CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

MAY/JUNE 2014

MATHEMATICS
GENERAL PROFICIENCY EXAMINATION

GENERAL COMMENTS

The General Proficiency Mathematics examination is offered in January and May/June each year. There was a candidate entry of approximately 90,100 in May/June 2014 and 50 per cent of the candidates earned Grades I–III. The mean score for the examination was 75 out of 180 marks.

DETAILED COMMENTS

Paper 01 — Multiple Choice

Paper 01 consisted of 60 multiple–choice items. It was designed to provide adequate coverage of the content with items taken from all sections of the syllabus. Approximately 70 per cent of the candidates earned acceptable grades on this paper; the mean score was 33 out of 60 marks. This year, 271 candidates earned the maximum available score of 60.

Paper 02 — Structured Questions

Paper 02 consisted of two sections. Section I comprised eight compulsory questions for a total of 90 marks. Section II comprised three optional questions: one each from Algebra, Relations, Functions and Graphs; Measurement, Trigonometry and Geometry; and Vectors and Matrices. Candidates were required to answer any two of the three questions from this section. Each question in this section was worth 15 marks. The mean score for this paper was 42 out of 120 marks.

Compulsory Section

<u>Question 1</u>

This question tested candidates' ability to

- use the calculator to divide decimals
- evaluate the square and the square root of rational numbers
- write a rational number correct to three significant figures
- solve problems involving hire purchase.

The question was attempted by 99 per cent of the candidates, 12.9 per cent of whom earned the maximum available mark. The mean mark was 7.21 out of 12.

In Part (a), candidates generally provided satisfactory responses. Those who used the calculator were able to perform the computations but did not approximate the result of the third computation to three significant figures.

In Part (b), a large number of candidates experienced difficulty in deriving the number of buckets of gravel that should be mixed with 4 buckets of cement. Two popular incorrect responses to this part of the question were: $\frac{4}{11} \times 6$ and $\frac{6}{11} \times 4$. Many candidates attempted to use the proportion 1:4:6 and any of the numbers given in the question to compute the required values.

In Part (c), most candidates were able to calculate the hire purchase price by adding the down payment to the total paid by instalments. However, some candidates incorrectly interpreted hire purchase to mean simple interest, and hence produced the following result: $\text{HP} = \frac{\text{P} \times \text{R} \times \text{T}}{100} = \frac{350 \times 120 \times 10}{100}$. Another common error in the solution to this part of the question was: Amt saved = Cost price − Hire purchase price.

Solutions

(a) **(i)** **350** **(ii)** **2.55** **(iii)** **15.7**

(b) **(i)** **24** **(ii) a)** **5** **b)** **30**

(c) **(i)** **\$1550** **(ii)** **\$251**

Recommendations

Teachers should provide students with a wide range of real life problems which require the appropriate use of mathematical computations including ratio and proportion. Students are encouraged to use calculators to perform mathematical calculations.

<u>Question 2</u>

This question tested candidates' ability to

- simplify algebraic fractions
- translate worded problems into algebraic expressions
- write an algebraic equation from information given in a flow diagram
- solve a pair of linear equations in two unknowns.

The question was attempted by 99 per cent of the candidates, 6.9 per cent of whom earned the maximum available mark. The mean mark was 5.69 out of 12.

The performance of candidates on this question was satisfactory. In Part (a), while most candidates were able to apply the algorithm for adding two algebraic fractions, they generally encountered problems with the use of directed numbers and hence did not simplify the numerator correctly.

In Part (b), candidates experienced difficulty with obtaining the right side of each equation. Two common incorrect responses were: $(i)\ x + 4 = \frac{x}{0.5}$, and (ii) $x^2 - 6 = x^2 + 9$.

In Part (c), most candidates were able to translate the information given in the flow charts into algebraic expressions and formulae but encountered difficulty with changing the subject of the formula to find the value of x when the value of y is given.

In Part (d), the majority of candidates identified a suitable strategy for solving the pair of simultaneous equations. However, when elimination was the chosen strategy, candidates who equated the coefficients of y proceeded to subtract the two equations even when they had $+3y\ and - 3y$ with which to work. Hence, the solution could not be completed.

Solutions

(a) $\frac{7x-5}{12}$

(b) **(i)** $x + 4 = \frac{x}{2} + 10$ **(ii)** $x^2 - 6 = 2x + 9$

(c) **(i)** $3x + 5$ **(ii)** **17** **(iii)** **1** **(iv)** $x = \frac{y-5}{3}$

(d) $x = 3,\ y = 1$

Recommendations

Teachers should emphasize the correct application of the distributive property, reinforce the distinction between an expression and an equation, provide candidates with opportunities to refresh their use of directed numbers and clarify in the minds and students when to add or subtract equations in the process eliminating a variable from a pair of simultaneous equations.

Question 3

This question tested candidates' ability to

- determine the number of elements in a set of integers bounded by two given integers
- list the elements in a defined set
- draw Venn diagrams to represent the relationship among three sets
- use a ruler to measure the length of a line segment
- use a protractor to determine the measure of an angle
- construct an angle of 60°
- construct a perpendicular to a line from a point outside the line.

The question was attempted by 98 per cent of the candidates, 2.4 per cent of whom earned the maximum available mark. The mean mark was 5.49 out of 12.

In Part (a), candidates were generally able to list the even numbers in their universal set but encountered difficulty with identifying the multiples of three from the same set. Most candidates were able to identify the intersection of the two sets they listed even when they misinterpreted the word "between". In addition, they demonstrated good proficiency in representing $A \cap B,\ A \cap B'$ and $B \cap A'$ on the Venn diagram but showed little understanding of what should occupy the region: $(A \cup B)'$.

In Part (b), while candidates were generally able to draw lines accurately and to construct an angle of 60°, they nevertheless encountered difficulty with identifying the angle to be measured and constructing a perpendicular to a given line from a point outside the line.

Solutions

(a) (i) 14 (ii) $\boldsymbol{A = \{12, 14, 16, 18, 20, 22, 24\}}$ **(iii)** $\boldsymbol{B = \{12, 15, 18, 21, 24\}}$

(b) (ii) $\boldsymbol{44°}$

Recommendations

In addition to teaching the required content, teachers should also focus on the mathematical terms in respective subject areas. Many students may know the content but experience challenges when exposed to terms such as between, inclusive, exclusive, at most, at least, in the context of mathematics.

Question 4

This question tested candidates' ability to

- use the scale of a map to determine lengths
- use the scale of a map to determine areas on a map
- use the scale of a map to determine actual distances and areas on the land.

The question was attempted by 92 per cent of the candidates, 1.1 per cent of whom earned the maximum available mark. The mean mark was 3.13 out of 10.

While most candidates interpreted the scale correctly, they were generally unable to transfer this understanding to the concept of area. Moreover, of those candidates who correctly computed that $1\ cm = 0.00001\ km$, few were able to continue the process to calculate the land represented by $8\ cm$ on the map.

In Part (b), there were errors in the measurement of the line segment LM with a large number of candidates stating the length as 7 cm. Similarly, in Part (c), determining the area of the forest reserve on the map proved difficult for many candidates. Even when they counted the number of squares correctly, they proceeded to square this value. Many candidates could not make the conversion from cm^2 to km^2 although they were given the conversion factor from kilometres to centimetres.

Solutions

(a) **(i)** **50 000 cm** **(ii)** $(\mathbf{50\ 000})^{\mathbf{2}}\ \boldsymbol{cm^2}$ **(iii)** $\mathbf{0.5}\ \boldsymbol{km}$

(b) **(i)** $\mathbf{8}\ \boldsymbol{cm}$ **(ii)** $\mathbf{4}\ \boldsymbol{km}$

(c) **(i)** $\mathbf{15}\ \boldsymbol{cm^2}$ **(ii)** $\mathbf{3.75}\ \boldsymbol{km^2}$

Recommendations

Students require additional practice in applying scales to the solution of problems related to distances and areas. It is suggested that authentic examples be developed using maps of the school and immediate community.

<u>Question 5</u>

This question tested candidates' ability to

- describe a translation in the plane
- draw the image of a triangle after undergoing an enlargement
- solve problems involving angles of elevation
- solve problems involving trigonometric ratios.

The question was attempted by 90 per cent of the candidates, 1.7 per cent of whom earned the maximum available mark. The mean mark was 2.19 out of 12.

The performance of candidates on this item was generally unsatisfactory. In Part (a), candidates demonstrated some competence in determining the image under an enlargement. However, many of them did not use the given centre of enlargement even though the correct identified the origin as the centre of enlargement. Moreover, they were generally able to describe in words the transformation that mapped triangle ABC onto triangle $A''B''C''$, although only few of them used the conventional vector notation.

In Part (b), candidates in general stated that the angle of elevation of T from P is $40°$ even though a small number chose the complement of that angle and stated the result as $50°$. Candidates, in an attempt to determine the length of FP, correctly chose the tangent ratio. However, to determine the measure of the angle of elevation of T from Q, some candidates used Pythagoras' theorem to find TP, and then proceeded to use the sine rule or the cosine rule to find the measure of angle TQP. The result was more easily obtained using the right-angled triangle TQF.

Solutions

(a) **(ii)** $\boldsymbol{M\ is\ a\ translation\ by\ the\ vector}\ \begin{pmatrix}\mathbf{4}\\ \mathbf{-5}\end{pmatrix}$

(b) **(i)** $\mathbf{40°}$ **(ii)** $\mathbf{95.3}\ \boldsymbol{m}$ **(iii)** $\mathbf{20.6°}$

Recommendations

Teachers should give attention to the description of transformations, ensuring that students are familiar with the language used to describe such transformations. Students should be guided as to when it is appropriate to use trigonometric ratios and when to use the more complex trigonometric formulae.

Question 6

This question tested candidates' ability to

- read coordinates of points plotted on a graph
- determine the gradient of a line drawn on a graph
- determine the equation of a line drawn on a graph
- determine the equation of a line which is parallel to a given line and which passes through the origin
- estimate the gradient of the tangent at a given point on the graph of a quadratic function.

The question was attempted by 80 per cent of the candidates, 2.5 per cent of whom earned the maximum available mark. The mean mark was 2.59 out of 11.

In Part (a), candidates were generally successful in determining the values of $x\ and\ y$ by taking readings from the graph. However, some candidates used the equation of the curve to derive these values. For example: $y = x^2\ so\ x = \sqrt{y} = \sqrt{9} = 3$ and $y = (-1)^2 = 1$.

In Part (b), most candidates used the formula: $m = \frac{\Delta y}{\Delta x}$ to calculate the gradient of MN. However, there were instances when they had correctly substituted into this formula and failed to determine the correct value of the gradient because of challenges with the subtraction of directed numbers. Moreover, when asked to write the equation of the line which passes through $(0, 0)$ and parallel to MN, some candidates did not recognize that the gradients were the same but proceeded to use the negative reciprocal of the gradient of MN as the gradient of the line whose equation was required.

In Part (c), candidates appeared not to be acquainted with concept of a tangent to a curve at a given point. Many of them simply drew a horizontal line at $(2, 4)$.

Solutions

(a) **(i)** $\boldsymbol{x = 3}$ **(ii)** $\boldsymbol{y = 1}$

(b) **(i)** **2** **(ii)** $\boldsymbol{y = 2x + 3}$ **(iii)** $\boldsymbol{y = 2x}$

(d) **The gradient of tangent at (2, 4) is 4**

Recommendations

Teachers should guide students through the concept of drawing the tangent to a curve at a point and to demonstrate some applications of such tangents. They should reinforce the distinction between the gradients of parallel and perpendicular lines.

Question 7

This question tested candidates' ability to

- complete a frequency table
- determine the mode of a given set of data
- estimate the mean of a frequency distribution
- determine the probability of a simple event from randomly sampling a data set.

The question was attempted by 95 per cent of the candidates, 4.8 per cent of whom earned the maximum available mark. The mean mark was 5.27 out of 11.

Candidates performed moderately on this question. The majority were able to complete the tally column and the associated frequency column from the raw data provided, as well as the column with $\boldsymbol{f} \times \boldsymbol{x}$. However, many candidates did not understand the significance of these values since they were not used to compute the mean of the data set. Moreover, when candidates correctly stated the formula for deriving the mean number of books per bag, many of them proceeded to divide the total number of books by a number different from their value of $\sum f$.

In Part (d), in determining the probability that a student chosen at random had less than 4 books in his/her bag, many candidates were able to correctly determine only one of the two quantities, that is, either the favourable outcomes or the possible outcomes. Several candidates quoted a value of 10 as the probability, which is a clear indication that they did not appreciate the possible range of a probability measure.

Solutions

(a)

$(\boldsymbol{x})$	**Tally**	**Frequency(f)**	$\boldsymbol{f} \times \boldsymbol{x}$
3	~~////~~	**5**	**15**
4	~~////~~ /	**6**	**24**
5	~~////~~ / /	**7**	**35**
6	////	**4**	**24**
7	///	**3**	**21**

(b) **mode = 5 text books**

(c) **Total number of books is 127** **(ii)** **mean = 4.2 books per bag**

(d) $\boldsymbol{P(text\ books} < 4) = \frac{\mathbf{10}}{\mathbf{30}} = \frac{\mathbf{1}}{\mathbf{3}}$

Recommendations

Teachers need to provide students with more practice in identifying the measures of central tendency of a data set, especially for data presented in a table. They should reinforce the convention for representation of tally. Emphasis should be given to the fact that a probability value lies between 0 and 1.

<u>Question 8</u>

This question tested candidates' ability to

- draw the fourth diagram in a given sequence of diagrams
- complete a table to show the values in a sequence of numbers
- derive the general rule representing the patterns in a sequence.

The question was attempted by 95 per cent of the candidates, 6.2 per cent of whom earned the maximum available mark. The mean mark was 6.60 out of 10.

Candidates' responses to this question were generally good. They were able to draw the fourth figure in the sequence of figures even though they did not always insert the correct number of dots on each side of the pentagon.

In Part (b) (i), most candidates were able to follow the pattern in the formula column and to determine the correct number of dots on each figure by counting even when they did not write the correct formula. Writing an algebraic expression for the number of dots associated with each term in the sequence proved challenging for most candidates.

Solutions

(b) **(i)** $\boldsymbol{formula: 5 \times 6 - 5; \quad n = 25}$ **(ii)** $\boldsymbol{formula: 5 \times 7 - 5; \quad n = 30}$

(c) $\boldsymbol{n = 5(f+1) - 5 = 5f}$

(d) $\boldsymbol{f = 29}$

Recommendations

Teachers need to extend the teaching of patterns and sequences to include deriving the general algebraic formulae for these sequences.

Optional Section

Question 9

This question tested candidates' ability to

- determine the value of x for which a rational function is undefined
- evaluate the composite of a function
- determine the inverse of a rational function
- use a table of values to draw the graph of a non-linear relation
- interpret a graph to determine unknown information.

The question was attempted by 73 per cent of the candidates, less than 1 per cent of whom earned the maximum available mark. The mean mark was 4.74 out of 15.

Candidates' performance on this question was generally unsatisfactory.

In Part (a), the majority of candidates were unable to determine the value of x for which a rational function is undefined. Further, although many of the candidates were able to correctly substitute $f(5)$ into $g(x)$, they lacked the algebraic skills to simplify the response. Many candidates, in an attempt to derive the inverse of $f(x)$, reached as far as interchanging $x \; and \; y$ in the original equation. However, they did not demonstrate the algebraic skills to correctly proceed to make y the subject of the formula in their reversed mapping.

In Part (b), the majority of candidates demonstrated a high level of proficiency in using the correct scales and plotting the given points. Nevertheless, a large number of these candidates connected the points with straight lines instead of drawing a parabola as required. Also, a large number of candidates were unable to determine the average speed of the ball during the first two seconds of its motion, and among those who provided an answer, the correct unit of speed was not used. Moreover, the majority of candidates could not determine the speed of the ball at $\mathrm{t} = 3$ since they did not associate this speed with the gradient of the tangent at the greatest height reached by the ball.

Solutions

(a) **(i)** $x = -1$ **(ii)** $gf(5) = \frac{43}{3}$ **(iii)** $f^{-1}(x) = \frac{x-7}{2-x}$

(b) **(ii)** a) $40\ ms^{-1}$ b) $0\ ms^{-1}$

Recommendations

Teachers should strengthen basic algebraic and computational skills of students prior to teaching the functions. Students must be able to differentiate between the composite and the inverse function. The concept of tangent at a point on a curve needs to be reinforced.

Question 10

This question tested candidates' ability to

- use the properties of circles and circle theorems to determine the measures of angles
- use cosine and sine rules to solve problems related to bearings.

The question was attempted by 43 per cent of the candidates, 2.2 per cent of whom earned the maximum available mark. The mean mark was 2.70 out of 15.

Candidates' performance on this question was generally unsatisfactory.

In Part (a), which required the use of the angle properties of triangles and circles, many candidates based their reasoning on the properties of triangles only instead of referring to circle theorems.

In Part (b), candidates lacked proper understanding of bearings. A common response to the calculation of the bearing of P from Q was $180° - (66° + 54°)$ instead of $(66° + 54°)$. In addition, candidates made the assumption that they were working with a right angled triangle and proceeded to employ trigonometric ratios in an attempt to calculate the length of PR and the measure of angle QPR. A simple application of the cosine formula followed by an application of the sine formula was required to complete these calculations.

Solutions

(a) **(i)** **140º** **(ii)** **22º** **(iii)** **110º**

(b) **(i)** $120°$ **(ii)** $PR = 83.64\ km$ **(iii)** $75°$

Recommendations

Teachers should use the appropriate jargon in the teaching of circle theorems. They should assist students in making the determination as to when to use trigonometric ratios and when to use trigonometric formulae.

Question 11

This question tested candidates' ability to

- solve for one unknown in a singular matrix
- use the matrix method to solve an equation with two unknowns
- recognize the position vector of a point in the plane
- write the coordinates of points in the plane as position vectors
- use vector geometry to determine the resultant of two or more vectors
- use the properties of equal vectors to solve problems in geometry.

The question was attempted by 37 per cent of the candidates, less than 1 per cent of whom earned the maximum available mark. The mean mark was 3.90 out of 15.

Candidates performed unsatisfactorily on this question. Most candidates seemed unaware of the condition under which the matrix would not have an inverse, that is, when $|M| = 0$. A few candidates were able to express the simultaneous equations in the required form and some continued to attempt to find a solution although this was not required.

In Part (c), candidates generally wrote the position vectors as required. However, challenges were experienced in determining the coordinates of the point R and identifying the type of quadrilateral.

Solutions

(a) $\boldsymbol{p = \frac{-7}{2}}$

(b) $\begin{pmatrix} \mathbf{4} & \mathbf{-2} \\ \mathbf{2} & \mathbf{3} \end{pmatrix} \begin{pmatrix} \boldsymbol{x} \\ \boldsymbol{y} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{4} \end{pmatrix}$

(c) **(i)** $\overrightarrow{\boldsymbol{OP}}$ **is the position vector of the point** $\boldsymbol{P}$

(ii) **a)** $\overrightarrow{\boldsymbol{OP}} = \begin{pmatrix} \mathbf{2} \\ \mathbf{4} \end{pmatrix}$; **b)** $\overrightarrow{\boldsymbol{OQ}} = \begin{pmatrix} \mathbf{8} \\ \mathbf{2} \end{pmatrix}$; **c)** $\overrightarrow{\boldsymbol{PQ}} = \begin{pmatrix} \mathbf{6} \\ \mathbf{-2} \end{pmatrix}$

(iii) $\boldsymbol{R}\ (\mathbf{6}, \mathbf{-2})$

(iv) $\boldsymbol{PQRO}$ **is a parallelogram. It is a quadrilateral with a pair of opposite sides equal and parallel.**

Recommendations

Teachers should encourage students to utilize diagrams to add clarity to their responses when solving problems on Vector Geometry. They should reinforce the concept that a resultant vector can be derived from the sum or difference of two or more position vectors. They should provide students with opportunity to develop the art of identifying the type of quadrilateral formed from a system of vectors in which there are equal or parallel combinations. They should provide students with more practice in writing the matrix equation corresponding to a pair of simultaneous linear equations.