



CARIBBEAN
EXAMINATIONS
COUNCIL

Caribbean Advanced
Proficiency Examination®

SYLLABUS

INTEGRATED MATHEMATICS

CXC A35/U1/22

Effective for examinations from May–June 2024



CAPE®

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NOTE TO TEACHERS AND LEARNERS

This document CXC A35/U1/22 replaces CXC A35/U1/15 issued in 2015.

Please note that the syllabus has been revised and amendments are indicated by italics.

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PLEASE NOTE



This icon is used throughout the syllabus to represent key features which teachers and learners may find useful.

Introduction

The Caribbean Advanced Proficiency Examination® (**CAPE**®) is designed to provide certification of the academic, vocational and technical achievement of students in the Caribbean who, having completed a minimum of five years of secondary education, wish to further their studies. The examinations address the skills and knowledge acquired by students under a flexible and articulated system where subjects are organised in 1-Unit or 2-Unit courses with each Unit containing three Modules. Subjects examined under **CAPE**® may be studied concurrently or singly.

The Caribbean Examinations Council offers three types of certification at the **CAPE**® level. The first is the award of a certificate showing each **CAPE**® Unit completed. The second is the **CAPE**® Diploma, awarded to candidates who have satisfactorily completed at least six Units, including Caribbean Studies. The third is the **CXC**® Associate Degree, awarded for the satisfactory completion of a prescribed cluster of *ten* **CAPE**® Units including Caribbean Studies, Communication Studies *and Integrated Mathematics*. *Integrated Mathematics is not a requirement for candidates pursuing the **CXC**® Associate Degree in Mathematics or pursuing Pure and Applied Mathematics Unit 1 OR 2.* The complete list of Associate Degrees may be found in the **CXC**® Associate Degree Handbook.

For the **CAPE**® Diploma and the **CXC**® Associate Degree, candidates must complete the cluster of required Units within a maximum period of five years. *To be eligible for a **CXC**® Associate Degree, the educational institution presenting the candidates for the award, must select the Associate Degree of choice at the time of registration at the sitting (year) the candidates are expected to qualify for the award.* Candidates will not be awarded an Associate Degree for which they were not registered.

Integrated Mathematics Syllabus

◆ RATIONALE

The Caribbean society is an integral part of an ever-changing world *that has been impacted by globalisation. As a result, the Caribbean region is encouraged to create opportunities to make the education and career prospects of current and future citizens dynamic and adaptable, and to create among its citizens a plethora of quality leadership with the acumen required to make meaningful projections and innovations for further development. It is therefore vital, that such leaders learn appropriate problem-solving techniques inherent to the study of mathematics. Mathematics promotes intellectual development, is utilitarian, and applicable to all disciplines. Additionally, its aesthetics and epistemological approaches provide solutions fit for any purpose. For that reason, Mathematics is the essential tool to empower people with the knowledge, competencies and attitudes which are required for academia as well as quality leadership for sustainability in this dynamic world.*

The Integrated Mathematics course of study *caters to a wide variety of disciplines. The teaching, learning and assessment strategies used will embolden learners to see Mathematics as interesting, relevant and recognisable by making connections to real world situations. The Integrated Mathematics course will provide students with the knowledge and skill sets required to model practical situations and provide workable solutions in their respective field of study. These skills which include critical and creative thinking, problem solving, logical reasoning, modelling ability, teamwork, communication, decision making, research techniques, and information and communication technology (ICT) are competencies for life-long learning. Additionally, the attitude and discipline which accompany the study of Mathematics will provide opportunities to nurture desirable character qualities. Such holistic development becomes useful for the transition into further studies.*

This syllabus will contribute to the development of the Ideal Caribbean Person *as articulated by the CARICOM Heads of Government. This individual will demonstrate multiple literacies, independent and critical thinking and innovative application of science and technology to problem solving. Such a person should also be emotionally secure with a high level of self-confidence and self-esteem, demonstrate a positive work ethic, and value and display creative imagination and entrepreneurship. Additionally, in keeping with the UNESCO Pillars of Learning, on completion of this course of study, students will learn to know, learn to do, learn to be, learn to live together and learn to transform oneself and society.*

◆ AIMS

The syllabus aims to:

1. empower students with the knowledge, competencies and attitudes which are precursors for academia as well as quality leadership for sustainability in the dynamic world;
2. improve *students'* mathematical knowledge, skills and techniques with an emphasis on *problem solving, logical thinking, efficiency and accuracy;*

3. provide students with the proficiencies required to model practical situations and provide *viable* solutions in their respective fields of work and study;
4. make Mathematics *fun*, interesting, recognizable, relevant *and applicable* to *daily life*;
5. develop competencies *geared towards collaboration, creativity*, critical thinking, *research, decision-making, effective communication, technological innovation, entrepreneurship and lifelong learning*; and,
6. *encourage* desirable character qualities that include *discipline*, self-confidence, self-esteem, ethics and emotional security.

◆ SKILLS AND ABILITIES TO BE ASSESSED

The skills and abilities that students are expected to develop on completion of this syllabus have been grouped under three headings:

- (a) Conceptual Knowledge;
- (b) Algorithmic Knowledge; and,
- (c) Reasoning.

Conceptual Knowledge

The examination will test candidates' ability to recall, and *understand* appropriate facts, concepts and principles in a variety of contexts.

Algorithmic Knowledge

The examination will test candidates' ability to manipulate mathematical expressions and procedures using appropriate symbols and language, logical deduction and inferences.

Reasoning

The examination will test candidates' ability to select appropriate strategy or select, use and evaluate mathematical models and interpret the results of a mathematical solution in terms of a given real-world problem and engage in problem-solving.

◆ PREREQUISITES OF THE SYLLABUS

Any person with a *fairly good* grasp of the contents of the syllabus of the Caribbean Secondary Education Certificate (CSEC®) General Proficiency course in Mathematics, or equivalent, should be able to undertake the course. However, successful participation in the course will also depend on the possession of good verbal and written communication skills.

◆ STRUCTURE OF THE SYLLABUS

The Integrated Mathematics Syllabus comprises three Modules, each requiring at least 50 hours. Students will develop the skills and abilities identified through the study of:

Module 1	-	FOUNDATIONS OF MATHEMATICS
Module 2	-	STATISTICS
Module 3	-	CALCULUS

◆ MODULE 1: FOUNDATIONS OF MATHEMATICS

GENERAL OBJECTIVES

On completion of this Module, students should:

1. acquire competency in the application of algebraic techniques;
2. appreciate the role of exponential *and* logarithmic functions in practical modelling situations;
3. understand the importance of *algebraic and trigonometric* functions and graphs in solving real-world problems;
4. appreciate the difference between a sequence and a series and their applications;
5. appreciate the need for *logical thinking and* accuracy in performing calculations; and,
6. understand the usefulness of different types of numbers.

SPECIFIC OBJECTIVES

CONTENT & SKILLS

Students should be able to:

1. ***Functions, Graphs and Inequalities***

- 1.1 *determine whether a given function is injective, surjective or bijective;*

Types of functions: injective, surjective and bijective.

Domain and range of functions.

Quadratic functions and graphs.

- 1.2 *determine the inverse function;*

Inverse functions.

Functions that are invertible for restricted domains.

- 1.3 *determine the composition of functions;*

Addition, subtraction, multiplication and division of functions for example

$$h(x) = \frac{f(x)}{g(x)}$$

Composite function: $h(x) = g[f(x)]$.

- 1.4 *determine the solution set for linear and quadratic inequalities; and,*

Graphical and algebraic solutions.

MODULE 1: FOUNDATIONS OF MATHEMATICS (cont'd)

SPECIFIC OBJECTIVES

Students should be able to:

Functions, Graphs and Inequalities (cont'd)

- 1.5 apply concepts of functions, graphs and inequalities to solve real life problems.

Worded problems relating to a variety of disciplines. For example, applications to agriculture, social sciences, physical sciences, medical sciences, engineering and other areas.

2. Exponents and Logarithms

- 2.1 apply properties of exponential and logarithmic functions;

Definition of exponential and logarithmic functions.

Properties of exponential and logarithmic functions.

Graphs of exponential (base e) and logarithmic functions (bases 10 and e).

Notation: $\log_e = \ln$ and $\log_{10} = \lg$

- 2.2 apply the laws of indices;

Equations of the form $af(x) = bg(x)$, where f and g are polynomials.

- 2.3 apply the laws of logarithm;

Laws of logarithm

For $a, c, \in \mathbb{R}$

1. $\log_a m + \log_a n = \log_a mn$

2. $\log_a m - \log_a n = \log_a \frac{m}{n}$

3. $\log_a m^n = n \log_a m$

4. $\log_a 1 = 0$

5. $\log_a a = 1$

Change of base

$$\log_a m = \frac{\log_c m}{\log_c a} \text{ where } a, c, m > 0$$

Equations of the form

$$a \log(x + b) = \log(c) - d$$

MODULE 1: FOUNDATIONS OF MATHEMATICS (cont'd)

SPECIFIC OBJECTIVES

Students should be able to:

Exponents and Logarithms (cont'd)

- 2.4 convert between the exponential and logarithmic equations;

Converting exponential equations to logarithmic equations and logarithmic equations to exponential equations

$$a^x = b \Leftrightarrow \log_a x = b.$$

- 2.5 convert exponential equations to linear form; and,

Converting equations to linear form:

$$y = ax^b \Leftrightarrow \log y = \log a + b \log x; \text{ and,}$$

$$y = ab^x \Leftrightarrow \log y = \log a + x \log b.$$

- 2.6 apply the concepts of exponents and logarithms to real life solutions.

Worded problems relating to a variety of disciplines. For example, applications to agriculture, social sciences, physical sciences, medical sciences, engineering and other areas.

3 Remainder and Factor Theorem

- 3.1 state the remainder and factor theorems;

If $f(a) = 0$, then $(x - a)$ is a factor of f .

If $f(b) \neq 0$, then $(x - b)$ leaves remainder $f(b) \in \mathbb{R}$ when it divides f .

Where $f(x)$ is a polynomial of degree 3 or less.

- 3.2 divide polynomials up to the third degree by linear expressions;

Methods of long division, synthetic division and inspection.

- 3.3 solve problems involving the factor and remainder theorems; and,

If $(x - a)$ is a factor of the polynomial $f(x)$, then $f(x)$ has a root at $x = a$.

Including finding coefficients of a polynomial given a factor or remainder when divided by a linear or quadratic expression.

Factorising cubic polynomials where one factor can be found by inspection.

- 3.4 apply the concepts of remainder and factor theorems to everyday life.

Worded problems relating to a variety of disciplines. For example, applications to the agriculture, social sciences, physical sciences, medical sciences, engineering and other areas.

MODULE 1: FOUNDATIONS OF MATHEMATICS (cont'd)

SPECIFIC OBJECTIVES

Students should be able to:

4 Sequences and Series

- 4.1 solve problems involving the binomial expansion;

- 4.2 distinguish between a series and a sequence;

CONTENT & SKILLS

$(a + b)^n$ where n is a positive integer not greater than 3, $a, b \in R$.

$$(a + b)^2 = a^2 + 2ab + b^2.$$

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3.$$

Problems involving finding the terms or the coefficient of a term of an expansion of linear expression such as $(ax + b)^3$.

While the students may become familiar with Pascal triangle and notations relating to $\binom{n}{r}$ or ${}^nC_r$, it is sufficient to know the binomial coefficients 1-2-1 and 1-3-3-1 for examination purposes.

Sequence.

Series and summation.

A series is represented as follows:

$$\sum_{n=a}^{n=b} T_n$$

Where a = the starting term

b = final term

T_n = formula (general term) for the series.

Identification of arithmetic and geometric progressions.

Common difference (d) and common ratio (r).

MODULE 1: FOUNDATIONS OF MATHEMATICS (cont'd)

SPECIFIC OBJECTIVES

Students should be able to:

Sequences and Series (cont'd)

- 4.3 determine the n^{th} term of arithmetic and geometric progressions;

CONTENT & SKILLS

n^{th} term of an Arithmetic progression:

$$T_n = a + (n - 1)d .$$

Where a is the 1st term, n is the number of terms, d is the common difference.

n^{th} term of a Geometric progression:

$$T_n = ar^{n-1}$$

Where r is the common ratio.

- 4.4 evaluate the sum of a finite arithmetic or geometric sequence;

Series.

The sum of the first n terms of a finite arithmetic sequence.

$$S_n = \frac{n}{2}(a + l)$$

l is the last term in the sequence

Or

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

The sum of the first n terms of a finite geometric sequence.

$$S_n = \frac{a(r^n - 1)}{r - 1}, (r < -1) \cup (r > 1)$$

Or

$$S_n = \frac{a(1 - r^n)}{1 - r}, -1 < r < 1$$

a is the 1st term, n is the number of terms, d is the common difference, r is the common ratio.

MODULE 1: FOUNDATIONS OF MATHEMATICS (cont'd)

SPECIFIC OBJECTIVES

Students should be able to:

Sequences and Series (cont'd)

- 4.5 determine the sum to infinity for a geometric series; and,

$$S_{\infty} = \frac{a}{1-r} \quad -1 < r < 1$$

a is the 1st term, r is the common ratio.

- 4.6 apply the concept of sequences and series to everyday life.

Worded problems including applications to a variety of disciplines. For example, applications to agriculture, social sciences, physical sciences, medical sciences, engineering and other areas.

5 Trigonometry

- 5.1 evaluate sine, cosine and tangent of an angle given in radians;

Converting between degrees and radian measure of an angle:

$$\pi = 180^{\circ}$$

- 5.2 solve equations involving trigonometric functions;

Functions of the form:

$$a \sin (x + b) = c$$

$$a \cos (x + b) = c$$

$$a \tan (x + b) = c$$

Where the domain is a subset of $-2\pi \leq x \leq 2\pi$ Principal, secondary and other solutions.

Problems involving trigonometric identity $\sin^2 x + \cos^2 x = 1$.

- 5.3 sketch the graph of the sine, cosine and tangent functions;

Characteristics of graphs of trigonometric functions with standard periods:

$$-2\pi \leq x \leq 2\pi$$

$$y = a \sin (x + b)$$

$$y = a \cos (x + b)$$

$$y = a \tan (x + b)$$

- 5.4 solve problems involving the graphs of sine, cosine and tangent functions; and,

Problems including locating intercepts and roots, minimum and maximum values for a domain.

MODULE 1: FOUNDATIONS OF MATHEMATICS (cont'd)

SPECIFIC OBJECTIVES

Students should be able to:

Trigonometry (cont'd)

- 5.5 *apply the concept of trigonometry to everyday life.*

Worded problems including applications to a variety of disciplines. For example, applications to agriculture, social sciences, physical sciences, medical sciences, engineering and other areas.

6 Matrices and Systems of Equations

- 6.1 perform the basic operations on matrices of order 3×3 ;

Addition, subtraction, multiplication, scalar multiplication.

- 6.2 represent a system of equations in matrix form;

Matrix form, $AX = B$

Augmented matrix $[A|B]$

- 6.3 evaluate the determinant of a 3×3 matrix;

The expansion of cofactors.

Sarrus' method.

- 6.4 solve a system of three linear equations with three unknowns; and,

Elementary Row operations.

Solutions by such as Cramer's rule and Row Echelon Form.

- 6.5 *apply concepts of matrices to solving real life problems.*

Worded problems including applications to a variety of disciplines. For example, applications to agriculture, social sciences, physical sciences, medical sciences, engineering and other areas.

7 Complex Numbers

- 7.1 *find complex solutions to quadratic equations which have no real solutions;*

Solving quadratic equations where the discriminant is negative.

Algebraic representation.

Real and imaginary parts.

Identifying the set of complex numbers as the superset of other numbers.

MODULE 1: FOUNDATIONS OF MATHEMATICS (cont'd)

SPECIFIC OBJECTIVES

CONTENT & SKILLS

Students should be able to:

Complex Numbers (cont'd)

- | | | |
|-----|---|---|
| 7.2 | <i>represent complex numbers graphically;</i> | <i>Argand diagram.</i> |
| 7.3 | <i>solve problems involving the properties of complex numbers; and,</i> | <i>Equality, conjugate, modulus and argument ($-\pi < \arg(z) \leq \pi$).

Addition, subtraction, multiplication and division (rationalise the denominator).</i> |
| 7.4 | <i>apply concepts of complex numbers to solve real life problems.</i> | <i>Worded problems relating to a variety of disciplines. For example, applications to agriculture, social sciences, physical sciences, medical sciences, engineering and other areas.</i> |

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives in this Module, teachers are advised to engage students in the following teaching and learning activities.

1. *Model the use of tools such as Microsoft Suite and graphical software for example Wolfram Mathematica (wolframalpha.com), Geogebra (geogebra.org) and Desmos (desmos.com) to draw graphs of functions and investigate their behaviour so that students will be able to utilize them.*
2. *Encourage students to participate in activities that involve constructing lines of symmetry and using vertical and horizontal line tests to investigate inverse of functions.*
3. *Guide students to make connections with Series (Arithmetic and Geometric) with different types of interest: simple, compound and depreciation.*
4. *Encourage students to work in small groups to research and present on how any of the concepts in Module 1 can be applied to real world situation to include but not limited to Exponents and Logarithm in Physics and Biology, and Matrices and Systems of Equations in Business, Economics, and Engineering.*
5. *Guide students in the use of online calculators. For example,*
 - (a) <https://www.symbolab.com/solver/>.
 - (b) <http://www.mathsisfun.com/algebra/matrix-calculator.html>.

MODULE 1: FOUNDATIONS OF MATHEMATICS (cont'd)

6. *Model the use of graphing software, for example Geogebra <https://www.geogebra.org/> to create models of trigonometric functions. Then adjust these models and observe how the parameters of the equations change.*
7. *Involve students in the review of case studies and guide them to apply mathematical skills and knowledge to identify solutions to problems outlined.*
8. Engage students in class discussions on the meaning of the square root of a negative number. *Students should be given an opportunity to express their understanding.*
9. Engage students in activities that facilitate the transfer of knowledge of real numbers and vectors to operations with complex numbers.
10. Engage students in activities that will allow them to relate complex solutions of quadratic equations to the quadratic graph that has no x-intercept.
11. Allow students to *work in pairs or groups* to apply geometrical triangle properties to determine the modulus and argument of a complex number. *They could then present the solutions to the class.*
12. *Encourage the use of elements of the performing arts (for example, role play, poetry and song) to reinforce concepts taught.*

RESOURCES

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Bahall, D.	<i>Pure Mathematics Unit 1 for CAPE Examinations.</i> London: Macmillan Publishers Limited, 2013
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Caribbean Examinations Council	Injective and Surjective Functions. Barbados, 1998.
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MODULE 1: FOUNDATIONS OF MATHEMATICS (cont'd)

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- Talbert, J.F., and Heng, H.H. *Additional Mathematics Pure and Applied. (6th Ed.). Singapore: Pearson Educational, 2010.*

Websites:

<https://www.symbolab.com/solver/>

<http://www.mathsisfun.com/algebra/matrix-calculator.html>

<https://tutorial.math.lamar.edu/Classes/Alg/Alg.aspx>

<https://tutorial.math.lamar.edu/Classes/Calcl/ReviewIntro.aspx>

<https://www.mathsisfun.com/algebra/polynomials-remainder-factor.html>

◆ MODULE 2: STATISTICS

GENERAL OBJECTIVES

On completion of this Module, students should:

1. understand the *importance of sampling, data* collection, description and analysis;
2. appreciate that the numerical and graphical representations of data *are* an important part of data analysis;
3. understand the concept of probability and its applications to real-world situations;
4. *understand the concept of correlation and linear regression in real-world situations; and,*
5. appreciate data-analysis processes for applications to real-world situations.

SPECIFIC OBJECTIVES

CONTENT

Students should be able to:

1. Data and Sampling

- 1.1 *explain the types of data;*

Quantitative and qualitative data.

Discrete and continuous data.

Nominal, ordinal, interval and ratio scales of data.

Primary and Secondary data.

- 1.2 distinguish between sample and population; and,

Sample survey and Population census, Statistics and Parameters.

Statistics:

$$\text{Sample mean } \bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

$$\text{Sample variance } s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$$

Parameter: mean (μ) and variance (σ^2)

$$\text{Population mean } \mu = \frac{\sum_{i=1}^N x_i}{N}$$

MODULE 2: STATISTICS (cont'd)

SPECIFIC OBJECTIVES

Students should be able to:

Data and Sampling (cont'd)

- 1.3 use an appropriate sampling method to select a sample from a population.

2. Coordinate Geometry

- 2.1 construct tables, given raw data;
- 2.2 present data in a variety of forms; and,
- 2.3 identify the shape of a distribution.

CONTENT

$$\text{Population variance } \sigma^2 = \frac{\sum_{i=1}^N (x_i - \bar{x})^2}{N}$$

Where **N** is the size of the population,

n is the size of the sample.

Random sampling.

Simple (Random number generators, use of the random number tables) stratified and systematic.

Non-random sampling: cluster and quota.

Advantage and disadvantage of the sampling methods.

Representative sample.

Sampling and non-sampling errors.

Tally tables, frequency tables, cumulative frequency tables.

Grouped and ungrouped data.

Bar chart, pie chart, line graph, histogram, cumulative frequency graph, stem-and-leaf, box-and-whiskers plot, scatter plots; cross-tabulations of nominal/categorical data.

Symmetric (*normal*), positively skewed (*skewed to the right*) and negatively skewed (*skewed to the left*).

MODULE 2: STATISTICS (cont'd)

SPECIFIC OBJECTIVES

Students should be able to:

3. Measures of Location and Spread

3.1 determine estimates for measures of location for grouped data;

3.2 *distinguish among* measures of spread for appropriate data types;

3.3 determine measures of spread for ungrouped data;

3.4 determine estimates for measures of spread for grouped data; and,

3.5 *apply measures of location and spread to real life situations.*

4. Permutations and Combinations

4.1 calculate permutations of *n* different objects;

CONTENT

Including *mean*, median and mode.

See formula sheet.

Measures of Dispersion: range, standard deviation and Interquartile range (IQR) where quartile one and quartile two are the median of the lower and upper halves respectively.

Advantages and disadvantages of different measures of spread.

The empirical rule for the standard deviation in relation to the mean.

Ranges, *interquartile range*, variance, standard deviation.

Variances and standard deviations.

$$s^2 = \frac{\sum_{i=1}^n f(x_i - \bar{x})^2}{(\sum f) - 1}$$

Measures of location (central tendencies): mean, mode and median; discrete and continuous variable; suitability of measures of location to nominal, ordinal, interval and ratio scales of data; advantages and disadvantages of different measures of location and spread.

Arrangement of n different objects ($n!$).

Arrangement of *r* objects from *n* distinct objects, (${}^n P_r$). Where,

$${}^n P_r = \frac{n!}{(n-r)!}$$

Consider also, cases where items are repeated.

Arrangement of n different objects p of which are alike, q of which are alike (but different from p) $\left(\frac{n!}{p!q!}\right)$.

MODULE 2: STATISTICS (cont'd)

SPECIFIC OBJECTIVES

Students should be able to:

Permutations and Combinations (cont'd)

- 4.2 calculate combinations of *objects*; and,

Number of *selections* of r *objects* from a total of n distinct objects, ${}^n C_r$:

$${}^n C_r = \frac{n!}{(n-r)!r!}$$

- 4.3 apply counting techniques to solve real- life problems.

Using permutations and combinations in calculating simple probabilities.

Worded problem relating to a variety of disciplines. For example, applications to agriculture, social sciences, physical sciences, medical sciences, engineering and other areas.

5. Probability and Probability Distributions

- 5.1 distinguish among *probability* terms;

Concept of probability.

Experiment, outcome, event, sample space and complementary events.

- 5.2 apply basic rules of probability;

Probability formulae: addition and multiplication.

Types of events:

1. mutually exclusive
 $P(A \cup B) = P(A) + P(B)$
or
 $P(A \cap B) = 0$
2. mutually exhaustive
 $P(A \cup B) = 1$
3. independent
 $P(A \cap B) = P(A) \times P(B)$

Conditional probability: contingency tables.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

MODULE 2: STATISTICS (cont'd)

SPECIFIC OBJECTIVES

Students should be able to:

Probability and Probability Distributions (cont'd)

- 5.3 *construct a probability tree diagram from a given probability situation;*

CONTENT

Interpreting Probability tree diagram (with and without replacement).

Calculating conditional probabilities from a tree diagram.

Worded problem relating to a variety of disciplines. For example, applications to agriculture, social sciences, physical sciences, medical sciences, engineering and other areas.

- 5.4 *distinguish between discrete and continuous random variables;*

Concept of a random variable.

Discrete random variable; *continuous random variable.*

- 5.5 *calculate probabilities from a discrete probability distribution table;*

Discrete probability distribution.

$$\sum P(X = x) = 1$$

$$0 \leq P(X = x) \leq 1$$

- 5.6 *investigate discrete probability distributions;*

Distribution table of a random variable with maximum 5 possible values.

Expectation:

$$E(X) = \sum xP(X = x).$$

Variance and standard deviation:

$$\text{Var}(X) = E(X^2) - [E(X)]^2.$$

Solving problems involving probabilities and expected values where, for example, the probability of an event is unknown.

MODULE 2: STATISTICS (cont'd)

SPECIFIC OBJECTIVES

Students should be able to:

Probability and Probability Distributions (cont'd)

- 5.7 justify the suitability of binomial distribution for modelling a given random experiment;

CONTENT

Properties for binomial distribution:

- (a) *finite number of trials;*
- (b) *two possible outcomes (success and failure);*
- (c) *probability of success is constant; and,*
- (d) *trials are independent.*

$X \sim \text{Bin}(n, p)$
where $E(X) = np$
and $\text{Var}(X) = npq$
where $q = 1 - p$

- 5.8 determine probabilities of a binomial distribution;

Binomial formula of Probabilities for at the most ten trials:

$$P(X = x) = \binom{n}{x} p^x q^{n-x};$$

Where $x = 0, 1, \dots, n$; and,

$p = \text{probability of success}.$

- 5.9 describe the features of the normal distribution;

Properties of the normal distribution.

The mean, mode and median are all equal.

The curve is symmetric around the mean (μ).

The total area under the curve is 1.

- 5.10 justify the suitability of a normal distribution for modelling a given random experiment;

Properties of the normal distribution curve
 $X \sim N(\mu, \sigma^2).$

MODULE 2: STATISTICS (cont'd)

SPECIFIC OBJECTIVES

Students should be able to:

Probability and Probability Distributions (cont'd)

- 5.11 find probabilities using the standard normal distribution; and,

$Z \sim N(0,1)$.

Using the standard normal table to find probabilities from z-values and vice versa.

(See CAPE formula booklet for table.)

- 5.12 use probabilities to solve real-world problems.

Worded problems relating to a variety of disciplines. For example, applications to agriculture, social sciences, physical sciences, medical sciences, engineering and other areas.

6. Correlation and Linear Regression

- 6.1 explain correlation in terms of bivariate data;

Independent and dependent variables.

Concept of correlation.

Negative, positive and no correlation.

Relating correlation to scatter plots.

- 6.2 interpret the value of the correlation coefficient;

Magnitude (strong, moderate or weak) and direction (negative or positive) of the correlation.

Correlation Coefficient:

0 = no correlation

$0 < |r| \leq 0.2$ (very weak)

$0.2 < |r| \leq 0.4$ (weak)

$0.4 < |r| \leq 0.6$ (fair/moderate)

$0.6 < |r| \leq 0.8$ (strong)

$0.8 < |r| \leq 1$ (very strong)

1 = Perfect correlation

Correlation coefficient value will be provided in the exam.

- 6.3 draw the regression line of y on x (by inspection), passing through $(\bar{x}, \bar{y})$ on a scatter plot;

Scatter plots.

Regression line of best fit.

MODULE 2: STATISTICS (cont'd)

SPECIFIC OBJECTIVES

CONTENT

Students should be able to:

Correlation and Linear Regression (cont'd)

- 6.4 interpret the equation of the regression line; and,

Regression coefficients.

Estimation from the regression line and regression equation.

The regression equation will be provided in the exam.

- 6.5 use statistics to solve real-world problems.

Interpretation of results from statistical calculations.

Case studies and worded problems relating to a variety of disciplines. For example, applications to agriculture, social sciences, physical sciences, medical sciences, engineering and other areas.

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives in this Module, teachers are advised to engage students in the following teaching and learning activities.

1. Encourage students to use computer applications *for example Microsoft Excel, Geogebra and Minitab* to draw graphs and charts and perform calculations.
2. *Encourage students to work in small groups to research the various methods of presenting numerical data, their advantages and disadvantages, their suitability and how to construct the various graphs and charts. Allow students to peer teach based on their research.*
3. *Encourage the use ICT devices such as projectors, tablets, laptops, and software such as G-suite and Microsoft Suite to make presentations and share findings.*
4. *Expose students to documentaries or movies (for example, the movie Moneyball). Students should be encouraged to discuss their understanding of the application of statistics in real-life.*
5. *Guide students in the collection, presentation and analysis of data from a cricket match using various statistical techniques to compare the performance of both teams. Students should be encouraged to present their findings to the class, using an appropriate strategy or tool.*

MODULE 2: STATISTICS (cont'd)

6. *Encourage students' participation in a 'lucky dip' or other practical activities to reinforce concepts such as probability.*
7. *Encourage students to analyse case studies on real-life events, such as, examination results, company reports, police reports, commodity prices, census data, and Covid-19 data which can be obtained from government offices, printed media or Internet to teach related concepts.*
8. *Incorporate the use of field trips to reinforce how Mathematics is applied in organisations and departments such as the central statistical office, government finance department, workplace of actuaries, or research companies. Students should be given guided questions and asked to report on their observations.*
9. *Encourage students to work in small groups where they will select a research topic from a relevant discipline. Students should identify a population of interest, use a suitable sampling technique to select an appropriate sample, gather relevant data, present the data using numerical and graphical techniques and analyse, interpret and draw inferences from the data.*
10. *Encourage students to participate in statistics day, mathematics clubs, mathematics expos and career day.*
11. *Invite a guest speaker, for example, a lecturer from a university, to talk to the students about Mathematics, its uses, abuses, misunderstandings, relevance, benefits, and careers.*
12. *Utilize various learning management systems and platforms to engage in web conferencing among teachers as well as students from across the region.*
13. *Organise visits with a teacher and class from another school. Your class can visit (face-to-face or virtually) the other school where you and another teacher can team teach a topic to both groups. At another time, invite the other class over to your school and again team teach.*
14. *Encourage students to complete worksheets individually or in groups to reinforce and practice different subtopics.*
15. *Incorporate the use of YouTube videos to reinforce concepts.*
16. *Guide students in the use of online calculators for example, <https://www.symbolab.com/solver/>.*
17. *Teachers are encouraged to become a member of a statistical association.*

For example:

- American Statistical Association (ASA) – <http://www.amstat.org/index.cfm>
The American Statistical Association publishes scholarly journals; statistical magazines; and a variety of conference proceedings, books, and other materials related to the practice of statistics.
- Royal Statistical Society <http://www.rss.org.uk/>.

MODULE 2: STATISTICS (cont'd)

18. *Encourage students and peers to use websites and online journal articles.*

For example:

- Ethic in Statistics <http://www.amstat.org/about/ethicalguidelines.cfm>.
- Journal of Statistics Education <http://www.amstat.org/publications/jse/> is a free, online, international journal focusing on the teaching and learning of statistics. This site also contains links to several statistical education organisations, newsletters, discussion groups and the JSE Dataset Archives.
- Useful sites for teachers: <http://www.amstat.org/education/usefulsitesforteachers.cfm>.

19. *Encourage students to visit <http://stattrek.com/> for various online tutorials.*

20. *Encourage statistics magazine subscriptions. For example, Significance magazine <https://www.significancemagazine.com>, articles are written by statisticians for anyone with an interest in the analysis and interpretation of data.*

21. *Encourage the use of elements of the performing arts (for example, role play, poetry and song) to reinforce concepts taught.*

RESOURCES

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Chambers, J., Crawshaw, J. & Balaam, P.	<i>Probability and Statistics 1 for Cambridge International A Level.</i> Nelson Thornes, 2012.
Dobbs, S. and Miller, J.	<i>Advanced Level Mathematics: Statistics 1.</i> Cambridge, United Kingdom: Cambridge University Press, 2002.
Haeussler, E., Paul, R., Wood, R.	<i>Introductory Mathematical Analysis for Business, Economics and the Life and Social Sciences (14th Ed.).</i> Pearson, 2018.
Mann, P.S.	<i>Introductory Statistics (9th Ed.).</i> New Jersey: Wiley, 2016.
Upton, G. & Cook, I.	<i>Introducing Statistics (2nd ed.).</i> Oxford: Oxford University Press, 2001.

Websites:

<http://www.worldofstatistics.org/>

<http://www.worldofstatistics.org/primary-secondary-school-teacher-resources/>



MODULE 2: STATISTICS (cont'd)

<http://www.examiner.com/article/community-college-students-and-the-international-year-of-statistics>

<https://www.youtube.com/watch?v=yxXsPc0bphQ&list=PLA6598DFE68727A9C>

<https://www.youtube.com/watch?v=HKA0htesJOA>

<https://www.mathsisfun.com/combinatorics/combinations-permutations.html>

<https://www.mathsisfun.com/data/index.html>

<https://www.amstat.org/>

<https://revisionmaths.com/advanced-level-maths-revision/advanced-level-level-statistics>

<https://www.mathsisfun.com/data/standard-normal-distribution.html>

<https://www.mathsisfun.com/data/binomial-distribution.html>

<https://www.mathsisfun.com/combinatorics/combinations-permutations.html>

◆ MODULE 3: CALCULUS

GENERAL OBJECTIVES

On completion of this Module, students should:

1. develop *an understanding* of limits and continuity;
2. *understand* the importance of differentiation and integration in analysing functions and graphs; and,
3. *appreciate* using calculus as a tool in solving real-world problems.

SPECIFIC OBJECTIVES

CONTENT

Students should be able to:

1. Limits and Continuity

- 1.1 describe the limiting behaviour of a function of x , as x approaches a given number;

Concept of the limit of a function.

- 1.2 use limit notation;

$$\lim_{x \rightarrow a} f(x) = L,$$

or

$$f(x) \rightarrow L \text{ as } x \rightarrow a$$

- 1.3 evaluate limits using limit theorems;

If $\lim_{x \rightarrow a} f(x) = F$, $\lim_{x \rightarrow a} g(x) = G$ and k is a

constant, then

$$\lim_{x \rightarrow a} kf(x) = kF$$

$$\lim_{x \rightarrow a} [f(x) \pm g(x)] = F \pm G$$

$$\lim_{x \rightarrow a} [f(x)g(x)] = FG$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{F}{G}$$

provided that $G \neq 0$

MODULE 3: CALCULUS (cont'd)

SPECIFIC OBJECTIVES

Students should be able to:

Limits and Continuity (cont'd)

- 1.4 apply factorisation to expressions whose limits are indeterminate;

Indeterminate form: $\frac{0}{0}$

Polynomials which can be factorised.

- 1.5 apply the concept of left- and right-handed limits to continuity;

Definition of continuity.

Graphs of continuous functions.

- 1.6 identify the points for which a function is discontinuous; *and*,

Discontinuous graphs.

Piece-wise functions.

- 1.7 *apply limits and continuity to solve real world problems.*

Worded problems relating to a variety of disciplines. For example, applications to agriculture, social sciences, physical sciences, medical sciences, engineering and other areas.

2. Differentiation

- 2.1 relate the derivative of a function to the gradient at a point on that function;

The gradient - derivative relationship.

The gradient function.

Notation for Derivatives:

$$y', f'(x), \frac{dy}{dx}, \frac{d}{dx}(f(x))$$

- 2.2 differentiate polynomials;

Differentiation from first principle for *linear functions only*

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx}(x^n) = nx^{n-1} \text{ where } n \text{ is any real number}$$

$$\frac{d}{dx}cf(x) = c \frac{d}{dx}f(x) \text{ where } c \text{ is a constant}$$

MODULE 3: CALCULUS (cont'd)

SPECIFIC OBJECTIVES

Students should be able to:

Differentiation (cont'd)

CONTENT

		$\frac{d}{dx}[f(x) \pm g(x)]$ $= \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$
2.3	differentiate expressions involving sine and cosine functions;	$\frac{d}{dx}(\sin x) = \cos x$ $\frac{d}{dx}(\cos x) = -\sin x$
2.4	apply the chain rule in the differentiation of composite functions;	Powers of a function; function of a function. If $y = [u]^n$ and u is a function of x , Then $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
2.5	differentiate exponential and logarithmic functions;	$\frac{d}{dx}(e^x) = e^x$ $\frac{d}{dx}(e^{f(x)}) = f'(x)e^{f(x)}$ $\frac{d}{dx}(\ln x) = \frac{1}{x}$ $\frac{d}{dx}\ln[f(x)] = \frac{f'(x)}{f(x)}$
2.6	differentiate products and quotients of functions;	Polynomials, sine, cosine, e^x and $\ln x$. Product Rule: If $y = uv$ where u and v are functions of x , then: $\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$

MODULE 3: CALCULUS I (cont'd)

SPECIFIC OBJECTIVES

Students should be able to:

Differentiation (cont'd)

CONTENT

Quotient Rule:

If $y = \frac{u}{v}$ where u and v are functions of x , then:

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

2.7 *use the sign of the derivative to investigate where a function is increasing or decreasing;*

Increasing when $f'(x) > 0$

Decreasing when $f'(x) < 0$

2.8 *determine the second derivative of a function;*

If $y = f(x)$, then:

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} (f'(x)) = f''(x)$$

2.9 *determine the stationary point(s) of a given function;*

Stationary point(s) occur when $f'(x) = 0$

2.10 *investigate the nature of the stationary point;*

Maximum and minimum points.

$f''(x_0) > 0$ indicates minimum value.

$f''(x_0) < 0$ indicates maximum value.

$f''(x_0) = 0$ possibly a point of inflexion.

If $f''(x_0) = 0$ then test the sign of the gradient on either side of the stationary point to determine its nature.

2.11 *apply the concept of stationary points and turning points to curve sketching;*

Polynomials of degree ≤ 3 .

2.12 *apply the concept of the derivative to rate of change;*

$$\frac{dy}{dx} \approx \frac{\Delta y}{\Delta x}$$

*Problems involving a variety of disciplines.
Related rates.*

MODULE 3: CALCULUS (cont'd)

SPECIFIC OBJECTIVES

Students should be able to:

Differentiation (cont'd)

- 2.13 find the first partial derivative of a function of two variables; and,

- 2.14 apply concepts above to solve real life problems.

3. Integration

- 3.1 recognise integration as the reverse process of differentiation;

- 3.2 compute indefinite integrals of polynomials;

- 3.3 integrate functions;

- 3.4 integrate composite functions by substitution;

- 3.5 compute definite integrals;

CONTENT

Notations: $f_x, f_y, \frac{\delta}{\delta x} [f(x, y)], \frac{\delta}{\delta y} [f(x, y)]$.

Interpret partial derivatives.

Worded problems relating to a variety of disciplines. For example, applications to agriculture, social sciences, physical sciences, medical sciences, engineering and other areas.

Derivative-integral relationship.

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1, n \in \mathbb{R}$$

Restricted to functions of these forms:

$$\int (a \sin x) dx = -a \cos x + C$$

$$\int (a \cos x) dx = a \sin x + C$$

$$\int \cos(ax + b) dx = \frac{1}{a} \sin(ax + b) + C$$

$$\int \sin(ax + b) dx = -\frac{1}{a} \cos(ax + b) + C$$

$$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln|ax+b| + c$$

$\int f[g(x)]dx$, where $g(x)$ is linear.

$$\int_a^b f(x) dx = F(b) - F(a).$$

$$F'(x) = f(x).$$

MODULE 3: CALCULUS (cont'd)

SPECIFIC OBJECTIVES

Students should be able to:

Integration (cont'd)

- 3.6 apply integration to determine the area between a curve and a straight line;

CONTENT

Points of intersection:

$$\int_a^b [f(x) - g(x)] dx$$

Area under a curve.

The area between a curve and a straight line.

- 3.7 *formulate a differential equation;*

Formulating differential equations of the form $y' = f(x)g(y)$.

- 3.8 *solve first order differential equations; and,*

Restricted to variables which are separable.

- 3.9 *apply integration to solving real life problems.*

Worded problems relating to a variety of disciplines. For example, applications to agriculture, social sciences, physical sciences, medical sciences, engineering and other areas.

MODULE 3: CALCULUS (cont'd)

Suggested Teaching and Learning Activities

To facilitate students' attainment of the objectives in this Module, teachers are advised to engage students in the following teaching and learning activities.

1. *Guide students in the use of online generic calculators, for example:*
 - (a) <https://www.symbolab.com>.
 - (b) <https://www.mathportal.org/calculators/calculus/>.
2. *Encourage the use of teaching tools to include, Microsoft Suite and graphical software such as Wolfram Mathematica (wolframalpha.com), Geogebra (geogebra.org) and Desmos (desmos.com) to draw graphs of functions and investigate their behaviour.*
3. *Organise debates on situations that involve mathematics, for example, the utility of mathematics to other disciplines.*
4. *Guide students in using and participating in fun calculus activities online, for example, at:*
 - (a) <http://teachinghighschoolmath.blogspot.com/2013/03/fun-calculus-activities.html>.
 - (b) <http://www.intmath.com>.
 - (c) <http://www.integral-calculator.com>.
 - (d) <http://www.mathway.com/calculus>.
5. *Incorporate the use of online teaching resources such as videos and presentations.*
6. *Encourage students to work in small groups to research how the concept covered in Module 3 (calculus) is applicable to disciplines outside of math. Students should use ICT devices such as tablets, laptops, projectors, document cameras, cellphones, to make presentations and share information.*
7. *Encourage the use of web conferencing on different platforms such as: **CXC**® Learning Institute (CLI), Zoom, Skype, Moodle, Schoology, Google Classroom, Blackboard LMS, Edmodo, Microsoft Teams, and Google Meet to engage learners.*
8. *Guide students in using worksheets to reinforce concepts taught in the Module.*
9. *Encourage students to collaborate to complete mini projects that make use of the concepts taught in calculus; for example, maximizing or minimizing volume or surface areas.*

MODULE 3: CALCULUS (cont'd)

RESOURCES

Bahall, D.	Pure Mathematics for CAPE Unit 1 and 2. Macmillan Caribbean, 2013.
Bostock, L., and Chandler, S.	Core Maths for Advanced Level (3 rd Ed.). London: Stanley Thornes (Publishers) Limited, 2000.
Campbell, E.	Pure Mathematics for CAPE: Volume 1. Kingston: LMH Publishing Limited, 2007.
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Edwards, K.	Integrated Mathematics Workbooks. (3 rd Ed.). Amsco School Pubns Inc, 2000.
Haeussler, E., Paul, R., Wood, R.	Introductory Mathematical Analysis for Business, Economics and the Life and Social Sciences (14 th Ed.). Pearson, 2018.
Neill, H. and Quadling, D.	Advanced Level Mathematics: Pure Mathematics 1. Cambridge: Cambridge University Press, 2002.
Neill, H. and Quadling, D.	Advanced Level Mathematics: Pure Mathematics 2 & 3. Cambridge: Cambridge University Press, 2002.
Stewart, J.	Calculus (7 th Ed.). Belmont: Cengage Learning, (2011).
Talbert, J.F., and Heng, H.H.	Additional Mathematics Pure and Applied. (6 th Ed.) Singapore: Pearson Educational, 2010.
Toolsie, R.	Additional Mathematics for the Caribbean. Caribbean Educational Publishers, 2003.

Websites:

<http://teachinghighschoolmath.blogspot.com/2013/03/fun-calculus-ap-activities.html>

<http://calculusresources.weebly.com>

<https://www.projectmaths.ie/teaching-calculus-for-understanding/>

<http://www.mathispower4u.com>

<https://www.youtube.com/nancypi>

<http://www.purplemath.com>



MODULE 3: CALCULUS (cont'd)

<https://www.khanacademy.org/>

<https://www.mathsisfun.com/calculus/index.html>

www.intmath.com/calculus

<https://tutorial.math.lamar.edu/Classes/Calcl/Calcl.aspx>

<https://www.mathsisfun.com/calculus/index.html>

◆ OUTLINE OF ASSESSMENT

The same scheme of assessment will be applied to each Module of this single-unit course. Candidates' performance will be reported as an overall grade and a grade on each Module.

The assessment will comprise two components:

1. External assessment undertaken at the end of the academic year in which the course is taken. This contributes 80% to the candidate's overall grade.
2. School-Based assessment undertaken throughout this course. This contributes 20% to the candidate's overall grade.

EXTERNAL ASSESSMENT

(80%)

The candidate is required to sit a multiple choice paper and a written paper for a total of 4 hours. *The alternate to the School-Based Assessment, Paper 032 will also be examined externally.*

Paper 01 (1 hour 30 minutes)	This paper comprises 45 compulsory multiple choice items, 15 from each module. Each item is worth 1 mark.	30 %
Paper 02 (2 hours 30 minutes)	This paper consists of three sections, each corresponding to a Module. Each section will contain two extended-response questions. Candidates will be required to answer all six questions.	50 %

SCHOOL-BASED ASSESSMENT

(20%)

Paper 031

The School-Based Assessment comprises a project designed and internally assessed by the teacher and externally moderated by **CXC®**. This paper comprises a single project requiring candidates to demonstrate the practical application of Mathematics in everyday life. In essence it should allow candidates to probe, describe and explain a mathematical area of interest and communicate the findings using mathematical symbols, language and tools. The topic(s) chosen may be from any Module or combination of different Modules of the syllabus. The project may require candidates to collect data (Project B), or may be theory based (Project A), requiring solution or proof of a chosen problem.

Project A is based on applying mathematical concepts and procedures from any module in the syllabus in order to understand, describe or explain a real-world phenomenon. The project is theory based. Project B is based on applying mathematical concepts and procedures from any module in the syllabus in order to understand, describe or explain a real-world phenomenon. The project is experiment based and involves the collection of data.

Candidates should complete one project, either Project A or Project B.

Paper 032 (Alternative to Paper 031), examined externally

Paper 032 is a written paper consisting of a case study based on the three modules.

This paper is an alternative for Paper 031 and is intended for private candidates. Details are on page 50.

MODERATION OF THE SCHOOL-BASED ASSESSMENT

School-Based Assessment Record Sheets are available online via the CXC®'s website www.cxc.org.

All School-Based Assessment Record of marks must be submitted online using the SBA data capture module of the Online Registration System (ORS). Assignments will be requested by CXC® for moderation purposes. These assignments will be re-assessed by CXC® Examiners who moderate the School-Based Assessment. Teachers' marks may be adjusted as a result of moderation. The Examiners' comments will be sent to schools. All *assignments* must be *submitted* by the stipulated deadlines.

Copies of the students' assignments that are not submitted must be retained by the school until three months after publication by CXC® of the examination results.

ASSESSMENT DETAILS

External Assessment by Written Papers (80% of Total Assessment)

Paper 01 (1 hour 30 minutes - 30% of Total Assessment)

1. Composition of papers

- (a) This paper consists of 45 multiple choice items and is partitioned into three sections (Module 1, 2 and 3). Each section contains 15 questions.
- (b) All items are compulsory.

2. Syllabus Coverage

- (a) Knowledge of the entire syllabus is required.
- (b) The paper is designed to test candidates' knowledge across the breadth of the syllabus.

3. Question Type

Questions may be presented using words, symbols, tables, diagrams or a combination of these.

4. Mark Allocation

- (a) Each item is allocated 1 mark.

- (b) Each Module is allocated 15 marks.
- (c) The total number of marks available for this paper is 45.
- (d) This paper contributes 30% towards the final assessment.

5. Award of Marks

Marks will be awarded for reasoning, algorithmic knowledge and conceptual knowledge.

<u>Reasoning:</u>	Clear reasoning, explanation and/or logical argument.
<u>Algorithmic knowledge:</u>	Evidence of knowledge, ability to apply concepts and skills, and to analyse a problem in a logical manner.
<u>Conceptual knowledge:</u>	Recall or selection of facts or principles; computational skill, numerical accuracy and acceptable tolerance in drawing diagrams.

6. Use of Calculators

- (a) Each candidate is required to have a silent non-programmable calculator and is responsible for its functioning.
- (b) The use of calculators with graphical displays will not be permitted.
- (c) Calculators must not be shared during the examination.

Paper 02 (2 hours 30 minutes - 50% of Total Assessment)

This paper will be divided into three sections, each section corresponding to a Module.

1. Composition of Paper

- (a) This paper consists of six questions, two questions from each Module.
- (b) All questions are compulsory.

2. Syllabus Coverage

- (a) Each question may require knowledge from more than one topic in the Module from which the question is taken and will require sustained reasoning.
- (b) Each question may address a single theme or unconnected themes.
- (c) The intention of this paper is to test candidates' in-depth knowledge of the syllabus.

3. Question Type

Paper 02 consists of six essay type questions which require candidates to provide an extended response involving higher order thinking skills such as application, analysis, synthesis and evaluation.



- (a) Questions may require an extended response.
- (b) Questions may be presented using words, symbols, diagrams, tables or combinations of these.

4. Mark Allocation

- (a) Each question is worth 15 marks.
- (b) The number of marks allocated to each sub-question will appear in brackets on the examination paper.
- (c) Each Module is allocated 30 marks.
- (d) The total marks available for this paper is 90.
- (e) The paper contributes 50% towards the final assessment.

5. Award of Marks

- (a) Marks will be awarded for conceptual knowledge, algorithmic knowledge and reasoning.

Conceptual Knowledge: Recall *and understand* facts or principles.

Algorithmic knowledge: Evidence of knowledge, ability to apply concepts and skills, and to analyse a problem in a logical manner. *Computational skill, numerical accuracy and acceptable tolerance in drawing diagrams.*

Reasoning: *Selection of appropriate strategy, evidence of clear reasoning, explanation and/or logical argument.*

- (b) Full marks are awarded for **correct** answers and the presence of **appropriate working**.
- (c) It may be possible to earn partial credit for a correct method where the answer is incorrect.
- (d) If an incorrect answer in an earlier question or part-question is used later in a section or a question, then marks may be awarded in the later part even though the original answer is incorrect. In this way, a candidate is not penalised twice for the same mistake.
- (e) A correct answer given with no indication of the method used (in the form of written work) may receive (one) mark only. Candidates are, therefore, advised to show all relevant working.

6. Use of Calculators

- (a) Each candidate is required to have a silent non-programmable calculator and is responsible for its functioning.
- (b) The use of calculators with graphical displays will not be permitted.
- (c) Answers found by using a calculator, without relevant working shown, may not be awarded full marks.
- (d) Calculators must not be shared during the examination.

7. Use of Mathematical Tables

A booklet of mathematical formulae and tables will be provided.

SCHOOL-BASED ASSESSMENT (20 per cent)

During the course of study for the subject, students obtain marks for the competence they develop and demonstrate in undertaking their School-Based Assessment assignments. These marks contribute to the final marks and grades that are awarded to the students for their performance in the examination.

The guidelines provided in this syllabus for selecting appropriate tasks are intended to assist teachers and students in selecting assignments that are valid for the purpose of School-Based Assessment. The guidelines provided for the assessment of these assignments are also intended to assist teachers in awarding marks that are reliable estimates of the achievements of students in the School-Based Assessment component of the course. In order to ensure that the scores awarded are in line with the **CXC®** standards, the Council undertakes the moderation of a sample of the School-Based Assessment assignments marked by each teacher.

School-Based Assessment provides an opportunity to individualise a part of the curriculum to meet the needs of the student. It facilitates feedback to the student at various stages of the experience. This helps to build the self-confidence of the students as they proceed with their studies. School-Based Assessment also facilitates the development of the critical skills and abilities emphasised by this CAPE subject and enhances the validity of the examination on which the students' performance is reported. School-Based Assessment, therefore, makes a significant and unique contribution to both the development of the relevant skills and the testing and rewarding of the student for the development of those skills. Note that groupwork should be encouraged and employed where appropriate; however, candidates are expected to submit individual assignments for the School-Based Assessment. *Small groups of no more than five persons.*

REQUIREMENTS OF THE SCHOOL-BASED ASSESSMENT

The School-Based Assessment is based on skills and competencies related specifically to the Modules of the Unit. However, students who repeat in a subsequent sitting may reuse their School-Based Assessment marks.

Skills to be assessed

<u>Conceptual knowledge:</u>	Recall <i>and understand</i> facts or principles.
<u>Algorithmic knowledge:</u>	Evidence of knowledge, ability to apply concepts and skills, and to analyse a problem in a logical manner. <i>Computational skill, numerical accuracy and acceptable tolerance in drawing diagrams.</i>
<u>Reasoning:</u>	<i>Selection of appropriate strategy, evidence of clear reasoning, explanation and/or logical argument.</i>

Managing the research project

The research project is worth 20% of the candidate's total mark. Teachers should ensure that sufficient time is allowed for teaching the research skills required, explaining the requirements of the School-Based Assessment, discussing the assessment criteria and monitoring and evaluating the project work.

Planning

An early start to planning project work is highly recommended. A schedule of the dates for submitting project work (agreed by both teachers and candidates) should be established.

Length of the report

The length of the report should not exceed 1500 words, not including bibliography, appropriate quotations, sources, charts, graphs, tables, pictures, references and appendices.

CRITERIA FOR THE SCHOOL-BASED ASSESSMENT (Paper 031)

This paper is compulsory and consists of a project.

AIMS OF THE PROJECT

The aims of the project are to:

1. promote self-learning *and collaboration*;
2. allow teachers the opportunity to engage in the formative assessment of their students;
3. enable candidates to use the methods and procedures acquired to describe or explain real-life phenomena; *and*,
4. foster the development of critical thinking skills among students.

Requirements of the Project

The project will be presented in the form of a report and should include the following:

Project A

1. Project title
2. *Introduction*
3. Mathematical Formulation
4. *Problem Solution*
5. Discussion of findings
6. *Bibliography*
7. *Appendix*

Project B

1. *Project title*
2. *Introduction*
3. Research Methodology
4. *Presentation of Data*
5. *Analysis of data*
6. *Bibliography*
7. *Appendix*

(a) Integration of Project into the Course

- (i) The activities related to project work should be integrated into the course to enable candidates to learn and practice the skills of undertaking a successful project.
- (ii) Time should be allocated for *in class* discussion of project work. For example, discussion of how data should be collected, analysed and presented.
- (iii) Class time should also be allocated for discussion between teacher and student, and student and student.

(b) Management of Project

Planning

An early start to planning project work is highly recommended and the schedule of the dates for submission should be developed by teachers and candidates.

Length

The length of the report of the project should not exceed 1500 words (excluding diagrams, graphs, tables and references). A total of 10 percent of the candidate's score will be deducted for any research paper in excess of 1500 words (excluding diagrams, graphs, tables and references). If a deduction is to be made from a candidate's score, the teacher should clearly indicate on the assignment, the candidate's original score before the deduction is made, the marks which are to be deducted, and the final score that the candidate receives after the deduction has been made.

Guidance

Each candidate should know the requirements of the project and its assessment process.

Although candidates may consult with resource persons besides the teacher, the candidate's submission should be his or her own work.

Candidates are not expected to work on their own. The teacher is expected to give appropriate guidance at all stages of project work, for example, chapters to read, alternative procedures to follow and other sources of information.

Authenticity

Teachers are required to ensure that all projects are the candidates' work. The recommended procedures are to:

1. engage candidates in discussion;
2. ask candidates to describe procedures used and summarise findings either orally or written; and,
3. ask candidates to explain specific aspects of the analysis.

ASSESSMENT CRITERIA FOR THE PROJECT

General

It is recommended that candidates be provided with assessment criteria before commencing the project.

1. The following aspects of the project will be assessed:

<i>Project A</i>	<i>Project B</i>
1 <i>Project title</i>	<i>Project title</i>
2 <i>Introduction</i>	<i>Introduction</i>
3 <i>Mathematical Formulation</i>	<i>Research Methodology</i>
4 <i>Problem Solution</i>	<i>Presentation of Data</i>
5 <i>Discussion of findings</i>	<i>Analysis of data</i>
6 <i>Overall Presentation</i>	<i>Overall Presentation</i>
7 <i>Bibliography</i>	<i>Bibliography</i>
8 <i>Appendix</i>	<i>Appendix</i>

2. For each component, the aim is to find the level of achievement reached by the candidate.
3. For each component, only whole numbers should be awarded.
4. It is recommended that the assessment criteria be available to candidates at all times.

ASSESSING THE PROJECT

The project will be graded out of a total of 60 marks and marks will be allocated to each task as outlined below. Candidates will be awarded 4 marks for communicating information in a logical way using correct grammar. These marks are awarded under *Overall Presentation*.

Allocation of Marks for the Research Project

Marks will be allocated according to the following scheme:

MARK SCHEME FOR THE SCHOOL-BASED ASSESSMENT

Project A

Project Descriptors	Allocation of marks	
Project Title [2]		
• Title is a clear and concise statement.	2	(2)
• Title is a concise statement.	1	
Introduction [7]		
• Background of project.	1	(1)
• Problem is clearly stated.	1	(1)
• Purpose of Project ○ Purpose is clearly stated. ○ Purpose relates to real-life situations OR Purpose relates to solving existing problem.	1 1	(2)
• <i>Research questions clearly stated and relate to purpose.</i>	2	(2)
• Summary of mathematical approach including the analysis used.	1	(1)
Mathematical Formulation [12]		
• Identifies all the important elements of the problem and shows a complete understanding of the relationships among them.	5 - 6	(6)
• Identifies some important elements of the problem and shows a general understanding of the relationships among them.	3 – 4	
• identifies some of the important elements of the problem and shows a very limited understanding of the relationships among them.	1 - 2	
• Good understanding of the problem's mathematical concepts/principles.	3	(3)
• Fair understanding of the problem's mathematical concepts/principles.	2	
• Limited understanding of the problem's mathematical concepts/principles.	1	
• Mathematical model(s)/method(s) applied is/are most suitable for the task.	3	(3)
• Mathematical model(s)/method(s) applied is/are appropriate but <i>not most</i> for the task.	2	
• Mathematical model(s)/method(s) applied is/are acceptable.	1	

Project Descriptors (cont'd)	Allocation of marks	
Problem Solution [23]		
- Assumptions are clearly identified and explained. - Assumptions <i>are</i> misidentified or stated in an unclear manner.	2 1	(2)
<i>Logical algorithms</i> and executed correctly. - Logical algorithms are used. <i>Logical Algorithms</i> are used but contain errors.	3 2 1	(3)
- All diagrams are appropriate to the problem. - Some of the diagrams are appropriate to the problem.	2 1	(2)
- All diagrams are clearly labelled. - Some diagrams are clearly labelled.	2 1	(2)
- Explanations are sufficient and clearly stated. - Some explanations are sufficient and clearly stated. - Most of the explanations are vague.	3 2 1	(3)
- All of the theorems and/or formulae are relevant to the solution and <i>are</i> correctly applied. <i>Most</i> of the theorems and/or formulae are relevant to the solution and correctly applied. - theorems and/or formulae are relevant to the solution and not correctly applied.	3 2 1	(3)
- More than 75% of calculations are accurate. - Between 50% and 75% of calculations are accurate.	2 1	(2)
- Adequate reference to previous work, <i>for example, books and journals</i>. (This includes at least THREE sources with supporting theorems and formulas). - Limited reference to previous work, <i>for example, books and journals</i>. (This includes ONE OR TWO sources with supporting theorems and formulas).	2 1	(2)
- Interpretation attempted. - Interpretations of results are <i>satisfactory</i> given the objectives. <i>Interpretations of results are aligned to desired targets.</i> <i>Interpretations of results are aligned to mathematical approach.</i>	1 1 1 1	(4)
Discussion of findings [10]		
Discusses the validity of the solution.	1	(1)
Applies the solution or proof correctly to the given real-world problem.	1	(1)
- Discussion is coherent, concise and relates to the purpose of the project. - Discussion is coherent, concise but does not fully <i>relate</i> to the purpose of the project.	2 1	(2)

Project Descriptors (cont'd)	Allocation of marks	
The limitations of the research are relevant and comprehensively discussed.	2	(2)
The limitations of the research are relevant but not fully discussed.	1	
Recommendations are relevant.	1	(2)
Recommendations are practical.	1	
Conclusion is succinct, fully reflects the objectives and is supported by data.	2	(2)
Conclusion is adequate, partially reflects the objectives and partially supported by data.	1	
Overall Presentation [4]		
Communicates information in a logical way using correct grammar and appropriate mathematical jargon all of the time.	4	(4)
Communicates information in a logical way using correct grammar and appropriate mathematical jargon most of the time.	3	
Communicates information in a logical way using correct grammar and appropriate mathematical jargon some of the time.	2	
Communicates information in a logical way using correct grammar and appropriate mathematical jargon in a limited way.	1	
Bibliography [2]		
In-text <i>citations are included in the Bibliography.</i>	2	(2)
Inclusion of bibliography only.	1	
Appendix		
TOTAL		60 marks

Project B

Project Descriptors	Allocation of marks	
Project Title [2]		
• Title is a clear statement.	1	(1)
• Title is a concise statement.	1	(1)
Introduction [7]		
• <i>Background of project.</i>	1	(1)
• Problem clearly stated.	1	(1)
• <i>Purpose of Project.</i>		
• <i>Purpose is clearly stated and relates to real-life situations OR Purpose relates to solving existing problem.</i>	2	(2)
• <i>Purpose is stated but does not relate to real-life situations OR Purpose does not relate to solving existing problem.</i>	1	
• <i>Research questions clearly stated and relates to purpose.</i>	2	(2)
• <i>Research question stated but not related to purpose.</i>	1	
• <i>Summary of methodology and analysis are included.</i>	1	(1)
Research Methodology [14]		
• Research design (experimental, quasi-experimental, non-experimental) is clearly and logically outlined.	2	(2)
• Research method/design (experimental, quasi-experimental, and non-experimental) is not clearly documented.	1	
• The limitations of the research are relevant and comprehensively discussed.	2	(2)
• The limitations of the research are relevant but not fully discussed.	1	
• Description of the sampling process/sample design ○ Identification of the target population. ○ Specification of the sampling frame or otherwise justify. ○ Description of sample selection methodology/selection of subjects (participants): - sampling method (non-random sampling) sample size is appropriate.	1 1 1	(3)
• Instrument Design ○ Selection of instrument (for example, questionnaires, interviews, case studies, tests, measures, observations, scales). ○ Selection is justified in a comparative manner.	1 1	(2)

Project Descriptors (cont'd)	Allocation of marks	
Instrument has relevant items which are clearly articulated and are logically outlined (Alternatively, if a previously designed instrument is used then it must be cited and justified).	2	(2)
Instrument has relevant items some of which are clearly articulated (Alternatively, if a previously designed instrument is used then it must be cited and justified).	1	
- Data Management <i>Logical</i> description of the data collection process. <i>Satisfactory description</i> of the data collection process. - Data Entry/ Data Recording methods clearly described.	2 1 1	(2) (1)
Presentation of Data [11]		
- Display of Results using graphs outlined in the syllabus <i>Using at least one table:</i> - appropriately used according to the data type and portray the data accurately and clearly. <i>Using Stem & Leaf Plot or Box and Whiskers Plot.</i> <i>Showing key/scale of Stem & Leaf Plot or Box and Whiskers Plot.</i> <i>Using at least TWO other graphs, figures or chart as it relates to the syllabus.</i> <i>Justification of choice for at least TWO of charts, graphs or figures.</i> <i>Appropriate use of tables, graphs, charts and figures.</i>	1 1 1 2 2 1	(8)
- Description of tables, charts and figures: <i>description of all tables.</i> <i>description of all graphs, charts or figures.</i> <i>description of some charts and figures.</i>	1 2 1	(1) (2)
Analysis of Data [9]		
- Statistical analysis tools <i>Measures of Location and Spread.</i> <i>Probability.</i> <i>Probability Distributions.</i> <i>Measures of Relative Position (e g percentiles and z-scores).</i>	1 1 1 1	(4)

Project Descriptors (cont'd)	Allocation of marks	
All relevant formulae stated.	2	(2)
Some relevant formulae stated.	1	
Relevant steps and calculation shown.	1	(1)
More than 75% of calculations are accurate.	2	(2)
Between 50% and 75% of calculations are accurate.	1	
Discussion of Findings [11]		
- Interpretations of results - Statement of at least THREE findings. - Statement of ONE to TWO findings. - Statement of findings follows from analysis of data. - Justification of at least ONE finding. - Identification of ONE trend and ONE pattern OR anomaly (1 mark awarded for the inclusion of 1 trend; and, 1 mark awarded for the inclusion of a pattern or anomaly).	2 1 1 1 2	(2) (1) (1) (2)
Conclusion is clearly stated.	1	(3)
Conclusion relates to the research questions/objectives.	1	
Appropriate conclusion is drawn from analysis/findings of data.	1	
- Recommendations for future development and research - Recommendations are relevant. - Recommendations are practical.	1 1	(2)
Overall Presentation [4]		
- Communication of information in a logical way - Communicates information in a logical way using correct grammar and appropriate mathematical jargon all of the time. - Communicates information in a logical way using correct grammar and appropriate mathematical jargon most of the time.	4 3	(4)

Project Descriptors (Cont'd)	Allocation of marks	
○ Communicates information in a logical way using correct grammar and appropriate mathematical jargon some of the time.	2	
○ Communicates information in a logical way using correct grammar and appropriate mathematical jargons in a limited way.	1	
Bibliography [2]		
• In-text <i>citations are included in the Bibliography.</i>	2	(2)
• Inclusion of bibliography only.	1	
Appendix		
TOTAL		60 marks

◆ REGULATIONS FOR PRIVATE CANDIDATES

Private candidates will be required to write Papers 01, 02 and 032. Detailed information on Papers 01 and 02 is given on pages 35–37 of this syllabus.

Paper 032 is the alternative paper to the School-Based Assessment. This paper is worth 20 per cent of the total mark for the Unit. Paper 032 will test the student's acquisition of the skills in the same areas of the syllabus identified for the School-Based Assessment. Consequently, candidates are advised to undertake a project similar to the project that the school candidates would normally complete and submit for School-Based Assessment to develop the requisite competences for this course of study. It should be noted that private candidates would not be required to submit a project document.

Paper 032 (1 hour 30 minutes – 20 % of Total Assessment)

1. Composition of Paper

Paper 032 is a written paper consisting of a case study based on the three modules. The paper consists of three compulsory questions which are divided into parts. The questions test skills similar to those in the School-Based assessment (Paper 031).

2. Syllabus Coverage

This paper is intended to test the knowledge and skills contained in Modules 1, 2 and 3 as outlined in the syllabus.

3. Question Type

Questions in this paper may be short answer or essay type, based on the case study.

4. Mark Allocation

- (a) This paper is worth 60 marks.
- (b) Each question is worth 20 marks and contributes 20 percent toward the final assessment.

◆ REGULATIONS FOR RESIT CANDIDATES

CAPE® candidates may reuse any moderated SBA score within a two-year period. In order to assist candidates in making decisions about whether or not to reuse a moderated SBA score, the Council will continue to indicate on the preliminary results if a candidate's moderated SBA score is less than 50% in a particular Unit. Candidates re-using SBA scores should register as "Re-sit candidates" and must provide the previous candidate number when registering.

◆ ASSESSMENT GRID

The Assessment Grid for this Course contains marks assigned to papers and to Modules, and percentage contributions of each paper to total scores.

Papers	Module 1	Module 2	Module 3	Total	(%)
	Foundations of Mathematics	Statistics	Calculus		
External Assessment					
Paper 01 (1 hour 30 minutes) Multiple Choice	15 (30 weighted)	15 (30 weighted)	15 (30 weighted)	45 (90 weighted)	30
Paper 02 (2 hours 30 minutes) Extended Response	30 (50 weighted)	30 (50 weighted)	30 (50 weighted)	90 (150 weighted)	50

Papers	Module 1	Module 2	Module 3	Total	(%)
	Foundations of Mathematics	Statistics	Calculus		
School-Based Assessment Paper 031 or Paper 032 (1 hour 30 minutes)	20	20	20	60	(20)
Weighted Module Total	100	100	100	300	100

◆ GLOSSARY OF EXAMINATION TERMS

WORD	DEFINITION	NOTES
Analyse	examine in detail	
Annotate	add a brief note to a label	Simple phrase or a few words only.
Apply	use knowledge/principles to solve problems	Make inferences/conclusions.
Assess	present reasons for the importance of particular structures, relationships or processes	Compare the advantages and disadvantages or the merits and demerits of a particular structure, relationship or process.
Calculate	arrive at the solution to a numerical problem	Steps should be shown; units must be included.
Classify	divide into groups according to observable characteristics	
Comment	state opinion or view with supporting reasons	
Compare	state similarities and differences	An explanation of the significance of each similarity and difference stated may be required for comparisons which are other than structural.
Construct	use a specific format to make and/or draw a graph, histogram, pie chart or other representation using data or material provided or drawn from practical investigations, build (for example, a model), draw scale diagram	Such representations should normally bear a title, appropriate headings and legend.
Deduce	make a logical connection between two or more pieces of information; use data to arrive at a conclusion	
Define	state concisely the meaning of a word or term	This should include the defining equation/formula where relevant.
Demonstrate	show; direct attention to	

WORD	DEFINITION	NOTES
Derive	to deduce, determine or extract from data by a set of logical steps some relationship, formula or result	This relationship may be general or specific.
Describe	provide detailed factual information of the appearance or arrangement of a specific structure or a sequence of a specific process	Description may be in words, drawings or diagrams or any appropriate combination. Drawings or diagrams should be annotated to show appropriate detail where necessary.
Determine	find the value of a physical quantity	
Design	plan and present with appropriate practical detail	Where hypotheses are stated or when tests are to be conducted, possible outcomes should be clearly stated and/or the way in which data will be analysed and presented.
Develop	expand or elaborate an idea or argument with supporting reasons	
Diagram	simplified representation showing the relationship between components	
Differentiate/ Distinguish (between/among)	state or explain briefly those differences between or among items which can be used to define the items or place them into separate categories	
Discuss	present reasoned argument; consider points both for and against; explain the relative merits of a case	
Draw	make a line representation from specimens or apparatus which shows an accurate relation between the parts	In the case of drawings from specimens, the magnification must always be stated.
Estimate	make an approximate quantitative judgement	
Evaluate	weigh evidence and make judgements based on given criteria	The use of logical supporting reasons for a particular point of view is more important than the view held; usually both sides of an argument should be considered.

WORD	DEFINITION	NOTES
Explain	give reasons based on recall; account for	
Find	locate a feature or obtain as from a graph	
Formulate	devise a hypothesis	
Identify	name or point out specific components or features	
Illustrate	show clearly by using appropriate examples or diagrams, sketches	
Interpret	explain the meaning of	
Investigate	use simple systematic procedures to observe, record data and draw logical conclusions	
Justify	explain the correctness of	
Label	add names to identify structures or parts indicated by pointers	
List	itemise without detail	
Measure	take accurate quantitative readings using appropriate instruments	
Name	give only the name of	No additional information is required.
Note	write down observations	
Observe	pay attention to details which characterise a specimen, reaction or change taking place; to examine and note scientifically	Observations may involve all the senses and/or extensions of them but would normally exclude the sense of taste.
Outline	give basic steps only	
Plan	prepare to conduct an investigation	
Predict	use information provided to arrive at a likely conclusion or suggest a possible outcome	

WORD	DEFINITION	NOTES
Record	write an accurate description of the full range of observations made during a given procedure	This includes the values for any variable being investigated; where appropriate, recorded data may be depicted in graphs, histograms or tables.
Relate	show connections between; explain how one set of facts or data depend on others or are determined by them	
Sketch	make a simple freehand diagram showing relevant proportions and any important details	
State	provide factual information in concise terms outlining explanations	
Suggest	offer an explanation deduced from information provided or previous knowledge. (a hypothesis; provide a generalisation which offers a likely explanation for a set of data or observations.)	No correct or incorrect solution is presumed but suggestions must be acceptable within the limits of scientific knowledge.
Use	apply knowledge/principles to solve problems	Make inferences/ conclusions.

◆ GLOSSARY OF MATHEMATICAL TERMS

WORDS	MEANING
Absolute Value	The absolute value of a real number x , denoted by $ x $, is defined by $ x = x$ if $x > 0$ and $ x = -x$ if $x < 0$. For example, $ -4 = 4$.
Algorithm	A process consisting of a specific sequence of operations to solve a certain type of problems. See Heuristic .
Argand Diagram	An Argand diagram is a rectangular coordinate system where the complex number $x + iy$ is represented by the point whose coordinates are x and y . The x -axis is called the real axis and the y -axis is called the imaginary axis.
Argument of a Complex Number	The angle, $\theta = \tan^{-1} \left(\frac{y}{x} \right)$ is called the argument of a complex number $z = x + iy$.
Arithmetic Mean	The average of a set of values found by dividing the sum of the values by the amount of values.
Arithmetic Progression	An arithmetic progression is a sequence of elements, $a_1, a_2, a_3, \dots$, such that there is a common difference of successive terms. For example, the sequence $\{2, 5, 8, 11, 14, \dots\}$ has common difference, $d = 3$.
Asymptotes	A straight line is said to be an asymptote of a curve if the curve has the property of becoming and staying arbitrarily close to the line as the distance from the origin increases to infinity.
Augmented Matrix	If a system of linear equations is written in matrix form $Ax = b$, then the matrix $[A b]$ is called the augmented matrix.
Average	The average of a set of values is the number which represents the usual or typical value in that set. Average is synonymous with measures of central tendency. These include the mean, mode and median.
Axis of symmetry	A line that passes through a figure such that the portion of the figure on one side of the line is the mirror image of the portion on the other side of the line.
Bar Chart	A bar chart is a diagram which is used to represent the frequency of each category of a set of data in such a way that the height of each bar is proportionate to the frequency of the category it represents. Equal space should be left between consecutive bars to indicate it is not a histogram.

WORDS

MEANING

Base	In the equation $y = \log_a x$, the quantity a is called the base. The base of a polygon is one of its sides; for example, a side of a triangle. The base of a solid is one of its faces; for example, the flat face of a cylinder. The base of a number system is the number of digits it contains; for example, the base of the binary system is two.
Bias	Bias is systematically misestimating the characteristics of a population (parameters) with the corresponding characteristics of the sample (statistics).
Biased Sample	A biased sample is a sample produced by methods which ensures that the statistics is systematically different from the corresponding parameters.
Bijjective	A function is bijjective if it is both injective and surjective; that is, both one-to-one and onto.
Bimodal	Bimodal refers to a set of data with two equally common modes.
Binomial	An algebraic expression consisting of the sum or difference of two terms. For example, $(ax + b)$ is a binomial.
Binomial Coefficients	The coefficients of the expansion $(x + y)^n$ are called binomial coefficients. For example, the coefficients of $(x + y)^3$ are 1, 3, 3 and 1.
Box-and-whiskers Plot	A box-and-whiskers plot is a diagram which displays the distribution of a set of data using the five number summary. Lines perpendicular to the axis are used to represent the five number summary. Single lines parallel to the axis are used to connect the lowest and highest values to the first and third quartiles respectively and double lines parallel to the axis form a box with the inner three values.
Categorical Variable	A categorical variable is a variable measured in terms possession of quality and not in terms of quantity.
Class Intervals	Non-overlapping intervals, which together contain every piece of data in a survey.
Closed Interval	A closed interval is an interval that contains its end points; it is denoted with square brackets $[a, b]$. For example, the interval $[-1, 2]$ contains -1 and 2. For contrast see open interval.

WORDS

MEANING

Composite Function

A function consisting of two or more functions such that the output of one function is the input of the other function. For example, in the composite function $f(g(x))$ the input of f is g .

Compound Interest

A system of calculating interest on the sum of the initial amount invested together with the interest previously awarded; if A is the initial sum invested in an account and r is the rate of interest per period invested, then the total after n periods is $A(1 + r)^n$.

Combinations

The term combinations refers to the number of possible ways of selecting r objects chosen from a total sample of size n if you don't care about the order in which the objects are arranged. Combinations is calculated using the formula. See **factorial**.

$${}^nC_r = \binom{n}{r} = C_{nr} = \frac{n!}{r!(n-r)!}$$

Complex Numbers

A complex number is formed by adding a pure imaginary number to a real number. The general form of a complex number is $z = x + iy$, where x and y are both real numbers and i is the imaginary unit: $i^2 = -1$. The number x is called the real part of the complex number, while the number y is called the imaginary part of the complex number.

Conditional Probability

The conditional probability is the probability of the occurrence of one event affecting another event. The conditional probability of event A occurring even B has occurred is denoted $P(A|B)$ (read "probability of A given B "). The formula for conditional probability is

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}.$$

Conjugate of a Complex Number

The conjugate of a complex number $z = x + iy$ is the complex number $\bar{z} = x - iy$, found by changing the sign of the imaginary part. For example, if $z = 3 - 4i$, then $\bar{z} = 3 + 4i$.

Continuous

The graph of $y = f(x)$ is continuous at a point a if:

1. $f(a)$ exists;
2. $\lim_{x \rightarrow a} f(x)$ exists, and,
3. $\lim_{x \rightarrow a} f(x) = f(a)$.

A function is said to be continuous in an interval if it is continuous at each point in the interval.

Continuous Random Variable

A continuous random variable is a random variable that can take on any real number value within a specified range. For contrast, see **Discrete Random Variable**.

WORDS	MEANING
Coterminal	Two angles are said to be coterminal if they have the same initial and terminal arms. For example, $\theta = 30^\circ$ is coterminal with $\alpha = 390^\circ$.
Critical Point	A critical point of a function $f(x)$ is the point $P(x, y)$ where the first derivative, $f'(x)$ is zero. See also stationary points .
Data	Data (plural of datum) are the facts about something. For example, the height of a building.
Degree	- The degree is a unit of measure for angles. One degree is $\frac{1}{360}$ of a complete rotation. See also Radian. - The degree of a polynomial is the highest power of the variable that appears in the polynomial. For example, the polynomial $p(x) = 2 + 3x - x^2 + 7x^3$ has degree 3.
Delta	The Greek capital letter delta, which has the shape of a triangle: Δ , is used to represent "change in". For example, Δx represents "change in x ".
Dependent Events	In Statistics, two events A and B are said to be dependent if the occurrence of one event affects the probability of the occurrence of the other event. For contrast, see Independent Events .
Derivative	The derivative of a function $y = f(x)$ is the rate of change of that function. The notations used for derivative include: $y' = f'(x) = \frac{dy}{dx} = \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}.$
Descriptive Statistics	Descriptive statistics refers to a variety of techniques that allows for general description of the characteristics of the data collected. It also refers to the study of ways to describe data. For example, the mean, median, variance and standard deviation are descriptive statistics. For contrast, see Inferential Statistics .
Determinant	The determinant of a matrix is a number that is useful for describing the characteristics of the matrix. For example if the determinant is zero then the matrix has no inverse.
Differentiable	A continuous function is said to be differentiable over an interval if its derivative exists for every point in that interval. That means that the graph of the function is smooth with no kinks, cusps or breaks.
Differential Equation	A differential equation is an equation involving the derivatives of a function of one or more variables. For example, the equation $\frac{dy}{dx} - y = 0$ is a differential equation.

WORDS

MEANING

Differentiation

Differentiation is the process of finding the derivative.

Discrete

A set of values are said to be discrete if they are all distinct and separated from each other. For example, the set of shoe sizes where the elements of this set can only take on a limited and distinct set of values. See **Discrete Random Variables**.

Discrete Random Variable

A discrete random variable is a random variable that can only take on values from a discrete list. For contrast, see **Continuous Random Variables**.

Estimate

The best guess for an unknown quantity arrived at after considering all the information given in a problem.

Even Function

A function $y = f(x)$ is said to be even if it satisfies the property that $f(x) = f(-x)$. For example, $f(x) = \cos x$ and $g(x) = x^2$ are even functions. For contrast, see Odd Function.

Event

In probability, an event is a set of outcomes of an experiment. For example, the event A may be defined as obtaining two heads from tossing a coin twice.

Expected Value

The average amount that is predicted if an experiment is repeated many times. The expected value of a random variable X is denoted by $E[X]$. The expected value of a discrete random variable is found by taking the sum of the product of each outcome and its associated probability. In short,

$$E[X] = \sum_{i=1}^n x_i p(x_i).$$

Experimental Probability

Experimental probability is the chances of something happening, based on repeated testing and observing results. It is the ratio of the number of times an event occurred to the number of times tested. For example, to find the experimental probability of winning a game, one must play the game many times, then divide by the number of games won by the total number of games played.

Exponent

An exponent is a symbol or a number written above and to the right of another number. It indicates the operation of repeated multiplication.

Exponential Function

A function that has the form $y = a^x$, where a is any real number and is called the base.

Extrapolation

An extrapolation is a predicted value that is outside the range of previously observed values. For contrast, see **Interpolation**.

Factor

A factor is one of two or more expressions which are multiplied together. A prime factor is an indecomposable factor. For example, the factors of $(x^2 - 4)(x + 3)$ include $(x^2 - 4)$ and $(x + 3)$, where $(x + 3)$ is prime but $(x^2 - 4)$ is not prime as it can be further decomposed into $(x - 2)(x + 2)$.

WORDS	MEANING
Factorial	The factorial of a positive integer n is the product of all the integers from 1 up to n and is denoted by $n!$, where $1! = 0! = 1$. For example, $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.
Function	A correspondence in which each member of one set is mapped unto a member of another set.
Geometric Progression	A geometric progression is a sequence of terms obtained by multiplying the previous term by a fixed number which is called the common ratio. A geometric progression is of the form $a, ar, ar^2, ar^3, \dots$
Graph	A visual representation of data that displays the relationship among variables, usually cast along x and y axes.
Grouped Data	Grouped data refers to a range of values which are combined together so as to make trends in the data more apparent.
Heterogeneity	Heterogeneity is the state of being of incomparable magnitudes. For contrast, see Homogeneity .
Heuristic	A heuristic method of solving problems involve intelligent trial and error. For contrast, see Algorithm .
Histogram	A histogram is a bar graph with no spaces between the bars where the area of the bars are proportionate to the corresponding frequencies. If the bars have the same width then the heights are proportionate to the frequencies.
Homogeneity	Homogeneity is the state of being of comparable magnitudes. For contrast, see Heterogeneity .
Identity	An equation that is true for every possible value of the variables. For example $x^2 - 1 \equiv (x - 1)(x + 1)$ is an identity while $x^2 - 1 = 3$ is not, as it is only true for the values $x = \pm 2$. The identity element of an operation is a number such that when operated on with any other number results in the other number. For example, the identity element under addition of real numbers is zero; the identity element under multiplication of 2×2 matrices is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.
Independent Events	In Statistics, two events are said to be independent if they do not affect each other. That is, the occurrence of one event does not depend on whether or not the other event occurred.
Inferential Statistics	Inferential Statistics is the branch of mathematics which deals with the generalisations of samples to the population of values.
Infinity	The symbol ∞ indicating a limitless quantity. For example, the result of a nonzero number divided by zero is infinity.

WORDS

MEANING

Integration	Integration is the process of finding the integral which is the antiderivative of a function.
Interpolation	An interpolation is an estimate of an unknown value which is within the range of previously observed values. For contrast, see Extrapolation .
Interval	An interval on a number line is a continuum of points bounded by two limits (end points). An Open Interval refers to an interval that excludes the end points and is denoted (a, b). For example, $(0,1)$. A Closed Interval is an interval which includes the end points and is denoted $[a, b]$. For example $[-1,3]$. A Half-Open Interval is an interval which includes one end point and excludes the other. For example, $[0, \infty)$.
Interval Scale	Interval scale refers to data where the difference between values can be quantified in absolute terms and any zero value is arbitrary. Finding a ratio of data values on this scale gives meaningless results. For example, temperature is measured on the interval scale: the difference between 19°C and 38°C is 19°C , however, 38°C is not twice as warm as 19°C and a temperature of 0°C does not mean there is no temperature. See also Nominal, Ordinal and Ratio scales .
Inverse	1. The inverse of an element under an operation is another element which when operated on with the first element results in the identity. For example, the inverse of a real number under addition is the negative of that number. 2. The inverse of a function $f(x)$ is another function denoted $f^{-1}(x)$, which is such that $f[f^{-1}(x)] = f^{-1}[f(x)] = x$.
Irrational Number	A number that cannot be represented as an exact ratio of two integers. For example, π or the square root of 2.
Limit	The limit of a function is the value which the dependent variable approaches as the independent variable approaches some fixed value.
Line of Best Fit	The line of best fit is the line that minimises the sum of the squares of the deviations between each point and the line.
Linear Expression	An expression of the form $ax + b$ where x is a variable and a and b are constants, or in more variables, an expression of the form $ax + by + c$, $ax + by + cz + d$ where a, b, c and d are constants.

WORDS

MEANING

Logarithm

A logarithm is the power of another number called the base that is required to make its value a third number. For example, 3 is the logarithm which carries 2 to 8. In general, if y is the logarithm which carries a to x , then it is written as $y = \log_a x$ where a is called the base. There are two popular bases: base 10 and base e .

1. **The Common Logarithm (Log):** the equation $y = \log x$ is the shortened form for $y = \log_{10} x$.
2. **The Natural Logarithm (Ln):** The equation $y = \ln x$ is the shortened form for $y = \log_e x$.

Matrix

A rectangular arrangement of numbers in rows and columns.

Method

In Statistics, the research methods are the tools, techniques or processes that we use in our research. These might be, for example, surveys, interviews, or participant observation. Methods and how they are used are shaped by methodology.

Methodology

Methodology is the study of how research is done, how we find out about things, and how knowledge is gained. In other words, methodology is about the principles that guide our research practices. Methodology therefore explains why we're using certain methods or tools in our research.

Modulus

The modulus of a complex number $z = x + iy$ is the real number $|z| = \sqrt{x^2 + y^2}$. For example, the modulus of $z = -7 + 24i$ is $|z| = \sqrt{(-7)^2 + 24^2} = 25$.

Mutually Exclusive Events

Two events are said to be mutually exclusive if they cannot occur simultaneously, in other words, if they have nothing in common. For example, the event "Head" is mutually exclusive to the event "Tail" when a coin is tossed.

Mutually Exhaustive Events

Two events are said to be mutually exhaustive if their union represents the sample space.

Nominal Scale

Nominal scale refers to data which names of the outcome of an experiment. For example, the country of origin of the members of the West Indies cricket team. See also Ordinal, Interval and Ratio scales.

Normal

The normal to a curve is a line which is perpendicular to the tangent to the curve at the point of contact.

Odd Function

A function is an odd function if it satisfies the property that $f(-x) = -f(x)$. For example, $f(x) = \sin x$ and $g(x) = x^3$ are odd functions. For contrast, see **Even Function**.

WORDS

MEANING

Ordinal Scale

Data is said to be in the ordinal scale if they are names of outcomes where sequential values are assigned to each name. For example, if Daniel is ranked number 3 on the most prolific goal scorer at the Football World Cup, then it indicates that two other players scored more goals than Daniel. However, the difference between the 3rd ranked and the 10th ranked is not necessarily the same as the difference between the 23rd and 30th ranked players. See also **Nominal, Interval and Ratio scales**.

Outlier

An outlier is an observed value that is significantly different from the other observed values.

Parameter

In statistics, a parameter is a value that characterises a population.

Partial Derivative

The partial derivative of $y = f(x_1, x_2, x_3, \dots, x_n)$ with respect to x_i is the derivative of y with respect to x , while all other independent variables are treated as constants. The partial derivative is denoted by $\frac{\partial f}{\partial x}$. For example, if

$$f(x, y, z) = 2xy + x^2z - \frac{3x^3y}{z}$$

$$\text{then } \frac{\partial f}{\partial x} = 2y + 2xz - \frac{9x^2y}{z}$$

Pascal Triangle

The Pascal triangle is a triangular array of numbers such that each number is the sum of the two numbers above it (one left and one right). The numbers in the n^{th} row of the triangle are the coefficients of the binomial expansion $(x + y)^n$.

Percentile

The p^{th} percentile of in a list of numbers is the smallest value such that $p\%$ of the numbers in the list is below that value. See also **Quartiles**.

Permutations

Permutations refers to the number of different ways of selecting a group of r objects from a set of n objects when the order of the elements in the group is of importance and the items are not replaced. If $r = n$ then the permutations is $n!$; if $r < n$ then the number of permutations is ${}^n P_r = \frac{n!}{(n-r)!}$.

Piecewise Continuous Function

A function is said to be piecewise continuous if it can be broken into different segments where each segment is continuous.

Polynomial

A polynomial is an algebraic expression involving a sum of algebraic terms with nonnegative integer powers. For example, $2x^3 + 3x^2 - x + 6$ is a polynomial in one variable.

Population

In statistics, a population is the set of all items under consideration.

Principal Root

The principal root of a number is the positive root. For example, the principal square root of 36 is 6 (not -6).

WORDS

MEANING

Principal Value	The principal value of the arcsin and arctan functions lies on the interval $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. The principal value of the arcos function lies on the interval $0 \leq x \leq \pi$.
Probability	1. The probability of an event is a measure of how likely it is for an event to occur. The probability of an event is always a number between zero and 1. 2. Probability is the study of chance occurrences.
Probability Distribution	A probability distribution is a table or function that gives all the possible values of a random variable together with their respective probabilities.
Probability Space	The probability space is the set of all outcomes of a probability experiment.
Proportion	1. A relationship between two ratios in which the first ratio is always equal to the second. Usually of the form $\frac{a}{b} = \frac{c}{d}$. 2. The fraction of a part and the whole. If two parts of a whole are in the ratio 2:7, then the corresponding proportions are $\frac{2}{9}$ and $\frac{7}{9}$, respectively.
Pythagorean Triple	A Pythagorean triple refers to three numbers, a, b & c , satisfying the property that $a^2 + b^2 = c^2$.
Quadrant	The four parts of the coordinate plane divided by the x and y axes are called quadrants. Each of these quadrants has a number designation. First quadrant – contains all the points with positive x and positive y coordinates. Second quadrant – contains all the points with negative x and positive y coordinates. The third quadrant contains all the points with both coordinates negative. Fourth quadrant – contains all the points with positive x and negative y coordinates.
Quadrantal Angles	Quadrantal Angles are the angles measuring $0^\circ, 90^\circ, 180^\circ$ & 270° and all angles coterminal with these. See Coterminal .
Quartic	A quartic equation is a polynomial of degree 4.

WORDS

MEANING

Quartiles	Consider a set of numbers arranged in ascending or descending order. The quartiles are the three numbers which divide the set into four parts of equal amount of numbers. The first quartile in a list of numbers is the number such that a quarter of the numbers is below it. The second quartile is the median. The third quartile is the number such that three quarters of the numbers are below it. See also Percentile .
Quintic	A quintic equation is a polynomial of degree 5.
Radian	The radian is a unit of measure for angles, where one radian is of a $\frac{1}{2\pi}$ $^5\sqrt{243} = 3$ $^4\sqrt{16} = 2$ $\sqrt{25} = 5$ $^q\sqrt{x}$ complete rotation. One radian is the angle in a circle subtended by an arc of length equal to that of the radius of the circle. See also Degrees .
Radical	The radical symbol used indicate the taking of a root of a number. $^q\sqrt{x}$ means the q^{th} root of x ; if $q = 2$ then it is usually written as $\sqrt{x}$. For example, $^4\sqrt{16} = 2$. The radical always means to take the positive value. For example, both 5 and -5 satisfy the equation $x^2 = 25$.
Random Variable	A random variable is a variable that takes on a particular value when a random event occurs. —
Ratio Scale	Data are said to be on the ratio scale if they can be ranked, the distance between two values can be measured and the zero is absolute, that is, zero means “absence of”. See also Nominal, Ordinal and Interval Scales.
Regression	Regression is a statistical technique used for determining the relationship between two quantities.
Residual	In linear regression, the residual refers to the difference between the actual point and the point predicted by the regression line. That is the vertical distance between the two points.
Root	1. The root of an equation is the same as the solution of that equation. For example, if $y = f(x)$, then the roots are the values of x for which $y = 0$. Graphically, the roots are the x-intercepts of the graph. 2. The n^{th} root of a real number x is a number which, when multiplied by itself n times, gives x. If n is odd then there is one root for every value of x; if n is even then there are two roots (one positive and one negative) for positive values of x and no real roots for negative values of x. The positive root is called the Principal root and is represented by the radical sign ($\sqrt{}$). For example, the principal square root of 9 is written as $\sqrt{9} = 3$ but the square roots of 9 are $\pm\sqrt{9} = \pm 3$.

WORDS	MEANING
Sample	A group of items chosen from a population.
Sample Space	The set of all possible outcomes of a probability experiment. Also called probability space.
Sampling Frame	In statistics, the sampling frame refers to the list of cases from which a sample is to be taken.
Scientific Notation	A shorthand way of writing very large or very small numbers. A number expressed in scientific notation is expressed as a decimal number between 1 and 10 multiplied by a power of 10 (for example, $7000 = 7 \times 10^3$ or $0.0000019 = 1.9 \times 10^{-6}$).
Series	A series is an indicated sum of a sequence.
Sigma	1. The Greek capital letter sigma, Σ, denotes the summation of a set of values. 2. The corresponding lowercase letter sigma, σ, denotes the standard deviation.
Significant Digits	The amount of digits required for calculations or measurements to be close enough to the actual value. Some rules in determining the number of digits considered significant in a number: - The leftmost non-zero digit is the first significant digit. - Zeros between two non-zero digits are significant. - Trailing zeros to the right of the decimal point are considered significant.
Simple Event	A non-decomposable outcome of a probability experiment.
Skew	Skewness is a measure of the asymmetry of a distribution of data.
Square Matrix	A matrix with equal number of rows and columns.
Square Root	The square root of a positive real number n is the number m such that $m^2 = n$. For example, the square roots of 16 are 4 and -4.
Standard Deviation	The standard deviation of a set of numbers is a measure of the average deviation of the set of numbers from their mean.
Stationary Point	The stationary point of a function $f(x)$ is the point $P(x_o, y_o)$ where $f'(x) = 0$. There are three type of stationary points, these are: 1. Maximum point is the stationary point such that: $\frac{d^2f}{dx^2} \leq u$

WORDS

MEANING

2. Minimum point is the stationary point such that:

$$\frac{d^2f}{dx^2} \geq u$$

3. Point of inflexion is the stationary where

$$\frac{d^2f}{dx^2} = u$$

and the point is neither maximum nor minimum point.

Statistic	A statistic is a quantity calculated from among the set of items in a sample.
Statistical Inference	The process of estimating unobservable characteristics of a population by using information obtained from a sample.
Symmetry	Two points A and B are symmetric with respect to a line if the line is a perpendicular bisector of the segment AB .
Tangent	A line is a tangent to a curve at a point A if it just touches the curve at A in such a way that it remains on one side of the curve at A . A tangent to a circle intersects the circle only once.
Theoretical Probability	The chances of events happening as determined by calculating results that would occur under ideal circumstances. For example, the theoretical probability of rolling a 4 on a fair four-sided die is $\frac{1}{4}$ or 25%, because there is one chance in four to roll a 4, and under ideal circumstances one out of every four rolls would be a 4.
Trigonometry	The study of triangles. Three trigonometric functions defined for either acute angles in the right-angled triangle are: Sine of the angle x is the ratio of the side opposite the angle and the hypotenuse. In short, $\sin x = \frac{O}{H}$. Cosine of the angle x is the ratio of the short side adjacent to the angle and the hypotenuse. In short, $\cos x = \frac{A}{H}$. Tangent of the angle x is the ratio of the side opposite the angle and the short side adjacent to the angle. In short, $\tan x = \frac{O}{A}$.
Z-Score	The z-score of a value x is the number of standard deviations it is away from the mean of the set of all values. $z - score = \frac{x - \bar{x}}{\sigma}$

Western Zone Office

14 July 2023



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INTEGRATED MATHEMATICS

Specimen Papers and Mark Schemes/Keys

Specimen Papers, Mark Schemes and Keys:

Paper 01

Paper 02

Paper 032

SPECIMEN 2022



TEST CODE **02167010**

CARIBBEAN EXAMINATIONS COUNCIL
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INTEGRATED MATHEMATICS

Paper 01

1 hour 30 minutes

SPECIMEN PAPER

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This test consists of 45 items. You will have 1 hour and 30 minutes to answer them.
2. In addition to this test booklet, you should have an answer sheet.
3. Do not be concerned that the answer sheet provides spaces for more answers than there are items in this test.
4. Each item in this test has four suggested answers lettered (A), (B), (C), (D). Read each item you are about to answer and decide which choice is best.
5. On your answer sheet, find the number which corresponds to your item and shade the space having the same letter as the answer you have chosen. Look at the sample item below.

Sample Item

Which of the following is equivalent to $y = e^{2x}$?

- (A) $y = 2e^x$
- (B) $\log y = 2x$
- (C) $\ln 2x = y$
- (D) $\ln y = 2x$

Sample Answer



The best answer to this item is “ $\ln y = 2x$ ”, so (D) has been shaded.

6. If you want to change your answer, erase it completely before you fill in your new choice.
7. When you are told to begin, turn this page and work as quickly and as carefully as you can. If you cannot answer an item, go on to the next one. You may return to that item later.
8. You may do any rough work in this booklet.
9. The use of silent, non-programmable scientific calculators is allowed.

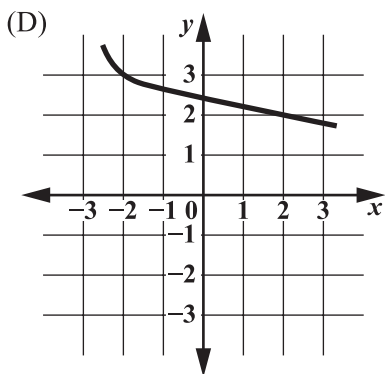
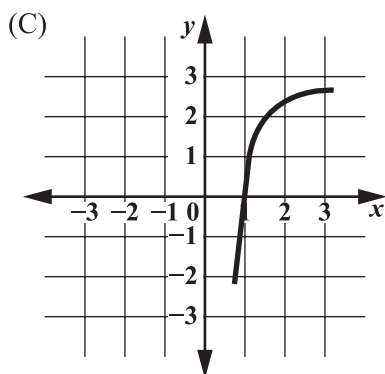
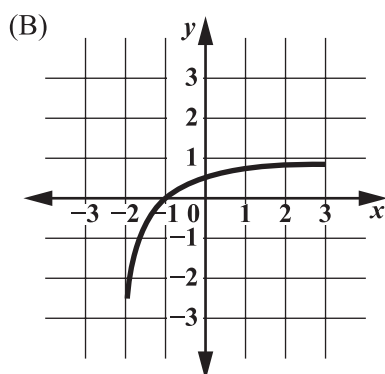
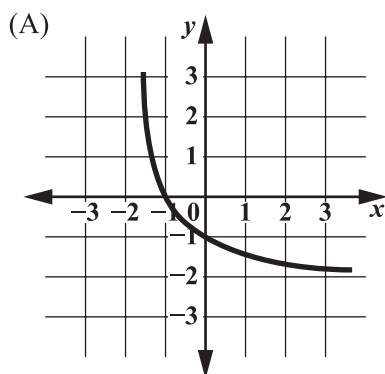
Examination Materials:

A list of mathematical formulae and tables. **(Revised 2012)**

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

1. Which of the following is defined as an injective function?
- (A) One-to-one
(B) One-to-many
(C) Many-to-one
(D) Many-to-many
2. If $f(x) = -3x + 1$ and $g(x) = -5x + 7$ then $f(g(x))$ is
- (A) $-8x + 8$
(B) $15x + 2$
(C) $15x - 20$
(D) $15x^2 - 26x + 7$
3. If $9 \times 3^{5x} = 54$, then x is
- (A) $\frac{\log_3 6}{5}$
(B) $\frac{\log_5 9}{54}$
(C) $\frac{\log_6 3}{9}$
(D) $\frac{\log_9 54}{15}$
4. $9 \log_t (r) - 6 \log_t (s^5)$ expressed as a single log is
- (A) $\log_t \left(\frac{r^9}{5s^6} \right)$
(B) $\log_t \left(\frac{9r}{6s^5} \right)$
(C) $\log_t \left(\frac{r^9}{s^{11}} \right)$
(D) $\log_t \left(\frac{r^9}{s^{30}} \right)$

5. Which of the following graphs represents the function $f(x) = 3 \ln(x + 2)$?



6. $7x^3 + 9x^2 - 19x + 10$ divided by $7x + 2$ is equal to

- (A) $(x^2 + x - 3) R 16$
 (B) $(x^2 - x + 3) R 16$
 (C) $(x^2 - x - 3) R 4$
 (D) $(x^2 + x + 3) R 4$

7. The n th term, T_n , in the arithmetic progression 5, 11, 17, 23, 29 is

- (A) $n - 6$
 (B) $6n - 1$
 (C) $5n + 6$
 (D) $6n + 11$

8. The coefficient of the x^2 term in the binomial expansion $(3x - 2)^3$ is

- (A) -54
 (B) -36
 (C) 36
 (D) 54

9. What is the sum to infinity of

$$1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \dots ?$$

- (A) 0
 (B) $\frac{3}{4}$
 (C) $\frac{20}{27}$
 (D) $\frac{3}{2}$

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10. 240° expressed in radians is

- (A) $\frac{3}{4} \pi$
- (B) $\frac{4}{3} \pi$
- (C) $\frac{2}{3} \pi$
- (D) $\frac{3}{2} \pi$

11. The solutions for x , $0 \leq x \leq 2\pi$ to the trigonometric function $-8 \cos (x + \pi) = 4$ are

- (A) $\frac{11}{12} \pi$ and $\frac{19}{12} \pi$
- (B) $\frac{1}{12} \pi$ and $\frac{17}{12} \pi$
- (C) $\frac{1}{3} \pi$ and $\frac{5}{3} \pi$
- (D) $\frac{7}{12} \pi$ and $\frac{11}{12} \pi$

13. Which of the following are the complex solutions to the equation $2x^2 + 3x + 4 = 0$?

- (A) $\frac{-3 \pm i\sqrt{19}}{4}$
- (B) $\frac{-3 \pm i\sqrt{41}}{4}$
- (C) $\frac{-3 \pm \sqrt{23}}{4}$
- (D) $\frac{-3 \pm i\sqrt{23}}{4}$

14. The conjugate of the complex number $z = -2 + 5i$ is

- (A) $2 - 5i$
- (B) $2 + 5i$
- (C) $-2 - 5i$
- (D) $\sqrt{29}$

Item 12 refers to the following matrices.

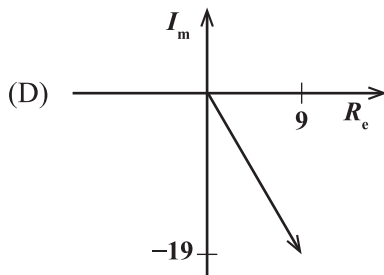
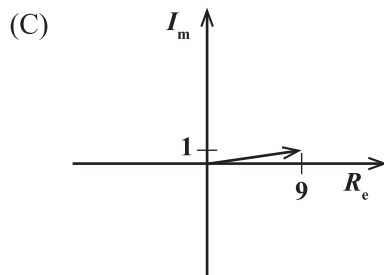
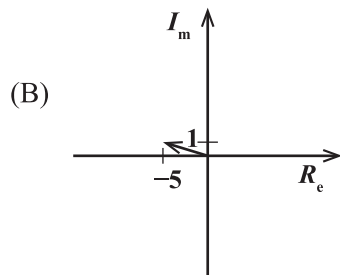
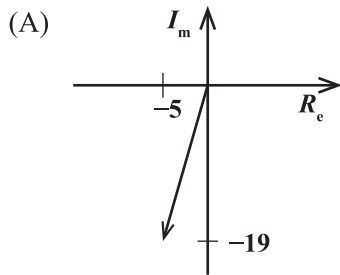
$$\begin{pmatrix} 72 & 24 & 120 \end{pmatrix} \begin{pmatrix} 25 & 48 & 75 \\ 50 & 26 & 125 \\ 27 & 28 & 144 \end{pmatrix}$$

12. When the following matrices are multiplied, the order of the resulting matrix is

- (A) 1×1
- (B) 1×3
- (C) 3×1
- (D) 3×3

GO ON TO THE NEXT PAGE

15. Given that $w = 2 - 9i$ and $v = -7 - 10i$, then the Argand diagram for $w + v$ is



16. Rebecca conducts a survey on the last general elections. She produces a questionnaire and distributes it to 50 males and 50 females. This type of survey is BEST described as

- (A) stratified random
(B) simple random
(C) cluster
(D) quota

17. The height of a toddler is measured and recorded every quarter (every three months) for four years from his birth to his fourth birthday. Which of the following techniques is BEST used to represent the heights over the four-year period?

- (A) A bar graph
(B) A histogram
(C) A line graph
(D) A stem-and-leaf plot

Item 18 refers to the results of a test which are summarized in the following distribution table.

Mark	1–3	4–6	7–9	10–12	13–15
Frequency	25	40	20	10	5

18. The mean of the data is

- (A) 5.9
- (B) 4.9
- (C) 3.7
- (D) 1.6

Item 19 refers to the following information which gives the number, in ascending order, of shoes sold in a store for two weeks.

35 36 36 39 42 45 47 48 48 52 56 57 76 83

19. The lower quartile is

- (A) 37.5
- (B) 39.0
- (C) 47.5
- (D) 56.0

20. How many 3-digit numbers can be generated from the digits 1, 2, 3, 6 and 7, if repetition is not allowed?

- (A) 5!
- (B) 3!
- (C) ${}^5C_3$
- (D) ${}^5P_3$

Item 21 refers to the information in the following table, which shows the national test summary scores of students at two schools: Gibbons and Rawlins.

	Schools	
	Gibbons	Rawlins
Mean	68	43
Median	61	45
Range	35	28
Standard deviation	9	6
Number of Students	97	104

21. The shape of the distribution curve for the national test scores at Gibbons would be BEST described as

- (A) negatively skewed
- (B) positively skewed
- (C) non symmetrical
- (D) normal

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22. Which of the following are characteristics of a binomial distribution for modelling a given random experiment?

- I. The trials are dependent.
- II. The number of trials is finite.
- III. There are only two possible outcomes.

- (A) I and II only
- (B) I and III only
- (C) II and III only
- (D) I, II and III

Items **23** and **24** refer to the table below which shows the number and brand of trucks from three cities.

	Make of Car			
City	Toyo	Nasca	Other	Total
A	30	50	20	100
B	100	60	90	250
C	40	80	30	150
Total	170	190	140	500

23. What is the probability that a car selected at random from city B is a Nasca?

- (A) 0.60
- (B) 0.32
- (C) 0.24
- (D) 0.12

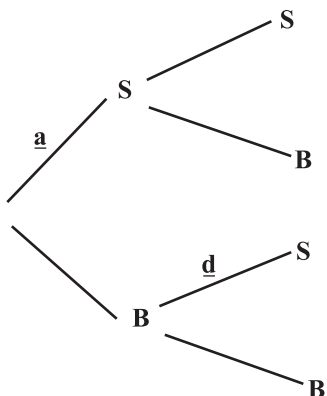
24. A car is randomly selected from among the three cities. What is the probability that it is from city C but is neither a Toyo nor a Nasca?

- (A) 0.056
- (B) 0.060
- (C) 0.200
- (D) 0.280

GO ON TO THE NEXT PAGE

Item **25** refers to the following information.

A box contains 10 items of which 6 are springs and the remaining are bells. Two items are selected without replacement. **S** represents the event that a spring is selected, and **B** represents the event that a bell is selected.



26. The normal distribution possesses the following features.

- I. The curve is symmetric about the mean value.
- II. Total area under the curve is 1.
- III. Mean = median = mode.

- (A) I and II only
- (B) I and III only
- (C) II and III only
- (D) I, II and III

25. The probability that the branch labelled **d** will occur is

- (A) $\frac{2}{3}$
- (B) $\frac{1}{3}$
- (C) $\frac{5}{9}$
- (D) $\frac{4}{9}$

27. If $X \sim \text{Bin}(10, 0.2)$, then $P(X > 2)$ is

- (A) $\binom{10}{0} (0.2)^0 (0.8)^{10} + \binom{10}{1} (0.2)^1 (0.8)^9$
- (B) $1 - \left[\binom{10}{0} (0.2)^0 (0.8)^{10} + \binom{10}{1} (0.2)^1 (0.8)^9 \right]$
- (C) $\binom{10}{0} (0.2)^0 (0.8)^{10} + \binom{10}{1} (0.2)^1 (0.8)^9 + \binom{10}{2} (0.2)^2 (0.8)^8$
- (D) $1 - \left[\binom{10}{0} (0.2)^0 (0.8)^{10} + \binom{10}{1} (0.2)^1 (0.8)^9 + \binom{10}{2} (0.2)^2 (0.8)^8 \right]$

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Items **28** and **29** refer to the following information.

The random variable **X** has the following discrete distribution table.

X	1	2	3	4	5
P(X = x)	0.30	0.20	0.10	y	0.25

28. The value of $P(X = 4)$ is

- (A) 0.25
- (B) 0.20
- (C) 0.15
- (D) 0.10

29. The expected value, $E(X)$, is

- (A) 1.00
- (B) 1.25
- (C) 1.50
- (D) 2.85

30. A correlation coefficient of $r = 0.17$ means

- (A) an inverse relationship
- (B) a weak positive relationship
- (C) a strong positive relationship
- (D) a moderate positive relationship

31. The value of $\lim_{x \rightarrow -3} \frac{x^2 - 9}{5}$ is

- (A) ∞
- (B) $\frac{18}{5}$
- (C) 0
- (D) $-\frac{18}{5}$

Items **32** and **33** refer to the piecewise function $f(x) = \begin{cases} 4x - 1 & : x < -3 \\ 3 + x & : x \geq -3 \end{cases}$

32. Determine $\lim_{x \rightarrow -3^+} f(x)$

- (A) -13
- (B) 0
- (C) 6
- (D) 11

33. For what value of x is $f(x)$ discontinuous?

- (A) 0
- (B) $\frac{1}{4}$
- (C) 1
- (D) 3

GO ON TO THE NEXT PAGE

34. Which of the following rules can be used to differentiate $f(x) = \frac{3x+1}{2-x^2}$?
- (A) Chain rule
(B) Power rule
(C) Quotient rule
(D) Logarithm rule
35. If $\frac{dy}{dx} = x^2 - 4x - 5$, then the stationary points are
- (A) 1 and -5
(B) -1 and 5
(C) -1 and -5
(D) 1 and 5
36. If $y = 6 \cos x - x^2$, then $y' =$
- (A) $-6 \cos x + 2x$
(B) $6 \cos x + 2x$
(C) $6 \sin x - 2x$
(D) $-6 \sin x + 2x$
37. If $y = -8x^4 + 5x^3 - 5$, then $\frac{d^2y}{dx^2} =$
- (A) -192
(B) $-192x + 30$
(C) $-32x^3 + 15x^2$
(D) $-96x^2 + 30x$
38. The stationary points of the function $g(x) = x^3 - 27x$ occurs at the values $x = -3$ and $x = 3$. What is the maximum value of the function?
- (A) -18
(B) 18
(C) 54
(D) 108
39. Which of the following operations is used to determine the gradient of $f(x)$ at the point (a, b) ?
- (A) $f'(a)$
(B) $f'(b)f'(b)$ or $f''(b)$
(C) $f(a)$
(D) $\int_a^b f(x) dx$
40. Given that $y = e^x \ln x$, the derivative of y is given as
- (A) $e^x \left(\frac{1}{x} + \ln x \right)$
(B) $e^x \left(\frac{1}{x} \right)$
(C) $e^x \ln x$
(D) $e^x (1 + \ln x)$
41. If 'C' is the constant of integration, then $\int (-9x^2 + 6x + 9) dx =$
- (A) $-9x^3 + 6x^2 + 9x + C$
(B) $-3x^3 + 3x^2 + 9x + C$
(C) $-18x + 6 + C$
(D) $-2x + 6 + C$

GO ON TO THE NEXT PAGE

42. Which of the following statements about integration is NOT true?

- (A) If $\frac{d}{dx}F(x) = f(x)$, then $\int f(x) dx = F(x) + c$.
- (B) The integral is the antiderivative.
- (C) Integration is differentiation in reverse.
- (D) Integration is the sum of 1-unit squared rectangles.

43. Which of the following derivatives would help you to find $\int -9x^2 dx$?

- (A) $\frac{d}{dx}(x^n) = nx^{n-1}$
- (B) $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$
- (C) $\frac{d}{dx}(e^x) = e^x$
- (D) $\frac{d}{dx}(\int f(x) dx) = f(x)$

44. $\int_1^{10} (-4x^2 + 2x^3) dx =$

- (A) 7.5
- (B) 1.83
- (C) 3667.5
- (D) 6331.5

45. The area between the curve $y = -6x^2 + 4x + 7$ and the x -axis between $x = 1$ and $x = 0$ is

- (A) 7
- (B) 5
- (C) -3
- (D) -8

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SUBJECT INTEGRATED MATHEMATICS – Paper 01

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SUBJECT INTEGRATED MATHEMATICS – Paper 02

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NAME OF SCHOOL/CENTRE

CANDIDATE'S FULL NAME (FIRST, MIDDLE, LAST)

DATE OF BIRTH

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I N T E G R A T E D M A T H E M A T I C S

Paper 02

2 hours 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of THREE sections. Each section consists of TWO structured questions.
2. Answer ALL questions.
3. Write your answers in the spaces provided in this booklet.
4. Do NOT write in the margins.
5. Unless otherwise stated in a question, all numerical answers MUST be given exactly OR to three significant figures as appropriate.
6. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra lined page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
7. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials Permitted

Mathematical formulae and tables (provided) – **Revised 2012**

Mathematical instruments

Silent, non-programmable, electronic calculator

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SECTION A

MODULE 1: FOUNDATIONS OF MATHEMATICS

Answer BOTH questions.

1. (a) Find the range of values of x for which $2x^2 + x - 15 \geq 0$.

[3 marks]

- (b) Solve the following equations for x .

$$3^{x-1} \times 27^{3x-1} = 81^{2x}$$

[3 marks]

GO ON TO THE NEXT PAGE

- (c) Expand the expression for $(2 - 3x)^3$ completely.

[3 marks]

- (d) Solve the equation $\cos\left(\theta + \frac{3\pi}{4}\right) = 0.5$, for the values of θ where $-\pi \leq \theta \leq \pi$.

[3 marks]

GO ON TO THE NEXT PAGE

- (e) Given that x and y are real numbers, calculate the value of x if

$$(x + y) + (x - y)i = 7 + 3i.$$

[3 marks]

Total 15 marks

GO ON TO THE NEXT PAGE

2. (a) Solve the following equation for x .

$$\log (x + 3) = 1$$

[2 marks]

- (b) Given that $g(x) = 2x^3 + 7x^2 + x - 10$, factorize $g(x)$ completely.

[4 marks]

GO ON TO THE NEXT PAGE

- (c) The first and fourth terms of the geometric series are 80 and 10 respectively. Find the sum to infinity.

[3 marks]

- (d) Kerry bought 1 pen, 2 erasers and 2 pencils for \$15.00. One month later, she bought 1 pen, 2 erasers and 3 pencils for \$17.00. In preparation for her final project, she purchased 3 pens, 1 eraser and 3 pencils for \$24.00. Using Cramer's rule, calculate the unit cost of the pencil. Assume the cost of each item was unchanged for each period.

[6 marks]

Total 15 marks

GO ON TO THE NEXT PAGE

SECTION B

MODULE 2: STATISTICS

Answer BOTH questions.

3. The nutrition teacher at a two-year college for adults complained to the principal that the eating habits of the Year 1 students were unhealthy, and as a result, the college should introduce a mandatory nutrition course for the next academic year. One hundred students enroll at the college each year. The principal collected data, as shown in the table below, on the masses in kilograms of 15 students in Year 1 and 15 students in Year 2.

Year 1	75.0	70.5	77.3	75.0	72.7	70.5	72.7	59.1	72.7	72.7	77.3	72.7	70.5	77.3	81.8
Year 2	75.0	65.9	68.2	70.5	59.1	75.0	68.2	65.9	68.2	63.6	70.5	68.2	68.2	90.9	75.0

- (a) Describe an appropriate process for selecting the sample of 30 students from the college.

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[2 marks]

- (b) Identify TWO suitable methods for graphically representing the data collected on the masses of students and give a reason for ONE of your selections.

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[3 marks]

GO ON TO THE NEXT PAGE

- (c) Calculate the standard deviation mass of the students in Year 1.

[4 marks]

- (d) Given that the mean and standard deviation masses of the students in Year 2 are 70.2 kilograms and 6.94 kilograms respectively, determine which year group has the more homogenous (less variable) weights. Give a reason for your answer.

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[2 marks]

GO ON TO THE NEXT PAGE

- (e) The principal plans to interview students from the sample of 30.
- (i) What is the probability of randomly selecting a student with a mass of at least 77.3 kilograms?

[2 marks]

- (ii) How many groups of 5 students can be selected for interview from among the 30 students if no restrictions are imposed?

[2 marks]

Total 15 marks

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4. (a) The probability of the random variable X is shown in the table below.

X	1	2	3	4	5
$P(X = x)$	0.1	0.1	0.3	y	0.2

Calculate

- (i) the value of y

[2 marks]

- (ii) the expected value $E(x)$.

[2 marks]

GO ON TO THE NEXT PAGE

- (b) The onion farmers in a certain community realized that being able to predict their yield will assist them in making better decisions on the best prices to get for their crops. They gathered data on the yield of onions (in tons) and rainfall (in millimetres), during the onion season, over a period of 10 years. Figure 1 shows the scatter plot diagram of the data collected over a period of 10 years.

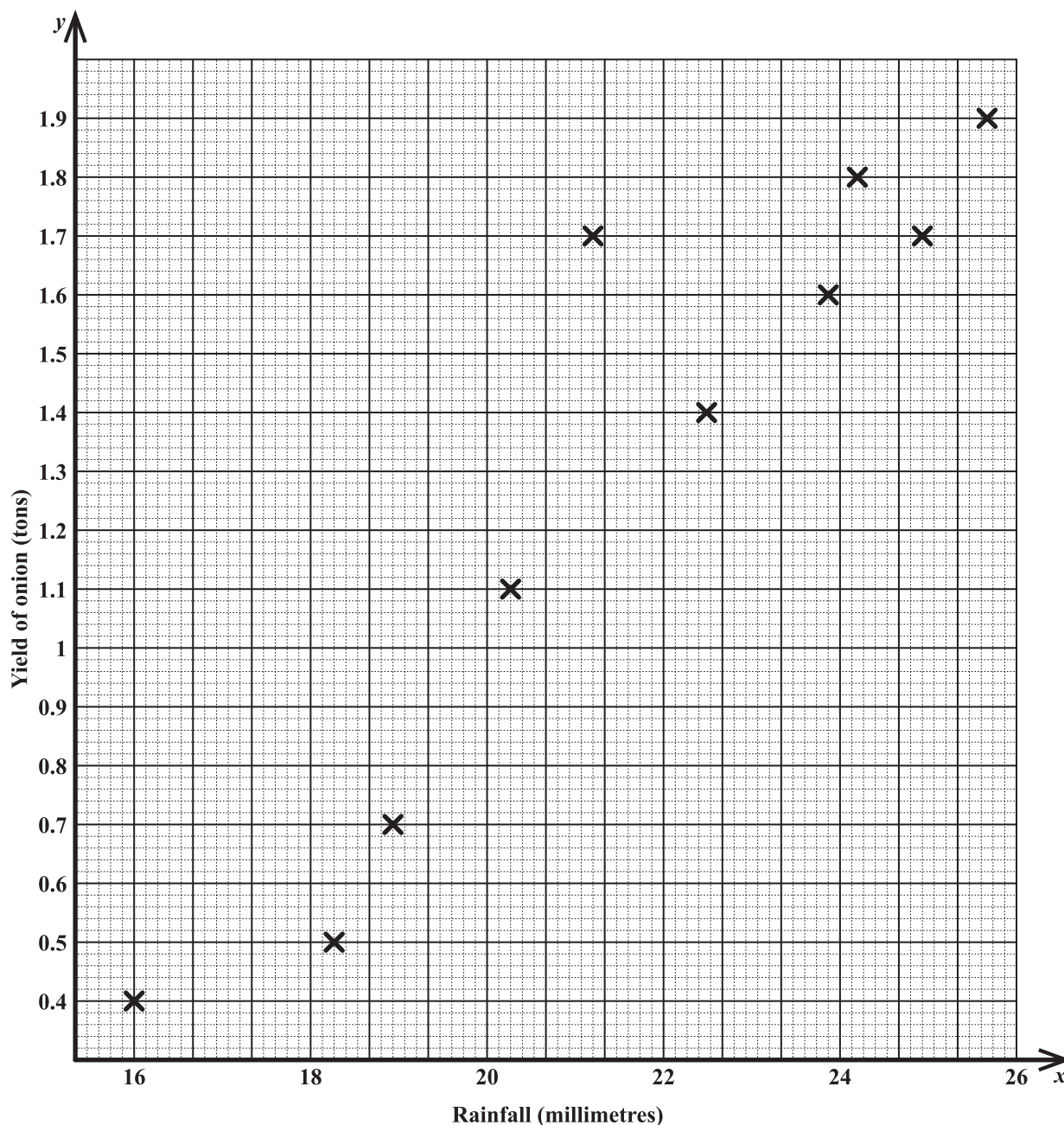


Figure 1. Scatter plot diagram of yield of onions (in tons) against rainfall (in millimetres)

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- (i) Given that the mean rainfall, $\bar{x}$, is 21.6 and the mean yield of onions, $\bar{y}$, is 1.28, draw the regression line on Figure 1.

[2 marks]

- (ii) Describe the correlation between the rainfall and yield of onions.

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[2 marks]

- (iii) Using your regression line in (b) (i), state an estimate for the yield of onions (in tons), when rainfall is 20 millimetres.

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.....

[1 mark]

- (iv) Given that the estimated regression equation is $\hat{y} = 1.65x - 2.29$, interpret the value of the slope.

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[2 marks]

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- (c) The meteorological office reported that during the onion season, rainfall in the community was normally distributed with a mean of 22 millimetres and a variance of 16 squared millimetres. Calculate the probability that during the onion season there will be between 19 mm to 24 mm of rainfall.

[5 marks]

Total 15 marks

GO ON TO THE NEXT PAGE

SECTION C

MODULE 3: CALCULUS

5. (a) Evaluate $\lim_{x \rightarrow 2} \frac{x-2}{x^2-x-2}$.

[3 marks]

- (b) Differentiate the following with respect to x .

$$y = \frac{e^{3x}}{5x + \sin x}$$

[4 marks]

GO ON TO THE NEXT PAGE

- (c) Find and classify the critical points of the function $g(x) = x^3 - 6x^2 + 9x$.

[6 marks]

- (d) The capacity of a cylinder tank, V , is given by $V_{(r,h)} = \pi r^2 h$.

Calculate the partial derivative $\frac{\partial V}{\partial r}$, the rate of change of V with respect to r .

[2 marks]

Total 15 marks

GO ON TO THE NEXT PAGE

6. (a) Determine $\int (2x - 4)^7 dx$.

[3 marks]

- (b) Evaluate $\int_{\frac{\pi}{2}}^{\pi} (12x^3 - 2\cos x) dx$.

[3 marks]

GO ON TO THE NEXT PAGE

- (c) The rate of increase of the population, P , of a parish is proportional to the population size (where t is the number of years after 2005).

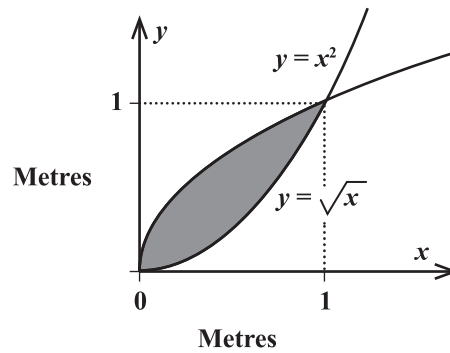
If in 2005 the population was 1400 and in 2015 it was 2100, show that

$$P(t) = 1400e^{\frac{1}{10}\ln\frac{3}{2}t}.$$

[6 marks]

GO ON TO THE NEXT PAGE

- (d) A design consists of a white square and a black shape, as shown in the diagram below.



Determine the area of the black/shaded interior.

[3 marks]

Total 15 marks

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SUBJECT INTEGRATED MATHEMATICS – Paper 02

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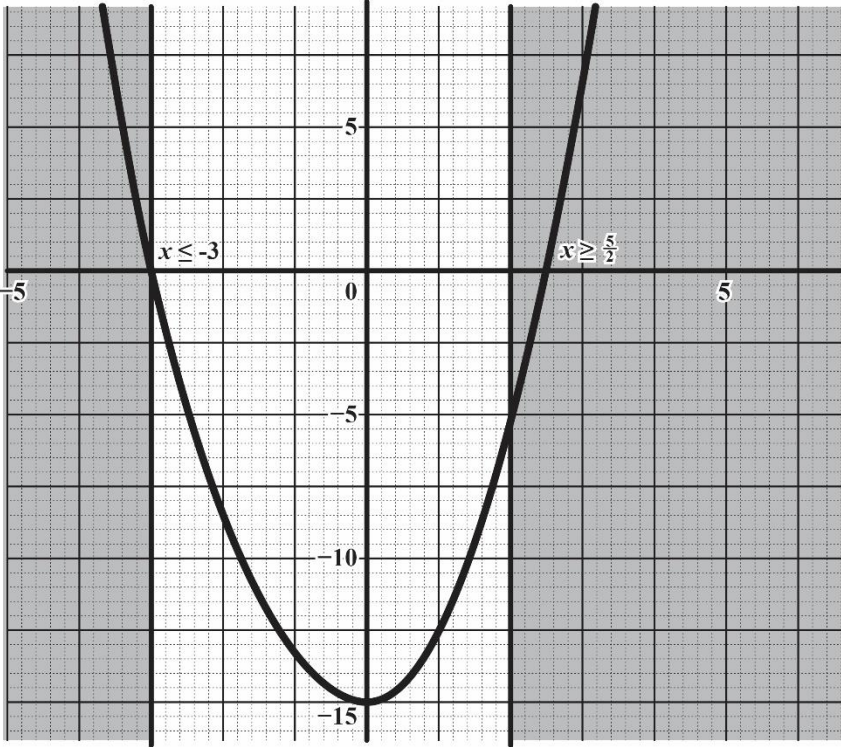
PAPER 02

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2022

INTEGRATED MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question			Syllabus Reference	Solution	Marking Scheme	CK	AK	R	Total
1	(a)		1.1.4	Find the range of values of x for which $2x^2 + x - 15 \geq 0$ $\Rightarrow (2x - 5)(x + 3) \geq 0$ $2x - 5 \geq 0 \quad \text{and} \quad x + 3 \geq 0$ $x \geq \frac{5}{2} \quad \text{and} \quad x \geq -3$ The solution is $x \leq -3$ or $x \geq \frac{5}{2}$ 	Factorization 1 mark for each correct answer	1	2		3

INTEGRATED MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question			Syllabus Reference	Solution	Marking Scheme	CK	AK	R	Total
1 cont'd									
	(b)		1.2.2	$3^{x-1} \times 27^{3x-1} = 81^{2x}$ $3^{x-1} \times 3^{3(3x-1)} = 3^{4(2x)}$ $3^{x-1} \times 3^{9x-3} = 3^{8x}$ $3^{x-1+9x-3} = 3^{8x}$ $3^{10x-4} = 3^{8x}$ $10x - 4 = 8x$ $2x = 4$ $x = 2$	Correct use of the power law Correct use of the sum law Equating the bases and solving powers correctly	1	1	1	3
	(c)		1.4.1	$(2 - 3x)^3 = \binom{3}{0} 2^3(-3x)^0 + \binom{3}{1} 2^2(-3x)^1 + \binom{3}{2} 2^1(-3x)^2 + \binom{3}{3} 2^0(-3x)^3$ $(2 - 3x)^3 = (1) (8) (1) + (3) (4) (-3x) + (3) (2) (9x^2) + (-27x^3)$ $(2 - 3x)^3 = 8 - 36x + 54x^2 - 27x^3$ OR $(2 - 3x)^3 = 2^3 + (3) (2^2) (-3x) + (3) (2) [(-3x)^2] + (-3x)^3$ $(2 - 3x)^3 = 8 - 36x + 54x^2 - 27x^3$	Expanding correctly Simplifying Correct answer	1	1		3

INTEGRATED MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question			Syllabus Reference	Solution	Marking Scheme	CK	AK	R	Total
1 cont'd									
	(d)		1.5.2	If $\cos\left(\theta + \frac{3\pi}{4}\right) = 0.5$, find θ in the range $-\pi \leq \theta \leq \pi$. $\cos\left(\theta + \frac{3\pi}{4}\right) = 0.5$ $\theta + \frac{3\pi}{4} = \cos^{-1}(0.5)$ $\theta + \frac{3\pi}{4} = \frac{\pi}{3}, \frac{5\pi}{3}$ $\theta = \frac{\pi}{3} - \frac{3\pi}{4}, \frac{5\pi}{3} - \frac{3\pi}{4} = \frac{3\pi}{4}$ $\theta = -\frac{5\pi}{12}, \frac{11\pi}{12}$	Calculating the inverse 1 mark for obtaining each correct angle	1			3
	(e)		1.7.3	$(x + y) + (x - y) i = 7 + 3i$ Equating real parts $x + y = 7$ Equation 1 Equating imaginary parts $x - y = 3$ Equation 2 Equation 1 + Equation 2 gives $x + x = 7 + 3$ $2x = 10$ $x = 5$	Equating real part Equating imaginary part Solving correctly	1		1	
						6	7	2	15

INTEGRATED MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question			Syllabus Reference	Solution	Marking Scheme	CK	AK	R	Total
2	(a)		1.2.1 1.2.4	$\log(x + 3) = 1$ $x + 3 = 10^1$ $x = 7$	Converting to exponential form Correct value of x		1	1	2
	(b)		1.3.1 1.3.2	$g(x) = 2x^3 + 7x^2 + x - 10$ $g(1) = 2(1)^3 + 7(1)^2 + (1) - 10 = 0$ $\therefore g(1) = 0$ $\Rightarrow x - 1$ is a factor $ \begin{array}{r} 2x^2 + 9x + 10 \\ x - 1 \overline{) 2x^3 + 7x^2 + x - 10} \\ \underline{2x^3 - 2x^2 - x - 10} \\ 9x^2 + 2x - 10 \\ \underline{9x^2 - 9x - 10} \\ 11x - 10 \\ \underline{11x - 11} \\ 1 \end{array} $ $\Rightarrow 2x^3 + 7x^2 + x - 10 \equiv (x - 1)(2x^2 + 9x + 10) \equiv (x - 1)(x + 2)(2x + 5)$ Alternative solution $2x^3 + 7x^2 + x - 10 \equiv (x - 1)(ax^2 + bx + c)$ Equating coefficients of x^2 : $a = 2$ Equating coefficients of constant: $-10 = -c$ $c = 10$ Equating coefficients of x : $1 = -b + c$ $b = 9$ $2x^3 + 7x^2 + x - 10 \equiv (x - 1)(2x^2 + 9x + 10)$ $= (x - 1)(x + 2)(2x + 5)$	Obtaining any ONE of the factors Long division Obtaining the other factor (quadratic) Factorizing completely Alternate Obtaining a factor 2 marks for getting all 3 coefficients of the quadratic (1 mark for getting at least 1 coefficient correct) 1 mark for factorizing completely	1 1	1	1	4

INTEGRATED MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question			Syllabus Reference	Solution	Marking Scheme	CK	AK	R	Total
2 cont'd									
	(c)		1.4.3 1.4.5	For G.P with a = 80 $ar^3 = 10$, then $r^3 = \frac{10}{80}$ $r = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$ Then $S_{\infty} = \frac{a}{1 - r}$ $= \frac{80}{1 - \frac{1}{2}}$ $= 160$	Correctly identifying the equation of the 4th term Solving for r Calculating S_{∞}	1		1	3

INTEGRATED MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

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PAPER 02
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Question		Syllabus Reference	Solution	Marking Scheme	CK	AK	R	Total
2 cont'd								
	(d)		Determinant of A_z $ A_z = \begin{vmatrix} 1 & 2 & 15 \\ 1 & 2 & 17 \\ 3 & 1 & 24 \end{vmatrix}$ $= [1(48 - 17) - 2(24 - 51) + 15(1 - 6)]$ $= 1(31) - 2(-27) + 15(-5)$ $= 85 - 75 = 10$ $z = \frac{ A_z }{ A } = \frac{10}{5} = 2$ Unit cost for pencils = \$2	Correct substitution in A_z formula Obtaining determinant of A_z Correct substitution in Cramer's Rule to obtain correct answer	1			
					6	7	2	15

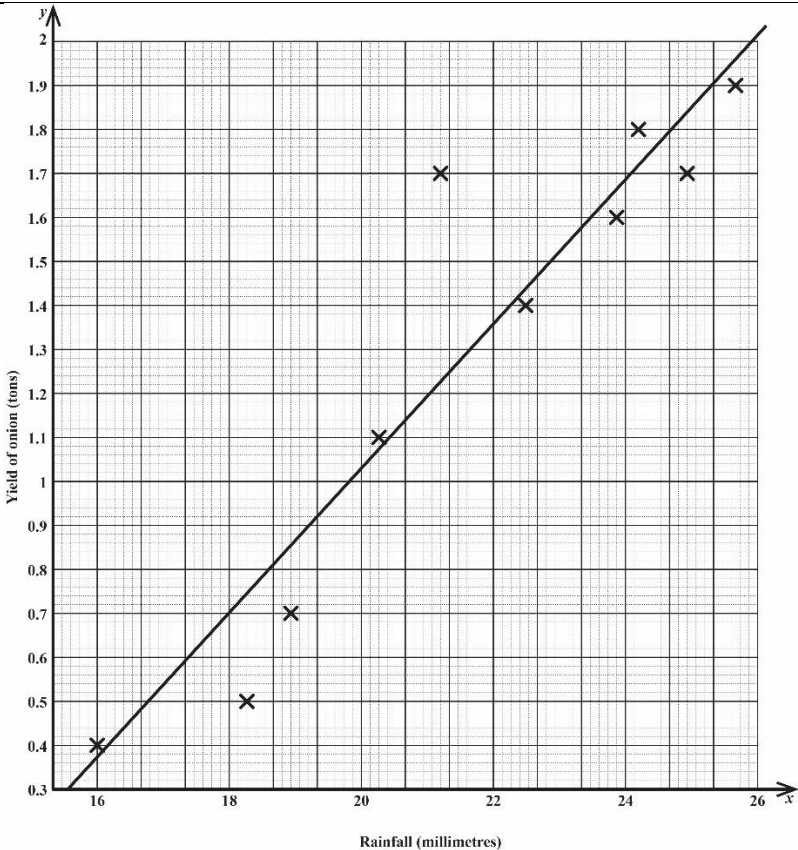
INTEGRATED MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question			Syllabus Reference	Solution	Marking Scheme	CK	AK	R	Total
3	(a)		2.1.3	Randomly select students. Process 1: Place student numbers in sequential order and number them from 1 to 200. Using a random number generator, provide 30 numbers between 1 and 200 inclusive, 15 for each year group. Process 2: Use stratified random sampling for each year and again for each program of each year.	Identifying a suitable sampling technique Describing the sampling technique	1			2
	(b)		2.2.2	A histogram/stem-and-leaf/box-and-whisker Both groups can be represented and easily compared.	Identifying any TWO suitable graphs Rationale for choice of ONE graph	2		1	3
	(c)		2.3.3	$= \frac{\sum x}{n} = \frac{1097.8}{15} = 73.19$ $\sigma = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} = \sqrt{\frac{80697.12}{15} - 5356.288}$ $= \sqrt{23.5198} = 4.85$	Calculating the mean Correct calculation of $\sum x^2$ Correct substitution into standard deviation formula Correct answer		1		4

INTEGRATED MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question			Syllabus Reference	Solution	Marking Scheme	CK	AK	R	Total
3 cont'd									
	(d)		2.3.5	Year 1 masses are more homogenous. Reason: Standard deviation is less than that of Year 2 masses. OR Standard deviation is a measure of spread Large standard deviation indicates non-homogeneity	Identifying the year Reason	1		1	2
	(e)	(i)	2.3.5	$P(\text{year 1} + \text{year 2} \geq 77.3) = \frac{\text{num 1} + \text{num 2}}{\text{total 1} + \text{total 2}}$ $= \frac{4 + 1}{15 + 15}$ $= \frac{5}{30} = \frac{1}{6}$	Correct numerator Correct denominator		1	1	2
		(ii)	2.4.3	Amount of groups = C_5^{30} $= \frac{30!}{5!25!}$ = 142506	Recognizing that a combination is being used Correct answer	1		1	2
						6	7	2	15

INTEGRATED MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question			Syllabus Reference	Solution	Marking Scheme	CK	AK	R	Total
4	(a)	(i)	2.5.5	$y = 1 - (0.1 + 0.1 + 0.3 + 0.2)$ $= 0.3$	Sum of the probability is 1 Correct answer	1	1		2
		(ii)	2.5.5	$E(X) = 1(0.1) + 2(0.1) + 3(0.3) + 4(0.3) + 5(0.2)$ $E(X) = 0.1 + 0.2 + 0.9 + 1.2 + 1.0$ $E(X)$	Calculating $E(X)$ Correct answer		1 1		2
	(b)	(i)	2.6.3		Line passing through $(\bar{x}, \bar{y})$ Appropriate best fit line	1	1 1 1		2

INTEGRATED MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question			Syllabus Reference	Solution	Marking Scheme	CK	AK	R	Total
4 cont'd									
	(b)	(ii)	2.6.4	Strong positive correlation	Stating that the correlation is positive Stating that the correlation is strong	1 1			2
		(iii)	2.6.4	$y = 1$	Accurate response based on regression line drawn		1		1
		(iv)	2.6.4	The slope, 1.6, indicates that when rainfall increases by 1 millimeter, the onion yield increases by 1.65 tons.	Appropriate interpretation			1	1
	(c)			$P(19 < X < 24) = P\left(\frac{19 - 22}{4} < Z < \frac{24 - 22}{4}\right)$ $P(-0.75 < Z < 0.5) = P(Z < -0.75) - P(Z < 0.5)$ $= [P(Z < 0.75) + P(Z < 0.5)] - 1$ $= (0.7734 + 0.6915) - 1$ $= 0.4649$	Identification of the interval Calculating that the standard deviation is 4 Standardization Reading values from the normal distribution table Correct answer	1	1 1 1	1	5
						6	7	2	15

INTEGRATED MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question			Syllabus Reference	Solution	Marking Scheme	CK	AK	R	Total
5	(a)		3.1.2 3.1.4	$\lim_{x \rightarrow 2} \frac{x-2}{x^2-x-2} = \frac{2-2}{4-2-2} = \frac{0}{0}, \text{ Indeterminate!!}$ $\lim_{x \rightarrow 2} \frac{x-2}{x^2-x-2} = \lim_{x \rightarrow 2} \frac{x-2}{(x-2)(x+1)}$ $\lim_{x \rightarrow 2} \frac{1}{x+1}$ $\frac{1}{2+1} = \frac{1}{3}$	Showing indeterminate form Factorizing and simplifying function Correct answer	1		1	3
	(b)		3.2.3 3.2.4 3.2.5 3.2.6	$y = \frac{e^{3x}}{5x + \sin x} = \frac{u}{v}$ $u' = 3e^{3x}$ $v' = 5 + \cos x$ Using the quotient rule $y' = \frac{3e^{3x}(5x + \sin x) - e^{3x}(5 + \cos x)}{(5x + \sin x)^2}$	Differentiating u correctly Differentiating v correctly Applying appropriate formula Correct answer	1 1 1		1	4

INTEGRATED MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question			Syllabus Reference	Solution	Marking Scheme	CK	AK	R	Total
5 cont'd									
	(c)		3.2.8 3.2.9 3.2.10	$g(x) = x^3 - 6x^2 + 9x$ $g'(x) = 3x^2 - 12x + 9$ $g''(x) = 6x - 12$ For stationary points $g'(x) = 3x^2 - 12x + 9 = 0$ $\Rightarrow x^2 - 4x + 3 = 0$ $(x - 3)(x - 1) = 0$ $x = 3, y = 0$ $x = 1, y = 4$ For nature of stationary points $g''(x) = 6x - 12$ $g''(3) > 0 \quad (3, 0) \text{ is a minimum point}$ $g''(1) < 0 \quad (1, 4) \text{ is a maximum}$	Finding $g'(x)$ Finding $g''(x)$ Solving $g'(x) = 0$ Determining critical points Determining the nature of first point Determining the nature of second point	1	1 1 1	1	6
	(d)		2.13	$V(r, h) = \pi r^2 h$ Treating h as a constant, $\frac{\partial V}{\partial r} = h \left(\frac{d}{dr} \pi r^2 \right) = 2\pi rh$	Treating h as a constant Differentiating correctly	1	1		2
						6	7	2	15

Question			Syllabus Reference	Solution	Marking Scheme	CK	AK	R	Total
6	(a)		3.4	Let $u = 2x - 4$ $du = 2dx$ $\int (2x - 4)^7 dx = \frac{1}{2} \int u^4 du$ $\frac{1}{2} \left(\frac{u^5}{5} \right) = c$ $= \frac{(2x - 4)^5}{8 \times 2} + c$ $= \frac{(2x - 4)^5}{16} + c$	Correct substitution for $2x - 4$ Integrating correctly	1	1		3
				Simplifying and including constant in solution	1				
	(b)		3.2 3.3 3.5	$2 \cos x) \, dx = [3x^4 - 2 \sin x]_{\frac{\pi}{2}}$ $= (3\pi^4 - 2 \sin \pi) - \left(3 \left(\frac{\pi}{2} \right)^4 - 2 \sin \left(\frac{\pi}{2} \right) \right)$ $= 292.2273 - 16.2642$ $= 275.9631$	Integrating both terms correctly Correctly substituting limits Correct answer	1	1 1		3

INTEGRATED MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question			Syllabus Reference	Solution	Marking Scheme	CK	AK	R	Total
6 cont'd									
	(c)		3.3.3 3.3.7 3.3.8	$\frac{dP}{dt} = kP$ $\int \frac{dP}{dt} = \int kdt$ $\ln P = kt + C$ $P(t) = Ae^{kt}$ Using $P(0) = 1400$ $Ae^0 = 1400$ $A = 1400$ $P(t) = 1400e^{kt}$ Using $P(10) = 2100$ $1400e^{10k} = 2100$ $e^{10k} = \frac{2100}{1400}$ $10k = \ln \frac{3}{2}$ $k = \frac{1}{10} \ln \frac{3}{2}$ Hence $P(t) = 1400e^{\frac{1}{10} \ln \frac{3}{2} t}$	Separating the variables Integrating correctly Solving for P(t) Solving for the constant using the initial condition given Taking ln on both sides Solving for k	1 			

INTEGRATED MATHEMATICS
PAPER 02
KEY AND MARK SCHEME

Question			Syllabus Reference	Solution	Marking Scheme	CK	AK	R	Total
6 cont'd									
	(d)		3.3.6	$\text{Area} = \int_0^1 [f(x) - g(x)] \, dx$ $\int_0^1 (\sqrt{x} - x^2) \, dx = \left[\frac{2}{3} x^{\frac{3}{2}} - \frac{1}{3} x^3 \right]_0^1$ $= \left(\frac{2}{3} - \frac{1}{3} \right) = \frac{1}{3}$	Subtracting the integral of g(x) from f(x) Integrating correctly Correct answer	1	1	1	2
						6	7	2	15

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FILL IN ALL THE INFORMATION REQUESTED CLEARLY IN CAPITAL LETTERS.

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SUBJECT: INTEGRATED MATHEMATICS – Paper 032

PROFICIENCY: ADVANCED

REGISTRATION NUMBER

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SCHOOL/CENTRE NUMBER					

NAME OF SCHOOL/CENTRE

CANDIDATE'S FULL NAME (FIRST, MIDDLE, LAST)

DATE OF BIRTH

D	D	M	M	Y	Y	Y	Y
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SPECIMEN 2022



TEST CODE **02167032**

C A R I B B E A N E X A M I N A T I O N S C O U N C I L

C A R I B B E A N A D V A N C E D P R O F I C I E N C Y E X A M I N A T I O N[®]

I N T E G R A T E D M A T H E M A T I C S

Paper 032

1 hour 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of a case study and SIX questions, some of which are structured.
2. Read the case study and answer ALL questions.
3. Write your answers in the spaces provided in this booklet.
4. Do NOT write in the margins.
5. Unless otherwise stated in the question, all numerical answers MUST be given exactly OR to three significant figures as appropriate.
6. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra lined page(s) provided at the back of this booklet. **Remember to draw a line through your original answer.**
7. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials Permitted

Mathematical formulae and tables (provided) – **Revised 2012**

Mathematical instruments

Silent, non-programmable, electronic calculator

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Instruction: Read the following case study and answer the questions that follow.

CASE STUDY

Rachel and her friend Michael attended the same college. They both enjoy consuming lots of drinks. Rachel is naturally curious and decides to investigate the nutritional contents of bottled drinks. She googled an explanation of how to interpret the nutritional facts on one of her favourite drink, which has nutritional contents as shown in Table 1.

TABLE 1: NUTRITIONAL CONTENT OF DRINK A

Drink A	
Sodium	1%
Added sugars	36%
Carbohydrates	4%

Rachel learns that the percentages represent the proportion of a nutrient in one serving of the drink which contributes to a total daily diet. For example, the 36% suggests that drink A provides 36% of the total added sugars for a daily diet.

1. (a) Interpret the 4% that is next to carbohydrates in Table 1

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[2 marks]

- (b) In terms of the nutritional contents, explain why Rachel should limit herself to having no more than three bottles of drink A on a given day.

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[2 marks]

GO ON TO THE NEXT PAGE

2. Rachel proceeds to check the nutritional information on sodium, added sugars and carbohydrates for two other favourite drinks, *B* and *C*. The results are shown in Table 2 and Table 3 below.

TABLE 2: NUTRITIONAL CONTENT OF DRINK *B*

Drink <i>B</i>	
Sodium	1%
Added sugars	14%
Carbohydrates	1%

TABLE 3: NUTRITIONAL CONTENT OF DRINK *C*

Drink <i>C</i>	
Sodium	2%
Added sugars	19%
Carbohydrates	3%

- (a) Using the information in Tables 1, 2 and 3, represent the nutritional facts for the three drinks, *A*, *B* and *C*, using a single 3×3 matrix.

[3 marks]

- (b) If Rachel consumes two of EACH bottle of her favourite drinks on Saturday, determine the percentage of EACH of the three nutrients (sodium, added sugars and carbohydrates) that she consumes from these drinks.

[4 marks]

GO ON TO THE NEXT PAGE

3. Michael investigates the nutritional content of his drinks (M , N and O) and obtained some information on his habits. This is summarized in the system of equations below.

Drink M : $x + 2y + z = 17$,

Drink N : $2x + 4y = 26$,

Drink O : $y + 3z = 17$,

where x , y and z represent the percentage sodium, added sugars and carbohydrates for three different drinks, M , N and O .

Using the system of equations provided by Michael,

- (a) state the percentage sodium of drink O

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[1 mark]

- (b) solve the system of equations for values of x , y and z .

[5 marks]

- (c) Complete the table below with the percentage for each nutritional fact for drink M .

TABLE 4: NUTRITIONAL CONTENT OF DRINK M

Drink M	
Sodium	
Added sugars	
Carbohydrates	

[2 marks]

GO ON TO THE NEXT PAGE

4. The two friends observed a trend with the percentages of added sugars and carbohydrates in most bottled drinks and, as such they started reviewing literature on added sugars and carbohydrates.

- (a) State an observation pertaining to the relationship between the added sugars and carbohydrate content of the drinks *A*, *B* and *C*.

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[1 mark]

GO ON TO THE NEXT PAGE

- (b) One of the articles read by Rachel and Michael reported an exponential relationship, $S = 12Ce^{-0.75C}$, where S is for added sugars and C represents carbohydrates. The graph of this function is shown in Figure 1 below.

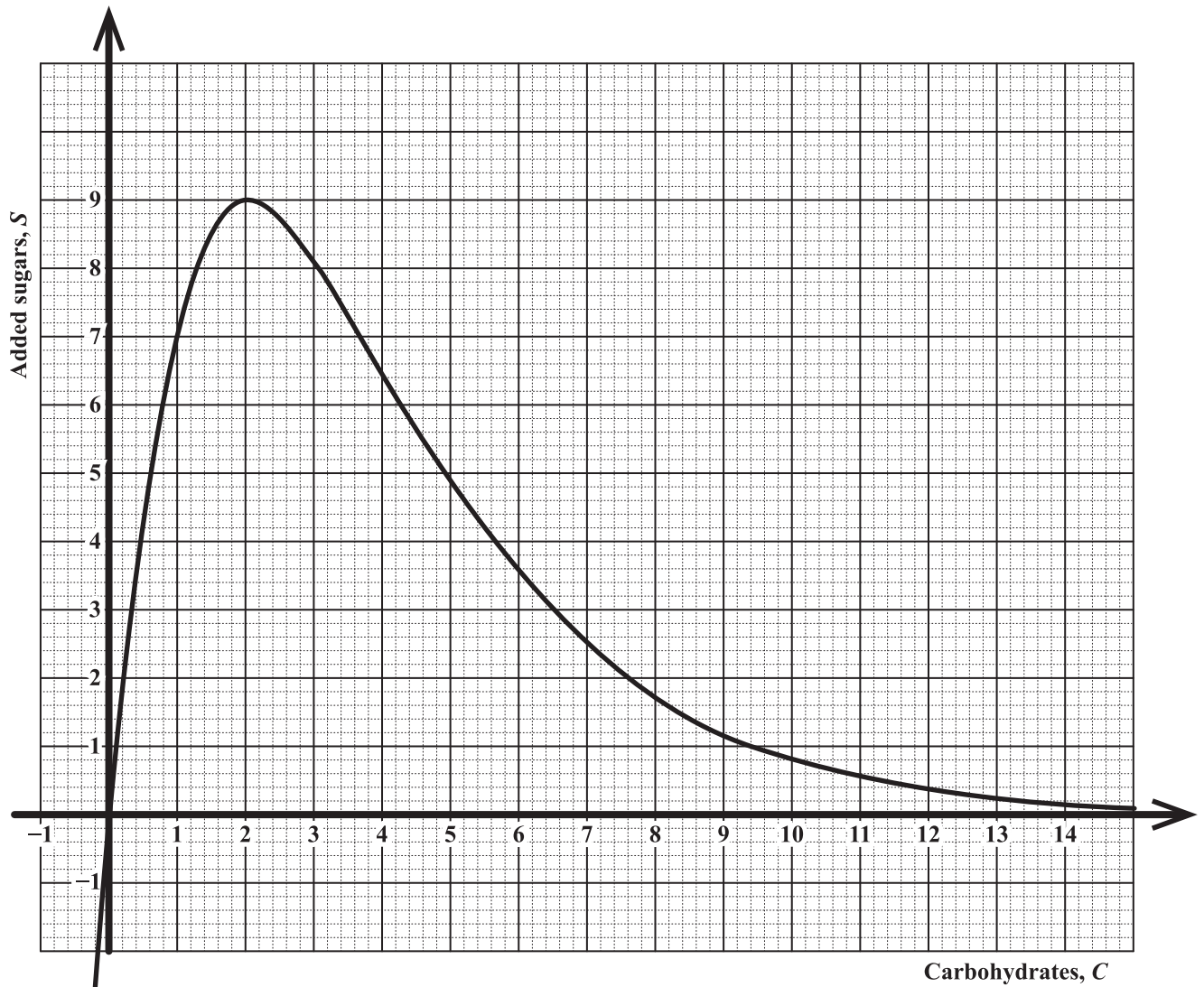


Figure 1. Graph showing the amounts for added sugars and carbohydrates

- Draw a rectangle around the portion of the curve where the added sugars level has its **greatest** rate of change. [2 marks]
- Mark and label with 'P' the point on the curve that represents a zero rate of change in added sugars. [2 marks]
- State the range of values for which the carbohydrates level is greater than that of added sugars.

[3 marks]

GO ON TO THE NEXT PAGE

5. Rachel and Michael decided to verify the exponential model by analysing the added sugars and carbohydrates contents of nine different drinks containing both added sugars and carbohydrates. The results are tabulated below.

TABLE 5: ADDED SUGARS AND CARBOHYDRATES CONTENT OF NINE DIFFERENT DRINKS

Carbohydrates Percentage	1	2	2	3	4	6	7	9	12
Added sugars Percentage	8	9	7	7	6	6	5	3	1

- (a) Use the results in Table 5 to draw a scatter plot on the graph paper provided **on page 11**. Use the horizontal axis for the carbohydrates percentages and the vertical axis for the added sugars percentages. **[4 marks]**

- (b) Compare the general trend of the scatter plot with that of the exponential model in 4 (b).

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[3 marks]

- (c) The linear regression equation was found to be $S = 9.1 - 0.65C$ with corresponding correlation coefficient = 0.964.

- (i) Interpret the value -0.65 in the equation above.

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[2 marks]

- (ii) Use the regression equation to estimate the added sugars content of a drink given that the carbohydrate content is 5%.

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[2 marks]

DO NOT WRITE IN THIS AREA

A full-page sheet of white graph paper featuring a uniform grid of small squares. The grid consists of 20 columns and 20 rows, creating a total of 400 small square units. The lines are thin and black, providing a clear guide for drawing or writing.

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- (iii) Give ONE reason why the result of the regression equation is an excellent estimate.

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[2 marks]

6. Rachel and Michael went on to gather data on the carbohydrates content of the favourite drinks of students of the science class and business class.

The carbohydrates contents for the favourite drinks of the 13 science students:

3 4 1 2 5 11 3 2 4 9 1 7 4

Using the data for the science students,

- (a) determine the median carbohydrates level

[2 marks]

- (b) calculate the mean daily percentage of carbohydrates in the favourite drinks

[2 marks]

- (c) show that the value of the standard deviation is 2.92.

[4 marks]

GO ON TO THE NEXT PAGE

7. The following are the summary carbohydrates statistics for the business students:

$$n = 30 \quad \bar{x} = 5.53\% \quad s.d. = 1.71\%$$

- (a) The median and mode are both 4.0% for the carbohydrates levels in the drinks of the business students.

Give ONE reason why the mean is much greater.

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[2 marks]

- (b) Compare the distribution of the carbohydrate levels in drinks for the science and the business students.

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[4 marks]

GO ON TO THE NEXT PAGE

8. (a) Provide a conclusion for this investigation.

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[4 marks]

- (b) Write a suitable title for the case study.

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[2 marks]

Total 60 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.

If you use this extra page, you MUST write the question number clearly in the box provided.

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Question No.

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CANDIDATE'S RECEIPT

INSTRUCTIONS TO CANDIDATE

1. Fill in all the information requested clearly in capital letters.

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SUBJECT INTEGRATED MATHEMATICS – Paper 032

PROFICIENCY ADVANCED

REGISTRATION NUMBER

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FULL NAME _____
(BLOCK LETTERS)

SIGNATURE _____

DATE _____

2. Ensure that this slip is detached by the Supervisor or Invigilator and given to you when you hand in this booklet.
3. Keep it in a safe place until you have received your results.

INSTRUCTION TO SUPERVISOR/INVIGILATOR

Sign the declaration below, detach this slip and hand it to the candidate as his/her receipt for this booklet collected by you.

I hereby acknowledge receipt of the candidate's booklet for the examination stated above.

Signature _____
Supervisor/Invigilator

Date _____

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PAPER 032

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INTEGRATED MATHEMATICS
PAPER 032
KEY AND MARK SCHEME

Question		Syllabus Reference	Solution	Mark Scheme	CK	AK	R	Total
1.	(a)	2.6.5	The 4% represents the proportion of the amount of carbohydrates in the drink for a daily diet.	A statement with 4% as a fraction of carbohydrates A statement referring to the daily percentage value	1 1			2
	(b)	2.6.5	Three bottles of drink A will exceed the total daily percentage of added sugars. Any amount greater than 3 will exceed the daily requirements of added sugars.	A statement with calculations of 3 bottles providing enough added sugars A statement pointing to the maximum number being exceeded		1	1	2
2.	(a)	1.6.2	Drinks Nutritional contents $\begin{bmatrix} 1 & 1 & 2 \\ 36 & 14 & 19 \\ 4 & 1 & 3 \end{bmatrix}$	A 3 × 3 matrix with one correct row or column A 3 × 3 matrix with two correct rows or columns A 3 × 3 matrix with all correct rows or columns		1 1 1		3
	(b)	1.6.1	$2 \times \begin{bmatrix} 1 & 1 & 2 \\ 36 & 14 & 19 \\ 4 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 36 & 14 & 19 \\ 4 & 1 & 3 \end{bmatrix} \times \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 138 \\ 16 \end{bmatrix}$ So, sodium = 8% Added sugars = 138% Carbohydrates = 16%	Multiplying contents by 2 Using a product by a 3 × 1 matrix or otherwise to obtain total Obtaining a total of each content Obtaining correct answer	1		1	4

INTEGRATED MATHEMATICS
PAPER 032
KEY AND MARK SCHEME

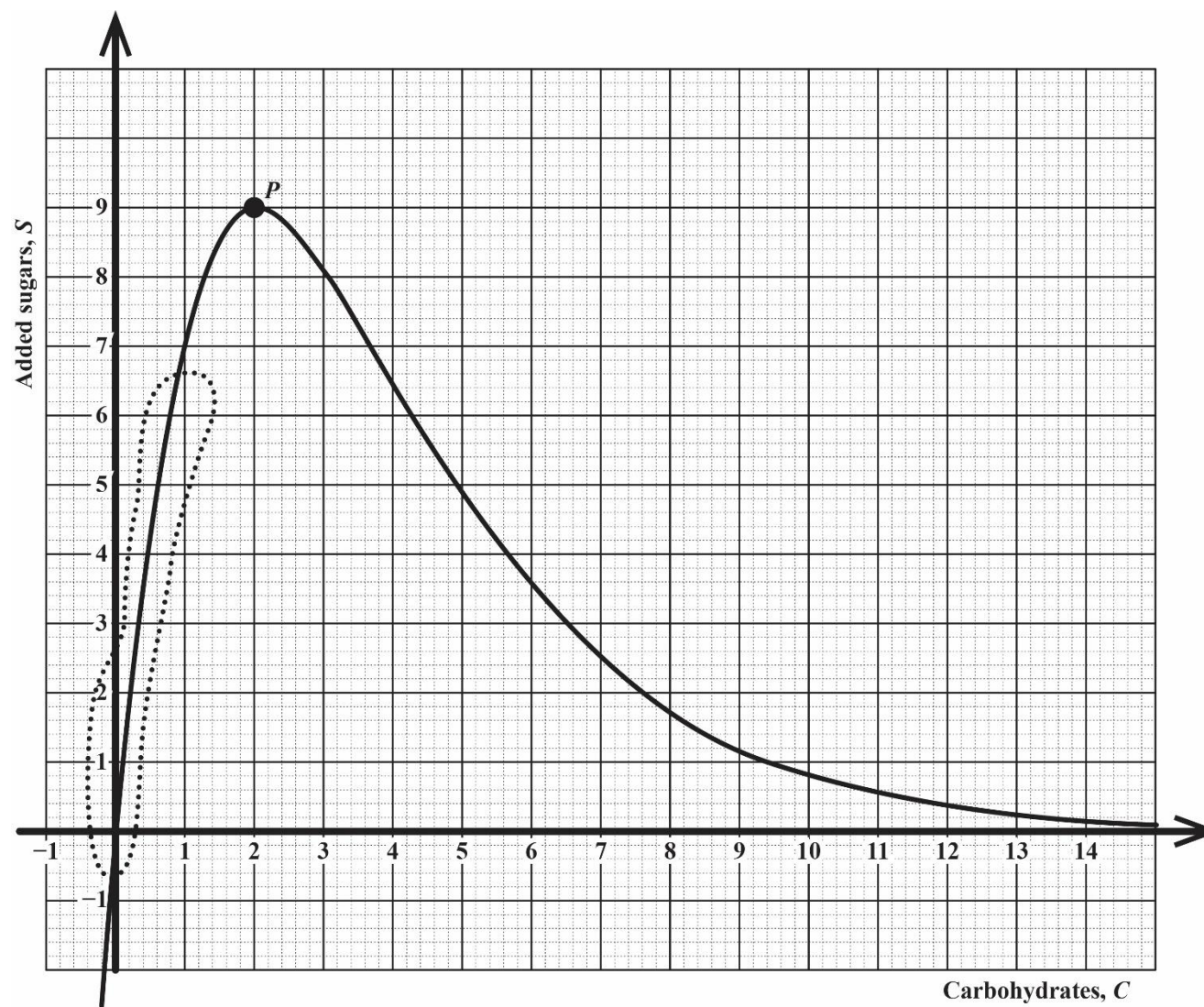
Question		Syllabus Reference	Solution	Mark Scheme	CK	AK	R	Total								
3.	(a)	1.6.5	0%	Correct answer			1	1								
	(b)	1.6.4	$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 0 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} 17 \\ 26 \\ 17 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & -2 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} 17 \\ -8 \\ 17 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 17 \\ 17 \\ 4 \end{bmatrix}$ z = 4 y + 3z = 17 y + 12 = 17 y = 5 x + 2y + z = 17 x + 10 + 4 = 17 x = 3	Setting up augmented matrix or other algorithm Correct elementary row operations to obtain triangular matrix Obtain value of one unknown Backward substituting to find two unknowns Correct answers		1 1 1 1	1	5								
	(c)	1.6.5	<table border="1"><tr><th colspan="2">Drink M</th></tr><tr><td>Sodium</td><td>3%</td></tr><tr><td>Added sugars</td><td>10%</td></tr><tr><td>Carbohydrates</td><td>4%</td></tr></table>	Drink M		Sodium	3%	Added sugars	10%	Carbohydrates	4%	One value correct All values correct based on previous result	1	1		2
Drink M																
Sodium	3%															
Added sugars	10%															
Carbohydrates	4%															

INTEGRATED MATHEMATICS
PAPER 032
KEY AND MARK SCHEME

Question		Syllabus Reference	Solution	Mark Scheme	CK	AK	R	Total
4.	(a)	2.5.3	The added sugars is high and carbohydrates low. OR Added sugars is lower for higher values of carbohydrates.	A statement commenting on the extreme values	1			1
	(b)	3.2.10	(i) See diagram on page 5	A loop including a portion of the curve near $x = 0$ A loop containing the portion of the curve from $x = 0$ to $x = 1$		1	1	2
		3.2.9	(ii) See dot on diagram on page 5	A point where rate of change is approximately zero Correct point where $x = 2$	1	1		2
		1.1.5	(iii) $C \geq 5$	An interval of values greater than 5 A finite interval that from 5 to some number k Correct interval from 5 onwards		1 1	1	3

INTEGRATED MATHEMATICS
PAPER 032
KEY AND MARK SCHEME

Question 4. (b) (i)

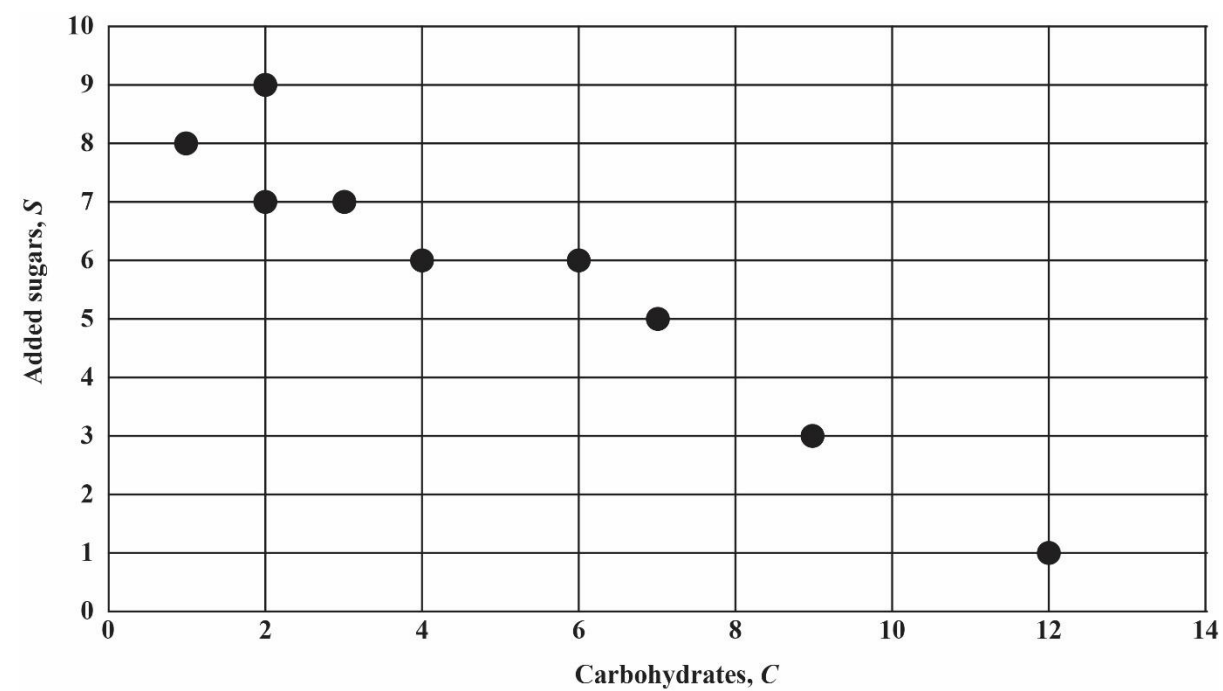


INTEGRATED MATHEMATICS
PAPER 032
KEY AND MARK SCHEME

Question		Syllabus Reference	Solution	Mark Scheme	CK	AK	R	Total
5.	(a)	2.2.2	See diagram on page 7	Correctly labelling the axes with Carbohydrates (x) and Added sugars (y) Three points plotted correctly Six points plotted correctly All points plotted correctly		1 1 1		4
	(b)	2.6.5	The shape of the scatter plot resembles the exponential model . However, it takes on a linear form that is without a curve nature .	Valid point of comparison Valid point of contrast A concise statements with valid comparison and contrast		1 1		3
	(c)	2.6.2	(i) The -0.65 indicates that for an increase by one percent in carbohydrates, the added sugar decreases by 0.65 of a percent.	A statement indicating that one increase the other decreases A statement explaining the rate of change	1		1	2
		2.6.4	(ii) $S(5) = 9.1 - 0.65(5) = 5.85\%$	Substituting 5 in regression equation given Evaluating correct answer	1		1	2
		2.6.2	(iii) Given that the correlation coefficient $r = 0.964$ is very large and close to 1, it suggests that the line is almost perfectly fitting the points used. Hence the suitability of the line given and the results obtained.	A statement with a reasonable justification of the suitability of the line. A statement justifying the suitability and the making use of the value of r.		1	1	2

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Question 5. (a)



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Question		Syllabus Reference	Solution	Mark Scheme	CK	AK	R	Total
6.	(a)	2.3.5	Carbohydrates level: 1 1 2 2 3 3 4 4 4 5 7 9 11	Ordering numbers correctly Correct answer	1	1		2
	(b)	2.3.5	$\bar{x} = \frac{1 + 1 + 2 + \dots + 9 + 11}{13} = \frac{56}{13} = 4.31$	Obtaining correct sum Correct answer	1	1		2
	(c)	2.3.3	$\sigma = \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2} = \sqrt{\frac{1 + 1 + 4 + \dots + 81 + 121}{13} - 4.31^2} = 2.92$	Use of correct formula Substituting correctly Evaluating the radicand correctly Correct answer	1	1 1 1		4

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Question		Syllabus Reference	Solution	Mark Scheme	CK	AK	R	Total
7.	(a)	2.6.5	Extreme large values result in the mean being higher than both the median and mode.	A statement explaining how extreme or large values affect the mean A statement explaining how extreme large values cause the mean to be higher than the median and mode	1		1	2
	(b)	2.6.5	The medians are the same; so too are the modes. The means are different where the business students have higher mean. The business students are more likely to exhibit positive skewness	One comparison e.g. median A second comparison e.g. mode A contrast e.g. mean Concise statement(s) including at least three comparisons and contrast in total.	1 1	1	1	4

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Question		Syllabus Reference	Solution	Mark Scheme	CK	AK	R	Total
8.	(a)	2.6.5	Drinks contain various amounts in the nutritional contents. While most drinks contain high added sugars levels there are also those with high carbohydrates levels. Further, when added sugars level is high, carbohydrates level is found to be low as they follow a negative linear relationship as compared to the exponential relationship of the literature. The average student across the two classes consume about the same amount of carbohydrates in their drinks. However, there are some drinks consumed by the Business students that are very high in carbohydrates.	A statement including the nutritional contents mentioned A statement including how the contents relate A statement including comparison with literature or comparison between classes A concise statement including all the above	1	1	1	4
	(b)	2.6.5	The distribution of nutritional contents in favourite drinks of students at a college. OR An analysis of the nutritional content of students' favourite drinks at a particular college.	A statement about investigating the nutritional contents A statement including some detail of demographics			1	2
					16	30	14	60



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