60.28 and standard deviation of 16.30, with scores ranging from 16 to 90.

## Paper 02 – Essay

### Module 1: Collecting and Describing Data

Question 1

The question tested candidates' ability to:

- Distinguish among the following sampling methods – simple random, stratified random and systematic random, and use these techniques to obtain a sample
- Construct and use box-and-whisker plots and cumulative frequency curves (ogives)
- Calculate the standard deviation of ungrouped data

This question was generally well done with approximately 55 per cent of candidates scoring more than 16 out of 25.

Parts (a), (b), (c) and (d) were satisfactorily done, with approximately 78 per cent getting their marks from this section. A few candidates confused the names of the sampling methods, or just did not know the names.

Part (e) seemed to be a challenge for some candidates. In Part (e) (i), some candidates forgot to multiply by 10,000 while others forgot to divide by 6. Part (e) (ii) was poorly done. The few candidates who attempted this part multiplied by 10,000 rather than $\sqrt{10000}$. Some also did not notice that the formula on the sheet is for grouped data.

Part (f) was fairly well done, but some candidates were unable to transition between the cumulative frequency and the class frequency. In Part f (ii), some found 60 per cent of 80 rather than 60 per cent of 50. Others found 60 per cent of 50 but did not interpolate to find the time. The median was also a challenge. Candidates either found half of 80 or used a $(n+1)$ formula.

The box-and-whisker plot was primarily well handled. Some candidates terminated the lower whiskers at ten rather than zero. Inaccurate calculations of the quartiles caused the 'box' to be incorrect though it was evident that they knew how to construct the diagram.

**Solutions:**

(a) (i) employees in the children clothing department
(ii) accessories made an average of $30 000
(b) (i) simple random; (ii) systematic; (iii) stratified
(c) method in (b) iii – stratified
(d) 2
(e) (i) $81 666.67; (ii) $421.96 $\approx$ $422.00
(f) (i) 34; (ii) 53 minutes; (iii) Median = 50 minutes

Question 2

This question tested candidates' ability to:

- Determine the class size and class boundary of the distribution.
- State the disadvantages of presenting data in a grouped frequency distribution.
- Determine the mean, variance and standard deviation of a grouped data.
- Draw a histogram and estimate the mode within the modal class.

For Part (a) (i), most candidates were able to identify the class and the boundaries. In Part (a) (ii), the majority of candidates knew that they had to subtract the two boundaries to obtain the class width. For Part (iii), the majority of candidates was able to explain the disadvantage. However, approximately 10 per cent of the candidates stated the advantage.

For Part (b), the majority of candidates was able to perform the various calculations to obtain the mean, variance and the standard deviation. Many of the candidates were able to use the two formulas that were suggested in the formula sheet. However, 10 per cent of the candidates chose to use other formulas such as the coded method and other variations of the ones given on the formula sheet, for example, $\frac{1}{\sum f}\left(\sum fx^2 - \frac{(\sum fx)^2}{\sum f}\right)$; $s^2 = \frac{\sum f(x-\bar{x})^2}{\sum f}$.
Approximately 10 per cent of candidates treated the data as if it were a sample, and used $(\sum f - 1)$ to respond to the question.

In Part (c), most candidates were able to draw the graph accurately. Several candidates did not use the graph sheet provided, which made the question difficult as they then had to find the correct scales.

In Part (d) (i), approximately 50 per cent of the candidates were able to estimate the mode using the histogram. Some candidates used the interpolation formula. Many candidates were able to identify the class but were unable to calculate the mode.

For Part (d) (ii), the majority of candidates was able to arrive at six for the number of consultations. However, approximately 40 per cent of the candidates were unable to get the correct solution.

**Solutions:**

(a) (i) 29.5, 39.5 (ii) 10 (iii) Data values are lost, further statistics must be estimated.
(b) (i) 32.9 min. (ii) 77.4 (iii) 8.8
(c) histogram
(d) (i) mode = 33.5 (ii) 6

**Module 2: Managing Uncertainty**

Question 3

This question tested candidates' ability to:

- Calculate the probability of events *A*, P (*A*), as the number of outcomes of *A* divided by the number of possible outcomes.
- Use the property that $P(\overline{A}) = 1 - P(A)$, where $P(\overline{A})$ is the probability that A does not occur.
- Calculate the $P(A \cup B)$ $and$ $P(A \cap B)$.
- Calculate the conditional probability $P(A/B)$ where $P(A/B) = \frac{P(A \cap B)}{P(B)}$ is the probability that A will occur given that B has already occurred.
- Identify independent and mutually exclusive events.
- Construct and use tree diagrams.
- Use the probability $P(A \cap B) = P(A) \cdot P(B)$ $or$ $P(A/B) = P(A)$ where *A* and *B* are independent events.

Part (a) (i) tested candidates' ability to complete the tree diagram. The question was well done. One common mistake made was in candidates writing 1 per cent and 2 per cent as 0.1 and 0.2 resulting in incorrect probabilities on the branches. Another common mistake was candidates adding probabilities along branches rather than multiplying. The majority of candidates did Part (a) (ii) correctly. The main mistake occurred with the placement of the decimal point.

For Part (a) (iii), most candidates recognized that they had to sum the defective probability from each machine. In Part (a) (iv), the majority of candidates recognized that it was a conditional probability. However, in most cases, they used the wrong event in the denominator or used the union rather than the intersection in the numerator. In addition, few candidates calculated P (defective) for the numerator. Part (a) (v) was very poorly done. Some candidates attempted to use the Binomial distribution, but this was not necessary to find a solution. Some other candidates recognized that exactly one means that the first was defective and the second was not defective, but they did not switch to say that the first was not defective and the second was defective.

Part (b) (i) was generally well answered. However, candidates had problems using algebra to simplify the expression and solve the equation. For Part (b) (ii), most candidates recognized it was a conditional probability and answered the question correctly. A few candidates used P (*N*) in the denominator instead of P (*M*). Part (b) (iii) was poorly done. Very few candidates constructed a Venn diagram to answer this question, but those who did, answered the question well.

For Part (c) (i), the majority of candidates correctly identified the events were not mutually exclusive; however, the explanation offered was inadequate. Many candidates confused mutually exclusive and independent events when offering their explanations. In Part (c) (ii), candidates appeared to be guessing and very seldom offered a mathematically sound explanation for the lack of independence.

**Solutions:**

(a) (i) 0.4, 0.02, 0.98, 0.01, 0.99

(ii) 0.012 (iii) 0.016 (iv) 0.75 (v) 0.0315

(b) (i) 0.45 (ii) 0.33 (iii) 0.4

(c) (i) Not mutually exclusive since $P(M \cap N) \neq 0$

(ii) Not independent since

$P(M \cap N) = 0.2,\ P(M) \cdot P(N) = 0.27 \neq 0.2 \text{ or } P(N/M) \neq P(N)$

Question 4

This question tested candidates' ability to:

- Identify the conditions of the binomial distribution and use it to calculate probabilities.
- Calculate the expected value, standard deviation and probability from a binomial distribution.
- Use the notation $X \sim Bin\ (n,\ p)$ where n is the number of independent trials and p is the probability of a successful outcome in each trial.
- Calculate the probabilities $P(X = a)$, $P(X > a)$, $P(X < a)$, $P(X \leq a)$, $P(X \geq a)$, or any combination of these where $X \sim Bin(n, p)$.
- Use the normal distribution as a model of data, as appropriate.
- Determine probabilities from tabulated values of the standard normal distribution.
- Solve problems $Z \sim N(0,1)$involving probabilities of normal distribution using Z scores.
- Use normal distribution as an approximation to the binomial distribution where appropriate $(np > 5,\ npq > 5)$ and apply a continuity correction.

Part (a) was generally well done by most candidates.

In Part (b) (i), most candidates knew the formula for the binomial expansion, however approximately 15 per cent of the candidates used it incorrectly. Candidates used the correct values for $p$ and $q$ but with the incorrect exponents, for example, $P(X=3) = {}^{12}C_3\, p^9q^3$instead of $P(X=3) = {}^{12}C_3\, p^3q^9$.

For Part (b) (ii), the majority of the candidates misinterpreted $P(X \geq 2)$, for example,
$1 - P(X=0) + P(X=1) + P(X=2)$ instead of $1 - P(X=0) + P(X=1)$.

A small number of candidates used the alternative method, that is, $P(X=2) + P(X=3) + \ldots + P(X=12)$ but they approximated each probability too early and this resulted in their answers being either truncated or greater than 1.

In Part (c) (i), most candidates observed that it was a normal distribution but some failed to standardize correctly. Other candidates standardized correctly and obtained the correct table value however did not subtract the table value from 1 to give the region specified. Some candidates also failed to obtain the correct table value.

Approximately 98 per cent of the candidates attempted Part (d) (i), which was well done. Part (d) (ii), was also well done as most candidates scored full marks. The minority, however, found the variance (*npq*) but failed to find its square root while others used an incorrect standard deviation formula. A few candidates used the correct formula but took the product of *np* and their *n* value in *npq*.

For example: $np = 200 \times 0.82 = 164$
$npq = 164 \times 0.82 \times 0.18$

instead of : $np = 200 \times 0.82 = 164$
$npq = 164 \times 0.18$

In Part (d) (ii), most candidates did not apply the continuity correction. They also used the standard error instead of the standard deviation when standardizing. A few candidates did not correctly read the tables to obtain the probabilities. Some candidates also subtracted their table value from 1 which was incorrect based on the required region.

**Solutions:**
(a) independent trials; finite number of trials; probability of success same for each trial; two outcomes only for each trial. (Any three correctly stated)
(b) (i) 0.012 (ii) (0.99968)
(c) 0.0668
(d) (i) 164 (ii) $\sqrt{5.43}$ (iii) 0.9732

**Module 3: Analysing and Interpreting Data**

<u>Question 5</u>

This question tested candidates' ability to:

- Calculate unbiased estimates for the population mean, proportion or variance.
- Calculate confidence intervals for a population mean or proportion using a large sample ($n \geq 30$) drawn from a population of known or unknown variance.
- Formulate a null hypothesis $H_0$ and an alternative hypothesis $H_1$.
- Apply a one-tailed test or a two-tailed test, appropriately.
- Relate the level of significance to the probability of rejecting $H_0$ given that $H_0$ is true.
- Determine the critical value from tables for a given test and level of significance.
- Identify the critical or rejection region for a given test and level of significance.
- Evaluate from sample data the test statistic for a given test and level of significance.
- Evaluate a t-test statistic.
- Determine the appropriate number of degrees of freedom for a given data set.
- Determine probabilities from t-distribution tables.
- Apply a hypothesis test for a population mean using a small sample ($n < 30$) drawn from a normal population of unknown variance.

Part (a) (i) a) was well done with most candidates producing the correct response of 7.42 minutes.

For Part (a) (i) b), many candidates found the variance of the set of data as opposed to the unbiased estimator for variance. Several different variations for the formula were used, however only approx. 55 per cent of the responses yielded the correct answer.

In Part (a) (ii) a), most candidates understood the concept of the null and alternative hypothesis. However, many neglected to state the answer using statistical symbols. Additionally, some candidates used $x$ or $\bar{x}$ as opposed to μ, while others stated the incorrect inequalities for the hypotheses.

Part (a) (ii) b) was not done well by most candidates. Most candidates did not realize that a t-test was required. Even those who correctly calculated the degrees of freedom went on to use the wrong distribution.

For Part (a) (ii) c), most of the candidates understood how to calculate the value of the test statistic. However, some confused the order of the numerator, using $\mu - \bar{x}$ (7- 7.42), or used $\hat{\sigma}$ as the denominator, neglecting to use $\frac{\hat{\sigma}}{\sqrt{n}}$.

For Part (a) (ii) d), most candidates were able to come to the correct conclusions based on their solutions from Parts (a) (ii) b) and c). Some candidates, even though they came to the correct conclusion, neglected to also state the decision (fail to reject $H_0$).

Part (a) (ii) e) was generally well done with approximately 87 per cent of the candidates stating the correct assumption of the distribution being normal, which was already stated in the question itself.

Part (b) was generally poorly done. Only approximately 20 per cent of the candidates were able to achieve full marks for the question. Most candidates did not treat the question as a confidence interval for proportions. However, most candidates did correctly state the correct Z value of 1.96.

**Solutions:**

5 (a) (i) a) 7.42mins b) 9.54

(ii) a) $H_o : \mu = 7$ $H_1 : \mu > 7$

b) $t > 1.796$

c) $t_{test} = 0.471$

d) Accept $H_o$, mean = 7

e) the distribution is normal

5 (b) (0.217, 0.475)

Question 6

This question tested candidates' ability to:

- Utilize regression analysis to solve application problems.
- Use chi-square analysis in problem solving.
- Develop null and alternative hypothesis test.
- Interpret the chi-square value as it relates to the null or alternative hypothesis.
- Calculate the product-moment correlation coefficient 'r' and interpret the 'r'.
- Calculate the expected frequency.

In Part (a) (i), most candidates were able to develop the correct null and alternative hypotheses and for Part (a) (ii), the majority of candidates was able to calculate the degrees of freedom correctly. Some of the candidates who attempted Part (a) (iii) had problems reading the $X^2$ values from the table. Many candidates used diagrams to represent their critical region correctly. Part (a) (iv) was well done. Some candidates struggled to write a clear conclusion. However, those who did, wrote excellent conclusions.

For Part b) (i), most candidates were able to calculate the regression equation correctly. For Part (b) (iii), many of the candidates struggled to interpret "*b*" in the regression equation as it relates to the given information. Candidates wrote that "*b*" is the gradient of the regression line $y=7.58 + 0.59x$. Approximately 25 per cent of the candidates were able to interpret the "*b*" value correctly. Part (b) (iv) was well done by most candidates however, some candidates had a problem interpreting the "*r*" value. Almost 95 per cent of the candidates were able to calculate a value for "*r*" correctly, but only 60 per cent were able to explain that "*r*" was a moderate and positive correlation.

Many candidates only stated that it was a positive correlation.

**Solutions:-**

(a) (i) $H_o$ : there is no association between predicted grade and actual grade
$H_1$ : there is an association between predicted grade and actual grade

(ii) (a) degrees of freedom 4 (b) critical region $\chi^2 > 9.488$

(iii) 23.0 (3 significant figures)

(iv) since $\chi^2 = 9.1625 < 9.488$, do not reject $H_o$
There is no association between the predicted grade and actual grade.

(b) (i) $a = 7.56$, $y= 7.56 + 0.587x$

(ii) 29.3

(iii) an increase of 1 mark on the aptitude test will give an increase of 0.587 on the productivity score.

(iv) $r = 0.442$; there is moderate to weak and positive correlation between the mark on the aptitude test and the productivity score.

## Paper 032 – Alternative to the School-Based Assessment (SBA)

### Module 1: Collecting and Describing Data

Question 1

This question tested candidates’ ability to:

- Distinguish between a population and a sample, a census and a sample survey, a parameter and a statistic.
- Distinguish between random and non-random samples.
- Identify sampling methods.
- Outline advantages and disadvantages of sampling methods.

Most candidates had an idea of how to respond to Parts (a) (i) and (ii). However, in Part (a) (i), some candidates referred to a social population rather than a statistical population. Generally, candidates were unable to distinguish between a parameter and a statistic as required in Part (a) (iii). Candidates did not understand Part (a) (iv).

Part (b) (i) was poorly attempted. Although most candidates attempted Parts (b) (ii) most failed to describe the entire process. Several students used a raffle method rather than simple random.

Part (c) (i) was generally well done. Parts (c) (ii) and (iii) were relatively well done. However, in finding the mean, several candidates used a total of seven rather than 100. The median was poorly done with most candidates failing to take into consideration the given frequencies. Part (c) (v) was done correctly by most candidates.

**Solutions:**

| | |
|---|---|
| (a) (i) – (iv) | definitions of terms |
| (b) (i) | classes not unique / overlap / students can be in more than one class |
| (ii) | Assign a two-digit number, starting at 00, to each student.<br>Start at a random point on the table, read, in a stated direction, 2-digit numbers.<br>Note numbers belonging to students until 20 valid values are obtained. |

(c) (i) 100 (ii) 221 (iii) 2.21 (iv) 2 (v) positively skewed

**Module 2: Managing Uncertainty**

Question 2

This question tested candidates' ability to:

- Use basic concepts of probability.
- Define mutually exclusive events.
- Calculate conditional probabilities.
- Identify and use the Binomial distribution.
- Use the normal distribution to calculate probabilities.

Although most candidates attempted Part (a), only a few got full marks. Some of the outcomes were not presented by candidates in Part (a) (i) and probabilities were not calculated accurately in Parts (a) (ii) and (iii).

Part (b) was well done by most candidates. However, most candidates were unable to correctly use the tables for more than two decimal points.

Parts (c) (i) and (ii) were generally well done.

**Solutions:**
(a) (i) BBB, BBR, BRB, RBB, BRR, RBR, BRR, RRR
(ii) 0.315 (iii) 0.813
(b) 0.0694
(c) (i) 0.237 (ii) 42

**Module 3: Analysing and Interpreting Data**

**Question 3**

This question tested candidates' ability to:

- Use the central limit theorem.
- Calculate confidence interval for a proportion from a large sample.
- Give practical interpretation of regression coefficients.
- Make estimates using the regression line.

For Parts (a) (i) to (iv), a few candidates used the proportion. Most candidates had an idea of what a confidence interval should look like and the corresponding $z$-value for CI of 95 per cent. However, substitution for the required proportion was poorly done indicating unfamiliarity with this type of question.

For Part (b) (i), many candidates failed to provide a complete solution with all of the components. The other two sections were generally well done. Candidates did demonstrate weakness in correctly using the tables for more than two decimal points.

For Part (c), candidates failed to interpret correctly the values with respect to the problem. Most candidates only used the terms *gradient* and *intercept*.

**Solutions:**

(a) (0.195, 0.302)

(b) (i) $\bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$. (ii) $\bar{x} \sim N\left(48, \frac{144}{75}\right)$ (iii) 0.0152

(c) (i) The life of the machine when it is new or at time zero.

(ii) For every unit increase in speed that the machine works, its life is reduced by 0.08 hours.

(iii) 3.4 hours

**Paper 031 – School-Based Assessment (SBA)**

The general presentation and standard of the samples submitted were satisfactory. Students selected topics that were generally suitable for their level, and were appropriate to the objectives of the syllabus. There were a few students who used techniques that went beyond the level expected.

Generally, the sizes of the samples submitted were adequate and marks were entered correctly onto the moderation forms.

<u>Project Title</u>

Approximately 30 per cent of the titles in the samples submitted were too vague. For instance titles like, ‘Free Throws’ or ‘Multiple Choice Test’ are too vague. Some students simply stated a mathematics topic such as ‘Critical Path Analysis’ or ‘Linear Programming’ for their title. Students should endeavour to relate their title to some real life situation. It is imperative that teachers explain this a bit more to students.

<u>Purpose of Project</u>

Variables were not clearly defined in many cases and in some cases, no variables were identified.

<u>Method of data collection</u>

Although this section was well done, students must describe how they collected the data and state which sampling technique is being used.

Presentation of Data

In this section, some of the diagrams and charts presented by students were unrelated to the purpose. Nonetheless, many students demonstrated mastery in the construction of histograms, pie charts, stem and leaf and box plots. Too many students presented long tables of data in this section. Students are reminded to append these at the end of the project in the appendix. In general, the presentation of data was well done.

Statistical Knowledge/Analysis of Data

The majority of students demonstrated a good grasp of statistical concepts in the syllabus and scored well in this area. However, a sizable minority of students did many calculations but gave no explanation for them. Some fundamental things about the chi-square test were of some concern:

- Using 2 x 2 contingency table without applying the Yates correction.
- Not merging cells with frequencies less than 5.

Discussion of Finding/Conclusion

This section was in many cases too verbose and much of it was irrelevant to the purpose of the project. A few students did not relate the findings to the purpose of the project.

Communication of Information

Only a small minority of students lost marks in this section. Students are reminded to proof read their projects before submission.

List of References

Most students were able to score well for this section but usually just one reference was given. Students should endeavour to use multiple references preferably using the APA style.

## UNIT 2

### Paper 01 – Multiple Choice

Performance on the 45 multiple-choice items on Paper 01 produced a mean of approximately 63.61 out of 90, standard deviation of 16.51 and scores ranging from to 90.

### Paper 02 – Essay

**Module 1: Discrete Mathematics**

Question 1

This question tested candidates' ability to:

- Determine the converse, inverse and contrapositive of a preposition.
- Establish the truth values of the converse, inverse and contrapositive of a preposition.
- Determine whether a preposition is a tautology or a contradiction.
- Construct the truth table values of compound prepositions that involve conjunctions, disjunctions and negations.
- Determine the truth values of conditional prepositions.
- Derive a Boolean expression from a given switching or logic circuit.
- Represent a Boolean expression by a switching or logic circuit.
- Use the laws of Boolean algebra to simplify Boolean expressions.

In Part (a), some candidates had difficulty in distinguishing between the contrapositive and the inverse.

For Part (b), the majority of candidates demonstrated competence in constructing a truth table but most candidates had difficulty formulating the inverse. Additionally, a significant minority of candidates were unable to provide the correct number of permutations of inputs to properly evaluate the outputs.

For Part (c), most candidates were able to generate the correct outputs and identify and justify the output as a tautology. However, some candidates were unable to provide the correct number of permutations of inputs to properly evaluate the outputs.

Generally, candidates did Part (d) well. However, rather than being consistent with their use of notation, a number of candidates used combinations of alternative Boolean operator notations, for example, $(a \wedge b) + c'$.

For Part (e), candidates were able to adequately answer the question, with the vast majority demonstrating a very good knowledge of the distributive law. However, many candidates were unable to represent B and C in parallel to A. Additionally, some candidates produced logic circuits instead of switching circuits.

**Solutions:**

(a) $q \rightarrow {\sim}p$

(b)

| p | q | ${\sim}p$ | ${\sim}p \rightarrow q$ |
|---|---|---|---|
| T | T | F | T |
| T | F | F | T |
| F | T | T | T |
| F | F | T | F |

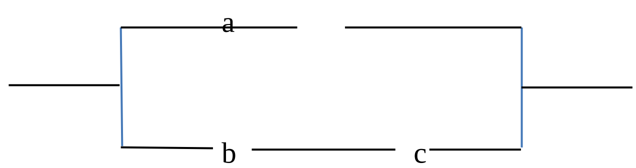
(c) (i)

| p | q | r | $p \rightarrow q$ | $q \rightarrow r$ | $(p \rightarrow q) \vee (q \rightarrow r)$ |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | T | F | T | F | T |
| T | F | T | F | T | T |
| T | F | F | F | T | T |
| F | T | T | T | T | T |
| F | T | F | T | F | T |
| F | F | T | T | T | T |
| F | T | F | T | T | T |

(ii) Tautology because all the truth vales are true.

(d) ${\sim}(a \wedge b) \vee {\sim} c$

(e) (i)




(ii) $(a \vee b) \wedge (a \vee c)$

<u>Question 2</u>

This question tested candidates' ability to:

- Use the activity network algorithm to draw a network diagram to model a real world problem.
- Calculate the earliest start time, latest start time and float time.
- Identify the critical path in an activity network.
- Derive a Boolean expression from a given switching or logic circuit.
- Establish the truth value of compound prepositions that involve conjunctions, disjunctions and negations.

Overall, the question was very well done with over 90 per cent of candidates obtaining a score between 16 and 25 and almost 40 per cent obtaining a perfect score.

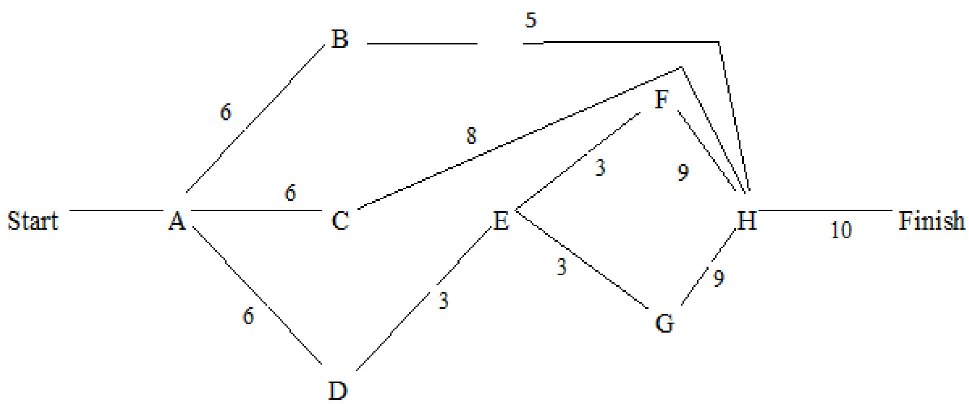
Part (a) (i) was relatively well done by candidates. However there were too many variations in the methodology used to construct the activity networks. The syllabus (Specific Objective (c) 4.) states that "activities will be represented by vertices and the duration of activities by edges".

For Part (a) (ii), most candidates successfully calculated the earliest start times, latest start times and float times from their diagrams.

Part (a) (iii) was very well done, with the majority of candidates identifying critical paths. However, many candidates placed simultaneous events on a single critical path, for example, start-A-D-E-F-G-H-finish, instead of start-A-D-E-F-H-finish and start-A-D-E-G-H-finish.

Part (b) was generally very well done, although a few candidates confused the symbols for disjunction and conjunction.

**Solutions:**




(a) (i)

(ii)

| Activity | EST | LST | FLOAT |
|---|---|---|---|
| A | 0 | 0 | 0 |
| B | 6 | 16 | 10 |
| C | 6 | 13 | 7 |
| D | 6 | 6 | 0 |
| E | 9 | 9 | 0 |
| F | 12 | 12 | 0 |
| G | 12 | 12 | 0 |
| H | 21 | 21 | 0 |

(iii) Start A D E G H Finish

Start A D E F H Finish

(b) (i) $p \wedge (p \vee \sim q)$

(ii)

| p | q | $\sim q$ | $p \vee \sim q$ | $p \wedge (p \vee \sim q)$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | T | T | T |
| F | T | F | F | F |
| F | F | T | T | F |

**Module 2: Statistical Analysis**

Question 3

This question tested candidates' ability to:

- Calculate the number of selections of n distinct objects taken r at a time.
- Calculate a number ordered arrangements with restrictions.
- Calculate probabilities of events using appropriate counting techniques.
- Calculate probabilities associated with conditional, independent or mutually exclusive events.

For Part (a), most candidates obtained the desired probability using $P(A' \cap B') = 1 - P(A \cup B)$. Marks were lost where candidates treated the events as being mutually exclusive.

For Part (b), candidates who used combinations generally obtained full marks while those using the fractional probability method usually did not recognize that there were three ways to get the solution and so did not multiply by three in Part (a) and similarly by six in Part (b). Many candidates failed to recognize that sampling was without replacement and solved these items as with replacement.

For Part (c), most candidates recognized that the conditional probability was required but the accuracy of calculations was weak. Fewer candidates used the combinations method in this part of the question. Similarly, many candidates failed to recognize sampling without replacement.

For Part (b) (ii), where full marks were not received in this item, candidates obtained partial credit for either obtaining part of the numerator or the denominator correct.

Few candidates achieved more than three marks for this item with many candidates being able to draw the diagram but unable to use the probabilities. The product of five and four factorial was seen more often than the two factorials and recognizing that having two directions required multiplying the probability by two.

**Solutions:**
(a) 0.34
(b) (i) a) 0.0975 b) 0.0887
(c) (i) 0.0702 (ii) 0.0542 (iii) 11520

Question 4

This question tested candidates’ ability to:

- Calculate and use the expected value and variance of linear combinations of independent random variables.
- Model practical situations using a Poisson distribution.
- Solve probability problems involving the normal distribution.

Part (c) (i) was not marked since the paper had a print error that rendered the problem unsolvable. The item should have read $P(X + Y = 3)$ and not $P(X + Y) = 3$. Where attempted, the majority of candidates did well.

For Parts (a) (i) and (ii), the majority of candidates scored at least three of the six marks. Several candidates used the Binomial distribution rather than the Poisson distribution.

For Part (b) (i), most candidates attempted to use the correct formula; however, some had difficulty getting the denominator correct. Nearly all candidates were able to read the table correctly but failed to interpret its use correctly (needing to subtract from one).

Part (b) (ii) was poorly done. Most candidates were unable to correctly manipulate the table value for the negative z-value.

Part (c) was done exceptionally well by most candidates. They were aware of how to evaluate the expected values and variances and in a few cases, after calculating the correct expected value, proceeded to further divide by ten.

For Part (c) (iii) a), the majority of candidates correctly interpreted $E(3x - 2y)$ and correctly substituted their values to evaluate the expression.

Most candidates who attempted Part (c) (iii) b) failed to change the subtraction to addition. A few candidates failed to take the square of the coefficients.

**Solutions:**

(a) (i) 0.168 (ii) 0.9999 = 1 (3 s.f.)
(b) (i) 0.271 (ii) 0.322
(c) (i) Ignored due to printed error
(ii) (a) 1.3 (b) 0.61 (c) 2.05 (d) 1.75
(iii) (a) -0.2 (b) 12.49

**Module 3: Particle Mechanics**

Question 5

This question tested candidates' ability to:

- Distinguish between displacement and distance, velocity and speed.
- Draw and use a displacement time graph.
- Identify forces acting on a body in a given situation.
- Solve problems involving concurrent forces in equilibrium.

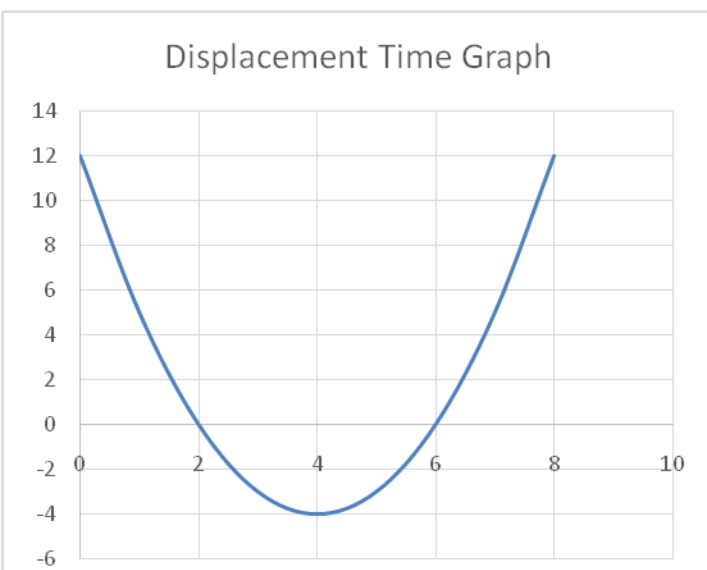
In Part (a), most candidates drew the graph accurately using appropriate scales. Some candidates got confused with distance vs displacement and velocity vs speed

In Part (a) (ii) c), candidates confused displacement = 0 for velocity = 0, that is, the velocity = 0 when the gradient of the displacement time graph is zero.

Part (b) was very poorly done. For Part (b) (i), candidates did not recognize that S was the resultant of the frictional and normal forces, thus this caused some confusion in the remainder of the question. In Part (b) (ii), candidates resolved vertically and horizontally but did not do so properly. The knowledge of trigonometry was not sufficient for this question. An alternative approach (Lami's theorem) would have simplified the question but this was not used by candidates. No candidate got full marks. Part (b) (iii) was poorly done.

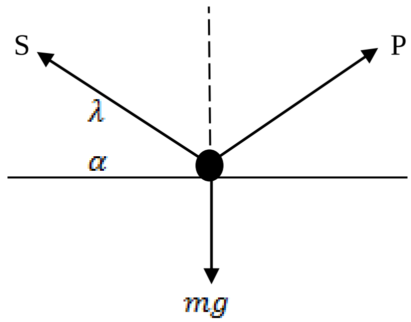
**Solutions:**

(a) (i)



(ii) a) $17\ m$ b) $-3ms^{-1}$ c) $t = 4$

(c) (i)




(ii) $P = mg\sin\lambda$; $\alpha = \lambda$ (iii) $P = 5m\ N$

Question 6

This question tested candidates' ability to:

- Formulate the equation of trajectory of a projectile.
- Use the equations of motion for a projectile to determine the angle of projection and time of flight.
- Formulate and solve first order differential equations as models of linear motion of a particle.

Part (a) was well done. Candidates generally were able to derive the equation of the trajectory. However, many candidates simply stated the trajectory rather than deriving it as asked in the question. Probably candidates did not understand the meaning of the word 'formulate'.

Part (b) (i) was poorly done. Candidates used the value of $y = 4$ instead of $-4$ and hence were unable to achieve the required equation. Although Part (b) (ii) was generally well done, many candidates chose to input values into their calculator instead of using the quadratic formula and showing appropriate steps.

In Part (b) (iii), some candidates used the simple formula $d = \frac{s}{t}$, instead of $x = (u\cos\theta)t$ and then proceeded to find *t*. Many candidates also did not approximate their value *t* to the nearest second.

In Part (c), many candidates assumed that the acceleration was constant and therefore used the incorrect formula. Candidates also integrated incorrectly by ignoring the constant of integration which led to obtaining the incorrect answer.

Very few candidates did Part (d) well. Resolution of forces was poorly done. Constant acceleration/force was incorrectly assumed. Many candidates also failed to calculate $\int vdv$.

**Solutions:**

(a) $y = x\tan\alpha - \frac{gx^2\sec^2\alpha}{2u^2}$ (b) (ii) $\alpha = 11^0$ or $77^0$ (iii) $t = 2s$ (c) $T = \frac{2}{3}s$

(d) $v = 5.04\ ms^{-1}$

**Paper 032 – Alternative to the School-Based Assessment (SBA)**

<u>Discrete Maths</u>

<u>Question 1</u>

This question tested candidates' ability to:

- Formulate simple propositions.
- Negate simple propositions.
- Construct compound propositions.
- Establish truth values of simple propositions.
- Compound propositions that involve conjunctions, disjunctions and negations.
- Represent a Boolean expression by a switching or logic circuit.
- Convert a maximization problem into a minimization problem.
- Solve a minimization assignment problem by the Hungarian Algorithm.
- Identify vertices and sequence of edges that make up a path.
- Determine the degree of a vertex.

Part (a) was generally well done by most candidates. Using either T and F or 0 and 1, candidates were able to complete the truth table successfully. However, many did not explicitly show or indicate the equivalent columns.

In Part (b), the logic diagram was correctly constructed by most candidates.

For Part (c), most candidates did not show all steps of the Hungarian algorithm with a few going directly to the allocations.

For Part (d), many candidates gave the paths from A to C rather than the requested paths from C to A.

**Solutions:**

(a) (i) $\sim p \wedge (p \rightarrow q)$

(ii) truth tables indicating equivalent columns

(b) logic gate diagram

(c) from Hungarian algorithm A assigned to IL, B to GX, C to AB, D to YG
(d) (i) CDA, CBA, CBDA, CDBA
(ii) 4

**Probability and Distributions**

Question 2

This question tested the candidates' ability to:

- Use the cumulative distribution function F(x).
- Use the result $P(a < X < b) = F(b) - F(a)$.
- Calculate expected value and variance from a given distribution.
- Calculate and use the expected value and variance from a linear combination of independent random variables.

For Parts (a) (i) to (v), most candidates were able to correctly calculate the constant in Part (a) (i) however, most of them did not seem to understand what was required for the remaining four parts of this section. In Part (a) (ii), candidates were required to find the area under the curve using integration rather that substituting the given values into the function and then evaluating. Most candidates knew that to find the median they were to use $F(x) = \frac{1}{2}$, but did not know what to do with it. In Part (a) (iv), candidates did not seem to recognize the need to differentiate in order to arrive at the probability density function. Similarly for Part (a) (v), candidates did not recognize the need to use integration to find the required probability.

Part (b) was well done.

**Solutions:**

(a) (i) $\frac{1}{3}$ (ii) $\frac{1}{3}$ (iii) 4.5

(iv) $f(x) = \begin{Bmatrix} \frac{1}{3} & 3 \leqslant x \leqslant 6 \\ 0 & otherwise \end{Bmatrix}$ (v) $\frac{9}{2}$

(b) (i) $3\frac{1}{2}$ (ii) $12\frac{1}{2}$

**Module 3: Particle Mechanics**

Question 3

This question tested the candidates' ability to:

- Calculate velocity, acceleration and time using the equations of motion.
- Calculate the work done by a constant force.
- Represent the contact force between two surfaces in terms of its normal and frictional component.
- Resolve forces on particles on an inclined plane.

This question was not well done as candidates had problems with resolving forces. Part (a) was generally well done though some candidates did not convert the speed of the body to $ms^{-1}$.

In Part (b), many candidates did not use the formula $W = Fs$. One candidate used $P = Fv$ to solve the question. However, the answer was left as the mass and not the weight.

Part (c) was poorly done. Candidates did not resolve forces at the $3m$ mass or the $4m$ mass and hence, could not solve the problem.

**Solutions:**
(a) (i) $a = -6.25\ ms^{-1}$ (ii) $t = 4s$ (b) $W = 450\ N$ (c) $a = 2.64\ ms^{-1}$

**Paper 031 – School-Based Assessment (SBA)**

Generally, the topics chosen were suitable for students at this level. The majority of the projects sampled were from the Discrete Mathematics and Probability and Distribution segments of the syllabus. A few interesting projects were seen in mechanics as well. These projects were appropriate and were generally well done by candidates.

Teachers' assessment of the projects was generally satisfactory. There was generally close agreement between the marks awarded by teachers and those by the CXC Moderator.

Statement of Task

The majority of students scored full marks in this section. Approximately 30 per cent of students did not identify the variables. A small minority of students gave irrelevant information in this section and seemed unclear on what was required.

Data Collected

This section was generally well done but in a small number of cases, the data collected was not realistic.

Mathematical knowledge/Analysis

Most students scored well in this area and demonstrated a good grasp of the mathematical principles taught.

Evaluation

No insights into the nature of and resolution of problems encountered in the tasks were seen for most students but most were able to score the three marks for the conclusion.

Communication of Information

This section was generally well done by most students.

## STRENGTHS AND WEAKNESSES

Strengths

- Recognition of conditional probability
- Knowing when and how to standardize
- Use of the z-table to find critical values
- Differentiating between distance and displacement
- Ability to calculate '*r*' the product moment correlation co-efficient correctly

Weaknesses

- Confusion about the concepts of independence and mutually exclusive;
- Confidence interval for proportionality — proportion was treated by many as if it was a mean;
- Ignoring the constant of integration
- Resolution of forces in the particle mechanics module
- Deficiency in knowledge of the Lami's Theorem; this method was attempted by a negligible number of students, who proceeded to use it incorrectly.

## RECOMMENDATIONS

- Candidates should practice more problems involving the use of the chi-squared test, normal distribution and mechanics.
- Candidates need to have a comprehensive understanding of the various modules, paying special attention to the mechanics module of Unit 2.
- Candidates are still having difficulty with (i) algebraic manipulations and (ii) problem solving especially in the mechanics module. Pre-requisite skills — namely finding the mean and standard deviation for statistics, and trigonometry for Particle Mechanics — need to be reviewed before starting the modules.
- It would appear from the performance of Module 3 in both Units, that these modules are hurriedly done. More time needs to be allocated to these modules.
- Tables and formula sheets that are used in the examination should be the same ones used in the classroom.
- Candidates need to get used to writing their final answers correct to three significant figures. They however, should be cautioned against premature approximation.
- Teachers should instruct students to cite references to inculcate good research skills from early.

Candidates also need to:

- Familiarize themselves more with different sampling methods.
- Know how to calculate the mode from a histogram.
- Use the normal approximation to the binomial distribution.
- Recognize and use the continuity correction accurately.
- Identify when and how to use the t-test.
- Use the equations of motion for a projectile to determine the angle of projection and time of flight.
- Be able to distinguish between logic gates and switching circuits.
- Familiarize themselves with Lami's Theorem.
- Be able to distinguish between logic gates and switching circuits.