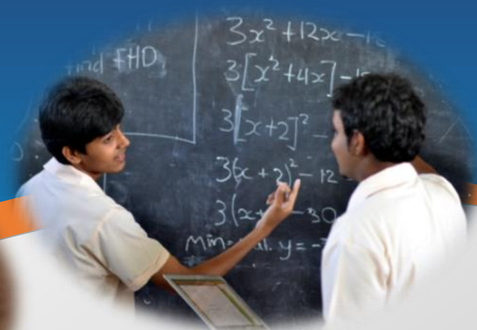
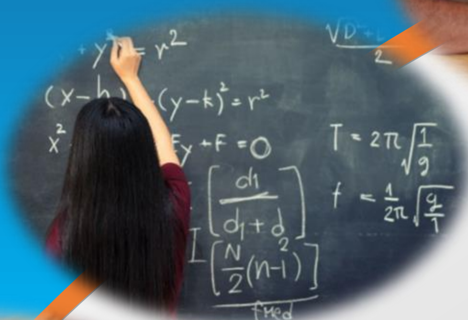
CARIBBEAN EXAMINATIONS COUNCIL

CSEC® MATHEMATICS





Subject Report with Exemplars

January 2021

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN SECONDARY EDUCATION CERTIFICATE® EXAMINATION

JANUARY 2021

MATHEMATICS
GENERAL PROFICIENCY

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INTRODUCTION

The CSEC Mathematics examination is offered in January and May/June of each year. The examination comprises the following papers.

- Paper 01 — Multiple Choice
- Paper 02 — Structured Essay
- Paper 031 — School-Based Assessment (SBA)
- Paper 032 — Alternative to the SBA

In January 2021, approximately 4793 candidates sat the Mathematics General Proficiency examination. The examination was designed to cater to the varying abilities of candidates. Overall, the mean score was 59.31 out of 200 marks and the standard deviation 28.88. Approximately 18 per cent of candidates earned Grades I–III. Although many candidates were well prepared, some candidates experienced difficulty answering questions for which they were required to use algebraic manipulations. Such candidates were unable to score full marks.

Three profiles were assessed on each paper. These profiles were Profile 1 — Knowledge, Profile 2 — Comprehension and Profile 3 — Reasoning. The mean score on each profile was as follows.

- Profile 1 — 18.21 out of 60 (30.35 per cent)
- Profile 2 — 21.00 out of 80 (26.24 per cent)
- Profile 3 — 13.93 out of 60 (23.22 per cent)

Candidates' performance on each paper is detailed in this report.

PAPER 01 — MULTIPLE CHOICE

Paper 01 consisted of 60 multiple–choice items. The paper was designed to provide adequate coverage of the syllabus content as the items were taken from all sections of the syllabus.

In January 2021, the mean score was 28.01 out of 60 marks (46.69 per cent) and the standard deviation 9.52. Three candidates (0.06 per cent of candidates) earned the maximum available score of 60, while 82 of them (or 1.70 per cent of candidates) scored zero.

PAPER 02 — STRUCTURED ESSAY

Paper 02 consisted of two sections which comprised structured and/or problem-solving questions. Section I comprised seven compulsory questions which were worth a total of 64 marks. Section II comprised three compulsory questions based on the following topics.

- Algebra, and Relations, Functions and Graphs
- Measurement, and Geometry and Trigonometry
- Vectors and Matrices

There was one question per topic and the paper was worth a total of 100 marks. The mean score was 17.07 and the standard deviation 15.28. No candidate earned the maximum available score; however, 13 (0.27 per cent of candidates) scored zero.

Question 1

This question tested candidates' ability to

- perform basic operations on real numbers (fractions)
- compute powers of real numbers (squares and cube roots)
- express a number to a given number of significant figures and decimal places
- write any rational number in scientific notation
- solve problems involving money (currency conversion).

Candidates' performance

More than 99 per cent of candidates attempted to provide a response. The mean was 3.64 out of 9 marks (40.44 per cent) and the standard deviation 2.46. Approximately 75 per cent of candidates scored less than 50 per cent of the available marks. Of the 4793 candidates taking the examination, 480 scored zero while 139 scored full marks.

Areas of good performance

- Many candidates answered Part (a) (i) correctly. A few candidates were able to obtain the correct answer by using the calculator effectively.
- Part (b) (ii) b) was well done. Most candidates were able to approximate the figure they obtained to one decimal place correctly.
- Part (c) was also answered correctly by most candidates. Some candidates wrote $\frac{4500}{630}$ which was correct but performed the calculation for $\frac{630}{4500}$.

Areas of weak performance

- Part (a) (ii) was poorly done by candidates. Few candidates answered this section correctly. Many candidates did not know how to calculate $\sqrt[3]{27}$. Some candidates who knew how to do the calculation did not give their answer in its lowest terms.
- For Part (b) (i), many candidates attempted to expand 485 x 10^{-2} first even though doing so was unnecessary.
- Part (b) (ii) was poorly done.
  - a) Many candidates were unable to approximate the answer to two significant figures correctly.
  - c) Many candidates were unable to express their answers in standard form and some interpreted standard form as expanded form.

Answers

The following are the expected answers for each part of Question 1.

(a) (i) $\frac{17}{42}$ (ii) $\frac{1}{27}$

(b) (i) 485 x 10^{-2} x 75 (ii) a) 360 b) 363.8 c) 3.6375 x 10^{2}

(c) 7.14

Recommendations

Teachers should

- encourage students to use their calculators properly
- emphasize to students that they need to read questions carefully so that they provide a complete answer for what has been asked
- give students sufficient practice on how to show proofs
- encourage students to write their answers in different forms for example, as a fraction or decimal.

Question 2

This question tested candidates' ability to

- apply the distributive law to factorize algebraic expressions
- apply the distributive law to expand algebraic expressions
- simplify algebraic fractions
- prove two algebraic expressions to be identical
- solve quadratic equations algebraically
- change the subject of formulae
- translate between algebraic symbols and worded expressions
- represent direct and inverse variation symbolically
- solve problems involving direct variation and inverse variation.

Candidates' performance

Overall, the performance of candidates was unsatisfactory. Even though more than 99 per cent of candidates attempted this question, the mean was 0.81 out of 9 marks (0.09 per cent) and the standard deviation of 1.56. Approximately 96 per cent of candidates scored less than half of the full mark. Of the 4793 candidates taking the examination, 3131 (65.32 per cent of candidates) scored zero while 30 (0.63 of candidates) scored full marks.

Areas of good performance

- Most candidates who attempted Part (a) were able to factorize the given expression correctly.

Areas of weak performance

- Part (b) was poorly done.
  - Part (b) (i): Candidates made several attempts to identify the correct denominator. Many candidates failed to do so and to distribute and collect the terms correctly. Such candidates did not apply the distributive law.
  - Part (b) (ii): Most candidates failed to equate the simplified fractional expression to zero. Instead, they equated the original expression. Of those who attempted to solve the equation, some of them were only able to find one solution.
- Part (c) was not well answered. Most candidates who attempted to remove the square root sign did so incorrectly.
- Most candidates did not respond to Part (d). Very few candidates who attempted it were able to write the correct equation.

Answers

The following are the expected answers for each part of Question 2.

(a) 4n (3n – m)

(b) (ii) 0 and $\frac{3}{4}$

(c) v = $\frac{p^2 - 5}{t}$

(d) 52.2

Recommendations

Students require more practice in the following areas.

- Simplifying and solving algebraic fractions
- Transposing formulae involving square root
- Solving variation problems

Question 3

This question tested candidates' ability to

- explain concepts relating to geometry
- bisect an angle using appropriate instruments
- draw and measure line segments accurately using appropriate instruments
- describe a transformation given an object and its image.

Candidates' Performance

Over 99 per cent of candidates attempted this question. However, the overall performance of candidates was unsatisfactory. The mean was 1.56 out of 9 marks (17.33 per cent) and the standard deviation 1.61. Approximately 94 per cent of candidates scored less than half of the full mark. Of the 4793 candidates taking the examination, 1214 (25.33 per cent of candidates) scored zero while 28 (0.58 per cent of candidates) scored full marks. The modal mark was one and the median mark was two.

Areas of good performance

Part (a) (i) was attempted by most candidates. More than 50 per cent of them chose the correct answer (congruent). In Part (a) (ii), it was clear that many candidates did not understand the meaning of the term *congruent* as very few of them were able to give an acceptable reason for choosing it in the previous part.

Areas of weak performance

- Part (b) (i) was poorly done. It was evident that many candidates were not familiar with the parallelogram rule with respect to interior angles.
- Approximately 50 per cent of the candidates who got Part (b) (i) correct were unable to find the total sum of interior angles in a pentagon as 540^o in Part (b) (ii). Many candidates omitted Part (b) (ii) or used 360^o, 450^o or 600^o instead of 540^o. Other candidates added three angles instead of four. A small percentage of candidates used construction lines to divide the shape into two quadrilaterals or a triangle and a quadrilateral. They then applied basic geometric concepts of the sum of angles in triangles, quadrilaterals and supplementary angles to correctly calculate the required angle $W\hat{X}Y$. An even smaller number of candidates completed a parallelogram and successfully used the angles on a straight line and the exterior angle of a triangle to obtain the correct answer. Such candidates arrived at $W\hat{X}Y = 82° + 66° = 148°$or $W\hat{X}Y = 180° - 32° = 148°$.
- Fewer than 50 per cent of candidates were able to produce an enlargement of the figure. Most candidates who produced the correct shape placed it in the incorrect location on the grid. Very few candidates used projection lines in determining the location of the enlarged figure. However, a large percentage of candidates knew that a translation preserved the shape and orientation, and they were able to gain the mark for this. However, they did not earn the mark for placing it in the correct location because they placed it incorrectly.

Answers

The following are the expected answers for each part of Question 3.

(a) (i) congruent (ii) same shape, size and interior angles

(b) (i) 66 (ii) 148

(c) (i) (0, −2), (2, 2), (4, −2) (ii) (0, 4); ((1, 6); (2, 4)

Recommendations

Teachers should take note of the following.

- Students must be taught the meaning of terms such as *congruent* and other related terms.
- Students should become familiar with different polygons and their interior angles.
- Students should be provided with more opportunities to experiment and explore transformation activities using various geometric shapes, a variety of mirror lines for reflections, different centres for rotations and enlargements as well as varying scale factors for enlargements.
- Students must be taught the concepts of isometric transformations and similarity transformations.
- Students should be given more opportunities to use mathematical language to communicate their ideas, reasons or explanations, especially in geometry.

Question 4

This question tested candidates' ability to

- determine a value for f(x) given x
- determine the inverse of a function
- derive the composition of function
- identify the relationship between a function and its inverse
- draw graphs of linear functions
- draw graphs to represent linear inequalities.

Candidates' Performance

Candidates' performance on this question was unsatisfactory. Even though more than 99 per cent of candidates attempted to answer this question, only 31 of them (0.65 per cent of candidates) completed the entire question successfully. The mean was 1.12 out of 9 marks (12.44 per cent) and the standard deviation 1.64. Approximately 94 per cent of candidates scored less than half the available marks. Of the 4793 candidates taking the examination, 2195 (45.8 per cent of candidates) scored zero, which was also the modal mark.

Areas of good performance

- Most candidates were able to substitute the value of *x* into $f(x)$ in Part (a) (i) to obtain the answer for $f(-1)$.
- In Part (a) (iii), candidates who were able to find the inverse of $f(x)$ were able to find $f(f^{-1}(-2))$ by finding $f^{-1}(-2)$ first, then substituting that answer into $f(x)$. Most candidates who attempted this part and got it correct were not able to make the connection that this was an identity function.
- Most candidates who attempted Part (b) (i) were able to successfully draw at least one line correctly.
- For Part (b) (ii), candidates who were able to draw all three lines correctly were able to shade the correct region, *R*.

Areas of weak performance

- For Part (a) (ii) a), candidates had difficulty finding the inverse of $f(x)$. It was clear that many candidates did not remember how to find the inverse and others had problems transposing the relevant information.
- In Part (a) (ii) b), candidates also had difficulty finding $f^2(x)$. Many candidates found the square of $f(x)$.

Answers

The following are the expected answers for each part of Question 4.

(a) (i) 5 (ii) a) $\frac{3-x}{2}$ (ii) b) $4x-3$ (iii) -2

Recommendations

The following suggestions can be used to improve students' performance.

- Teachers could review changing the subject of formulae to improve students' skill in transposing a function to find its inverse.
- Students need to be exposed to various ways composite functions could be expressed and how to interpret them.
- Teachers could allow students to investigate the composition of a function and its inverse so that students could conclude that $ff^{-1}(x)$ results in *x*.
- Students need to practise drawing linear graphs.

Question 5

This question tested candidates' ability to

- determine measures of central tendency
- determine the probability of a simple event
- make inferences from a pie chart.

Candidates' Performance

More than 99 per cent of candidates attempted to provide a response. However, less than one per cent of candidates (34 candidates or 0.71 per cent of candidates) scored full marks. The mean was 2.22 out of 9 marks (24.67 per cent) and the standard deviation 1.96. Approximately 88 per cent of candidates scored less than half the available marks. Of the 4793 candidates taking the examination, 208 (4.34 per cent of candidates) scored zero. The modal mark was one.

Areas of Good Performance

- Candidates had a good understanding of how to read a table.
- Candidates recognized that the probability is a fraction between 0 and 1.
- Part (a) (i) was well done. Approximately 80 per cent of candidates were able to provide the correct answer. This, however, was the only part they got correct.
- Candidates who attempted Part (b) were able to get Part (b) (i) correct. Candidates used several strategies but by far the most popular was to divide 360 by 3. Candidates used proportional reasoning to perform this task but they often did so without providing any working.
- Many candidates attempted Part (b) (iii) even when Part (b) (ii) was incorrectly done.

Areas of Weak Performance

- Part (a) (iii) proved to be the most challenging part. Candidates did not know what the term *median* means. They simply assumed that the median was the middle of the 60 data points so they found the thirtieth item.
- Most candidates (80 per cent) were unable to state the mode.
- Many candidates did not express the probability as a fraction. These candidates were able to find the number of students with at least 12 correct answers but they did not divide by 60.
- In Part (b) (ii), candidates disregarded the fact that Grades I, II and IV came from the 243° and not the whole 360°.

Answers

The following are the expected answers for each part of Question 5.

(a)	(i) 11	(ii) 9	(iii)	12.5	(iv)	$\frac{8}{15}$
(b)	(i) 120	(ii) 93	(iii)	54		

Recommendations

Teachers should

- allow students to practise using a frequency table. Teachers should also give students a frequency table and ask them to create possible questions based on the table. This will cause the students to anticipate the possible questions that could be based on a frequency table.
- deconstruct the frequency table into raw data in order to emphasize the mode and the median. It is easy for the students to get confused when they try to interpret frequency tables with numbers in both columns.
- focus on improving students' literacy in mathematics. The meaning of key terms such as *at least* and *at most* must be emphasized. Teachers should also use simple examples, starting with raw data then moving gradually to frequency tables. Teachers should also provide practice for students in the area of cumulative frequency tables. These data representations should not be taught in isolation from other relevant concepts.

For Part (b), it may be necessary for teachers to get students to use their prior knowledge. This can be done by showing students the following steps.

If $117° = 39$

Then $1° = \frac{39}{117}$

So $360° = \frac{39}{117} \times 360$

Then the proportion formula can be introduced to students.

Question 6

This question tested candidates' ability to

- calculate the volume of a cuboid
- use the volume of a cylinder and cuboid to determine the volume of water in a tank
- calculate the depth/height of water in a tank given its volume
- solve problems involving measurement.

Candidates' Performance

This question was attempted by approximately 99 per cent of candidates sitting the exam. The performance of candidates was below average. Approximately 75 per cent of candidates scored less than half the available marks. The mean was 2.47 out of 9 marks (27.44 per cent) and the standard deviation 3.01. Of the 4 793 candidates taking the examination, 1935 (40.37 per cent of candidates) scored zero while 423 (8.83 of candidates) scored full marks.

Areas of good performance

- In Part (a), most candidates were able to calculate the correct volume of the water in the tank.
- For Part (b), many candidates were able to calculate the correct volume of the water in the cylinder.
- For Part (c), most candidates who achieved full marks in Part (b) were able to show $\left(\frac{56\ 400}{2\ 000} = 28.2 \text{ cm}^3\right)$ or verify $\left(50 \times 40 \times 28.2 \text{ cm}^3 = 56400 \text{ cm}^3\right)$. This means that they correctly calculated that the depth of the water in the tank was 28.2 cm. Some candidates used creative ways of showing the new depth of water in the tank was 28.2 cm; for example, they divided the volume of the cylinder, 26 400 cm^3, by the base area of the tank, 2000cm^3, to get 13.2 cm and then added the value they obtained to 15 cm.

Areas of weak performance

- In Part (b), candidates either calculated 20^2 to be 20×2 in the correct formula or used the formula $V = 2\pi rh$ thereby obtaining 2640 as their answer.
- For Part (c), many candidates did not know how to proceed. This resulted in some candidates providing no response.
- Part (d) was poorly done by most candidates. Additionally, many candidates did not provide a response. Some candidates failed to calculate the difference in volumes or the difference in heights before the division. As a result, $\frac{96\ 000}{24\ 600} = 3.63$ or $\frac{48}{13.2} = 3.63$ was often given by candidates. A few candidates rounded up the correct answer of 1.5 containers to 2 containers because of the false belief that the number of containers should be a whole number.

Answers

The following are the expected answers for each part of Question 6.

(a) 30 000 cm^3

(b) 56 400 cm^3

(c) $\frac{56\ 400}{50 \times 40}$ = 28.2 cm

(d) 1.5 containers

Recommendations

Teachers need to give students more opportunities to practise

- solving problems involving the volumes of different solids
- calculating any dimension when given the volume of a solid.

Question 7

This question tested candidates' ability to

- compute terms of a sequence, given a rule
- derive an appropriate rule, given the terms of a sequence.

Candidates' Performance

Candidates were familiar with this type of reasoning question and most attempted to provide answers to the various parts. Most candidates provided a response and scored at least the first three marks. The mean was 2.96 out of 10 marks (29.6 per cent) and the standard deviation 1.88. Approximately 90 per cent of candidates scored at most half of the available marks. Of the 4793 candidates taking the examination, 268 candidates (5.59 per cent of candidates) scored zero while 17 candidates (0.35 per cent of candidates) scored full marks.

Areas of good performance

- Most candidates were able to obtain the solution for Part (a) (i).
- Most candidates attempted Part (b) and were able to obtain the mark for comprehension because they equated 4n + 1 to 541.
- Approximately 50 per cent of candidates were able to solve the equation and obtained the correct answer (135 circles). Some candidates divided 541 by 4 to obtain 135.25 and then rounded down as they recognized the answer could not include one-quarter of a dot. They too were awarded full marks.

Areas of weak performance

- In both sections of Part (a), candidates were not awarded marks because they omitted the letter representing pi (π) from their answers. They provided answers such as '9' and '30' instead of 9π and 30π.
- In Part (a) (ii), some candidates omitted the brackets from their answer. For example, candidates wrote '2n − 1 π' instead of $(2n - 1)\pi$.
- A few candidates attempted Part (c) and very few candidates worked out the answer correctly. Candidates were required to substitute 3p for n in the expression $n^2\pi$. Most candidates neglected to square both terms which would have given either $(3p)^2\pi$ or $9p^2\pi$.

Answers

The following are the expected answers for each part of Question 7.

(a) (i) 21; 9π; 30π (ii) $4n+1$, $n^2 \pi$; $(2n-1)\pi$, $n(n+1) \pi$

(b) 135

(c) $9p^2\pi$

Recommendations

- Teachers should show students how to express the formula for the n^{th} term of various types of sequences. They should also show students what patterns to look for.

Question 8

This question tested candidates' ability to

- determine the intercept on the *x* and *y* axes of a straight line
- calculate the length of a line segment and the midpoint between two points
- solve a pair of equations in two variables when one equation is quadratic and the other linear
- analyse a speed-time graph.

Candidates' Performance

Overall, candidates performed poorly. Approximately three per cent of candidates got five or more marks out of 12. The mean was 0.47 out of 12 marks (3.92 per cent) and the standard deviation 1.56. Approximately 98 per cent of candidates scored less than half the available marks. Of the 4793 candidates, 3997 (83.39 per cent of candidates) scored zero while 24 candidates (0.50 per cent of candidates) scored full marks.

Areas of good performance

- Candidates were able to find the length and midpoint fairly well. Candidates seemed to have a good grasp of the formulae used and how to manipulate them to get the correct outcome
- Candidates were able to simplify the quadratic.
- Finding the gradient of a line was a popular method used by candidates. They were able to set up and use the formula.
- Candidates knew that the average speed was total distance divided by time.

Areas of weak performance

- Candidates were not aware that when a straight line intersects the x-axis that the y coordinate is zero and vice versa.
- Candidates were not able to make a quadratic equation from the given functions. They seemed to be aware of only one method to obtain the quadratic.
- Candidates had little knowledge of speed-time graphs. They were not familiar with finding the average speed from that type of graph. Those who had some knowledge lacked the basic skill of finding areas of basic figures/shapes (for example, a triangle, a rectangle, a trapezium).

Answers

The following are the expected answers for each part of Question 8.

(a) (i) P (5, 0), Q $(0, \frac{5}{3})$ (ii) 5.27 (iii) $(\frac{5}{2}, \frac{5}{6})$

(b) (3, 2) and (−2, 7)

(c) (i) 4.8 (ii) 9.86

Recommendations

Teachers should

- ensure that students know the formulae used for finding the length of a line and the mid-point of a line because those formulae are not given on the formula sheet
- ensure that students are given practice in intersecting graphs. They must be given practice in the varied ways of solving these types of questions when dealing with linear and non-linear equations
- encourage students to find the point of intersection (x, y) values rather than only the x value
- ensure that students know that total distance is represented by the area under the curve
- give students adequate practice with different questions on distance-time graphs and velocity-time graphs, ensuring that they know the difference between the two types of graphs.

Question 9

This question tested candidates' ability to

- solve geometric problems using properties of circles and circle theorems
- relate objects in the physical world to geometric objects
- use the sine and cosine rules to solve problems involving triangles
- solve problems involving bearings.

Candidates' Performance

Candidates' performance was unsatisfactory. Generally, candidates performed better on this question than in previous years. The mean was 1.19 out of 12 marks (9.92 per cent) and the standard deviation 2.21. Approximately 96 per cent of candidates scored less than half the available marks. Of the 4793 candidates taking the examination, 2843 (59.32 per cent of candidates) scored zero while 24 (0.50 per cent of candidates) scored full marks.

Areas of good performance

- In Part (a), candidates were able to calculate the angle $B\hat{A}C$ correctly and in most cases, they were also able to give the correct reason for the size of the angle.
- Many candidates were also able calculate the value of q and recognize that $r = q = 2$.
- For Part (b), some candidates were able to correctly insert the angle $Q\hat{H}S$ in the diagram as requested.
- Candidates were able to identify that the cosine rule should be used to find the length *QS* in most cases even if they were not able to get the correct answer for *QS*.
- In finding the bearing of *S* from *Q,* candidates were also able to identify that the sine rule should be used.

Areas of weak performance

- For Part (a), too many candidates were not able to give the correct reason for why the angle $A\hat{B}C$ was 90°.
- Even though candidates were able to calculate the values of p (18°), too many candidates were not able to give $B\hat{A}C$ as 36°. In addition, many candidates did not realize that these angles were acute. In many cases, candidates did not gain the marks allocated for providing a reason for the size of the angle they obtained.
- Many candidates were unable to figure out that the angle $Q\hat{H}S$ was 126°. There were too many cases where 311° was used instead of 126°, which indicates that the candidates did not realize that the angle was obtuse.
- Candidates struggled to identify and calculate the bearing of *S* from *Q*.

Answers

The following are the expected answers for each part of Question 9.

(a) (i) angle at the circumference subtended by the diameter

(ii) a) 36 b) 36 (iii) 36

(b) (i) 126 (inserted correctly) (ii) 7.98 (iii) 151.8

Recommendations

Teachers should ensure that students

- fully understand the circle properties and theorems in order to apply them correctly
- can determine when to use the various trigonometrical ratios and rules appropriately
- are exposed to more problem-solving situations that involve geometry and trigonometry. This should include more examples where the student is required to locate the position of a bearing from a given point.

Question 10

The question tested candidates' ability to

- simplify expressions involving vectors
- use vectors to solve problems in geometry
- explain basic concepts associated with matrices
- solve problems involving matrix operations
- obtain the inverse of a nonsingular 2 x 2 matrix
- determine a 2 x 2 matrix associated with a specified transformation — transformation which is equivalent to the composition of two linear transformations in a plane (where the origin remains fixed).

Candidates' Performance

The performance of candidates was unsatisfactory. Candidates mainly focused on the matrices aspect and often did not answer the vector portion of the question. Most candidates scored zero, (3420 out of 4793 candidates or 71.35 per cent of candidates) while 11 (0.23 per cent) gained full marks. The mean was 0.63 out of 12 marks (5.25 per cent) and the standard deviation 1.49. Approximately 98 per cent of candidates scored less than half the available marks.

Areas of Good Performance

- Most of candidates who attempted Part (a) (i) were able to recognize that $L = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 3 & 6 \\ -2 & 5 \end{pmatrix} = \begin{pmatrix} -3 & -6 \\ 2 & -5 \end{pmatrix}$ and therefore they gained the mark.

  A few candidates who attempted this part stated that L = W or they were unable to correctly multiply the elements of W by (−1). Some candidates also attempted to find the determinant of Matrix W as a path to arriving at the answer. A few candidates either subtracted $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ from W or added $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ to W to get a value for L.
- For Part (a) (ii), a few candidates were able to calculate the determinant and use it to divide the adjoint to get the inverse. Many candidates who attempted to find the determinant did so correctly and earned the mark. Some candidates were able to gain a mark even though their determinants were incorrect. This was so because they used the determinant they obtained and the correct adjoint to get an inverse. A few students used their determinant with Matrix *W* to get their inverse. However, the approach of many candidates was to divide the identity matrix by *W*. Of those who attempted this part, most candidates were able to gain at least one mark.

- For Part (b), candidates were asked to determine the coordinates of the images of the three vertices of a triangle given the 2×2 transforming matrix. A few candidates attempted this part. Some candidates were able to gain the mark for stating the correct coordinates for at least two of the three vertices. However, only a few of them were able to gain the mark allocated for following the correct procedure for pre-multiplying by the transforming matrix/recognizing that it was a reflection in the line $y = x$ and as such interchanging the coordinates. A few candidates lost the mark because the set up for pre-multiplying was incorrectly stated. Candidates sometimes provided $\begin{pmatrix}1 & 1\\3 & 1\\3 & 4\end{pmatrix}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}=\begin{pmatrix}1 & 1\\1 & 3\\4 & 3\end{pmatrix}$.
- In Part (c) (i) a), candidates were asked to state $\overrightarrow{LM}$ in terms of *u* and v. Only a few candidates attempted this part. Of those candidates who attempted it, some were able to gain the mark for correctly stating the path from L to M. Many candidates tried to use a path other than the obvious one and as a result did not obtain the correct answer because they were not able to simplify the algebraic vector sum. Thus, they did not gain the second mark.
- Candidates were asked to state $\overrightarrow{PR}$ in terms of *u* and *v* in Part (c) (i) b). Some candidates were able to gain the mark. A few candidates who attempted Part (c) (i) were able to gain a maximum of three marks while most of them gained only one mark or two marks.

Areas of Weak Performance

- For Part (c) (ii), candidates were asked to prove, using vectors, that the points *L*, *M*, and *R* are collinear. Most candidates did not attempt this part. Of those who attempted it, a few of them gained the two marks for writing the route correctly, substituting the vectors, and simplifying and factorizing to find either $\overrightarrow{MR}$ or $\overrightarrow{LR}$. However, many candidates were unable to simplify the algebraic fractions after stating the correct path and substitute the vectors when finding $\overrightarrow{MR}$. As a result, they could not gain the second mark.
- Only a few candidates who successfully found $\overrightarrow{MR}$ or $\overrightarrow{LR}$ were able to gain the other two marks. Most of them were neither able to recognize or state the scalar relationship between the vectors $\overrightarrow{LM}$ and $\overrightarrow{MR}$ or $\overrightarrow{LM}$ and $\overrightarrow{LR}$ in the form $a = k\,b$, nor were many of them able to explain that the vectors are parallel with a common point, hence collinear. As a result, they lost the two marks.

Answers

The following are the expected answers for each part of Question 10.

(a) (i) $\begin{pmatrix} -3 & -6 \\ 2 & -5 \end{pmatrix}$ (ii) $\frac{1}{27}\begin{pmatrix} 5 & -6 \\ 2 & 3 \end{pmatrix}$

(b) X'(1, 1); Y'(1, 3); Z' (4, 3)

(c) (i) a) $\frac{1}{2}$ (v-u) b) 2v - 3u

(ii) $\frac{2}{3}$ (v-u) +u

(iii) LR = 4 LM; common point L

Recommendations

- Teachers need to teach students the additive inverse of 2×2 matrices
- Teachers need to teach candidates to recognize that when the product of two 2×2 matrices gives the identity matrix (I), then one matrix is the inverse of the other. In addition, the inverse is not found by finding the reciprocal of the matrix. To find the inverse, the determinant and the adjoint must be found.
- Teachers need to spend more time teaching and showing candidates how vectors can be used to solve problems in geometry.
- Teachers should ensure that students understand how to deduce collinearity between vectors, especially in identifying the scalar multiple.

PAPER 032 — ALTERNATIVE TO THE SCHOOL-BASED ASSESSMENT (SBA)

Paper 032 is an alternative to Paper 031 and is intended to be taken by private candidates. The paper comprises two compulsory questions. The topics tested may be taken from any section or a combination of sections of the syllabus.

The paper was worth a total of 20 marks, which is then scaled to 40 marks. The mean score was 3.09 (or 15.45 per cent) and the standard deviation 4.25. In January 2021, 28 candidates (0.94 per cent of candidates) earned the maximum available score while 779 (26.08 per cent of candidates) scored zero.

Question 1

This question tested candidates' ability to

- determine the perpendicular height of an equilateral triangle
- find the cross-sectional area and the volume of a container
- determine the net area of a figure
- solve problems involving measurement.

Candidates' Performance

This question proved to be challenging for most candidates. Most candidates scored zero (2357 out of 2987 candidates or 78.91 per cent of candidates) while 45 (1.51 per cent of candidates) gained full marks. The mean was 0.74 out of 10 marks (7.4 per cent) and the standard deviation 1.96. Approximately 5 per cent of candidates scored more than half the available marks.

Areas of good performance

- Candidates recognized that Pythagoras theorem or a select trigonometric ratio could be used to find *h*.
- For Part (e), candidates were able to use the hint that was given (*ABCD* is a square) to correctly find the area of the square, $3a \times 3a = 9a^2$.

Areas of weak performances

Candidates had difficulty

- applying Pythagoras theorem (or the trigonometric ratio) correctly and changing the subject of the formula
- showing proof that the area was $0.433a^2$ or $\frac{\sqrt{3}}{4}a^2$ and finding the volume of the container in terms of *a*
- calculating the depth of the oil in the container.

Answers

The following are the expected answers for each part of Question 1.

(a)	0.866 a
(b)	—
(c)	1.3 a^3
(d)	18.475

Recommendations

- Teachers should encourage students to practice using real world applications of mathematics in their lessons.

Question 2

This question tested candidates' ability to

- read coordinates correctly from a given graph
- determine the intercept on the *y* axis, *c*, and the gradient, *m*, of a straight-line graph
- determine the equation of a straight line when given the graph of that line
- interpret the meaning of *m* and *c* to a real-world problem
- compare the number of movies rented from two establishments to determine the better deal.

Candidates' Performance

Overall, candidates' responses were unsatisfactory. The question proved to be challenging for most candidates but them performed better than they did on Question 1. Of the 2987 candidates who attempted the question, 808 of them (27.05 per cent of candidates) scored zero while 119 of them (3.98 per cent of candidates) gained full marks. The mean was 2.35 out of 10 marks (23.5 per cent) and the standard deviation 2.74. Approximately 14 per cent of candidates scored more than half the available marks.

Areas of good performance

Candidates were able to

- read the values from the graph and complete the table satisfactorily.
- follow through and find the number of movies rented correctly.

Areas of weak performances

- Candidates were unable to relate *c* and *m* to the real-world situation.

Answers

The following are the expected answers for each part of Question 2.

(a) 0 and 165
(b) (i) 75 (ii) 1.8
(c) 75; 1.8
(d) (i) 340 (ii) 358; Rightstar

Recommendations

- Teachers should encourage students to practise questions based on real world applications of mathematics.

GENERAL RECOMMENDATIONS

Students need to

- read the instruction section at the front of the booklet carefully
- read the questions thoroughly to understand what is required of them
- write down all necessary working especially when questions ask them to show
- make use of the list of formulae given on page 4 of the booklet.

Teachers need to remind and encourage students to

- provide an answer related to what the questions requires
- follow the instructions given always
- realize when solutions make no sense and therefore check their working for obvious mistakes. For example, students should know that the answer is incorrect when they obtain a negative answer for the volume of an object or for the number of people
- show all necessary working especially for questions where they must show or verify a solution.

Teachers need to emphasize the use of correct mathematical terminology (for example, transpose, reflection, rotation, expand, factorize, etc.) in their teaching so that students would not encounter such terms for the first time in the examination.