CARIBBEAN EXAMINATIONS COUNCIL

ADDITIONAL MATHEMATICS

Subject Report
with
Exemplars

June/July 2022

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE
CARIBBEAN SECONDARY EDUCATION CERTIFICATE®
EXAMINATION

JUNE/JULY 2022

ADDITIONAL MATHEMATICS
GENERAL PROFICIENCY

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INTRODUCTION

This guide was compiled using candidate responses to the 2022 June/July examination in CSEC Additional Mathematics.

The examination included two externally marked papers: Paper 01 (Multiple Choice), which consisted of 45 items worth one mark each, and Paper 02 (Structured Essay), which consisted of six compulsory questions.

For Paper 031, School-Based Assessment (SBA), candidates in approved educational institutions were required to complete a project. Paper 032 was an alternative to the SBA and was taken by private candidates.

The year 2022 was the second year that the examination was designed using the revised syllabus, CXC 37/G/SYLL 18. In the new syllabus, all four sections became compulsory and could now be assessed across all papers. The four revised sections were

- Section 1: Algebra, Sequences and Series (previously Algebra and Functions)
- Section 2: Coordinate Geometry, Vectors and Trigonometry (same as previous syllabus)
- Section 3: Introductory Calculus (same as previous syllabus)
- Section 4: Probability and Statistics (previously Basic Mathematical Applications).

The questions in the examination were developed from the specific objectives of the Additional Mathematics syllabus, with the aim of ensuring that these objectives were tested across Papers 01 and 02. The questions were designed at the appropriate level to test the skills of conceptual knowledge, algorithmic knowledge and reasoning.

Specific objectives for the topic Kinematics, which were found in Section 4 of the previous syllabus, were now incorporated into Section 3 (Introductory Calculus). This meant that these topics, that is, both Kinematics, and Probability and Statistics could be assessed on Paper 01 and Paper 032. Previously, these topics could only be assessed on Paper 02 or on Paper 031 (SBA).

The requirements for Paper 031 (SBA) remained similar to those outlined in the previous syllabus. Candidates were required to choose one of two project types — mathematical modelling (Project A) or data handling/statistical analysis (Project B). The main revisions were to the marking scheme; thirty marks were now allotted to the paper rather than 20 marks.

The structure of the alternative to the SBA, Paper 032, remained similar to structure outlined in the previous syllabus. The paper consisted of one in-depth extended question. The question was broken into subparts which were used to test candidates on any aspect of the syllabus. The paper was also revised to be worth 30 marks rather than 20 marks.

In 2022, approximately 4422 candidates were registered for the Additional Mathematics General Proficiency examination, while 4208 candidates actually sat the examination. This number has

decreased by approximately 6 per cent from the 2021 examination. Overall, approximately 63 per cent of candidates earned Grades I–III, compared with 61 per cent in 2021. Twenty-three per cent of candidates attained Grade I, compared with 18 per cent in 2021.

PAPER 01 – MULTIPLE CHOICE

The mean and standard deviation were 36.09 and 13.79, respectively, compared with 35.07 and 13.36 in 2021. Hence, there was a slight improvement in candidates' performance when compared with 2021.

This paper consisted of four sections which comprised six compulsory questions. The mean and standard deviation were 38.09 and 27.86, respectively, compared with 34.00 and 25.46 in 2021.

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Question 1: Section 1
_____

This question tested candidates' ability to
- use the relationship between the sum and product of the roots of a quadratic equation to determine values for constants in a given equation and factorize a cubic function
- show that a given series is geometric and calculate its sum to infinity
- identify a problem set in context as a finite arithmetic progression and derive its general expression to solve the given problem.

The maximum score was 15 marks. The mean mark was 6.01 and the standard deviation 4.61. Two hundred and forty-one candidates were awarded full marks, while 438 scored zero.

Candidate's Response to Part (a) (i)

Consider the quadratic equation $qx^2 - (4p)x + pq^2 = 0$, where p and q are both **positive** integers.

(i)   Express the sum AND product of the roots of the equation in terms of p and q.

$$a = q, \quad b = -4p$$
$$c = pq^2$$

$$(\alpha + \beta) = \frac{-b}{a} \, \#$$

$$= \frac{-(-4p)}{q}$$

$$= \frac{4p}{q}$$

$$\alpha\beta = \frac{c}{a} = \frac{pq^2}{q} = pq$$

(3 marks)

Candidate's Response to Part (a) (ii)

(ii)   Determine the value for q such that the sum of the roots is equal to the product of the roots.

$$(\alpha + \beta) = \alpha\beta$$

$$\frac{4p}{q} = pq$$

$$\frac{4p}{p} = \frac{pq^2}{p}$$

$$4 = q^2$$

$$\therefore q = \sqrt{4}$$

$$q = 2$$

(2 marks)

Candidate's Response to Part (a) (iii)

(iii) If the sum of the roots of the equation is 20, use your answer from (a) (ii) to determine a value for p.

$(\alpha + \beta) = 20$

$\therefore \dfrac{4p}{q} = 20$

if $q = 2$,

$\dfrac{4p}{2} = 20$ | $4p = 40$ | $\therefore p = \dfrac{40}{4} = 10$ **(1 mark)**

Examiner's Comments

Candidates' performance on Part (a) (iii) was determined by how they performed on the previous parts.

Candidate's Response to Part (a) (iv)

(iv) . Hence, express the given quadratic equation in terms of its numerical coefficients.

$qx^2 - (4p)x + pq^2 = 0$

if $q = 2$ and $p = 10$,

$2x^2 - (4 \cdot 10)x + 10(2)^2 = 0$

$2x^2 - 40x + 10(4) = 0$

$2x^2 - 40x + 40 = 0$ **(1 mark)**

Examiner's Comments

Candidates' performance on Part (a) (iv) was determined by how they performed on Part (a) (i).

Candidate's Response to Part (b) (i)

(b)   A series is given by

$$25 - 5 + 1 - \frac{1}{5} + \frac{1}{25} \cdots$$

(i)   Show that the series is geometric.

geometric series ⟹ common ratio, r

r = $\frac{5}{}$

r = $\frac{a_2}{a_1}$ = $\frac{-5}{25}$ = $-\frac{1}{5}$ ∴ Series is geometric

                                                ⟹ r = $-\frac{1}{5}$

r = $\frac{a_4}{a_3}$ = $\frac{-\frac{1}{5}}{1}$ = $-\frac{1}{5}$

(2 marks)

Examiner's Comments

Part (b) and Part (c) were the parts on which most candidates were able to score. Although some candidates were able to show that each successive term was being divided by -5, many were unable able to make the connection that the common ratio $r = \frac{-1}{5}$.

Candidate's Response to Part (b) (ii)

(ii)   Calculate the sum to infinity of the series, giving the answer to **2 decimal places.**

$$S_\infty = \frac{a}{1-r} = \frac{25}{1-\left(-\frac{1}{5}\right)} = \frac{25}{1\frac{1}{5}} = \frac{25}{\frac{6}{5}} = \frac{25}{1} \times \frac{5}{6}$$

$$\therefore S_\infty = \frac{125}{6} = 20.83$$

(2 marks)

Examiner's Comments

Generally, candidates who were able to do Part (b) (i) were also able to successfully complete Part (b) (ii). The sum to infinity formula was usually correctly applied.

Candidate's Response to Part (c)

A recent university graduate was offered a starting salary of $720 000 for the first year, with increases of $5000 at the start of every year thereafter. Determine the number of years (to the nearest whole number) that it would take for her annual salary to be 20% greater than her salary in the first year.

Starting salary = $720 000

Salary 20% greater = $720 000 + \left(\dfrac{20}{100} \times 720\,000\right)$

$= 720\,000 + 144\,000$

$= \$864\,000$

$a_1 = 720\,000$

$a_n = 864\,000$

$d = 5000$

$n = ?$

$T_n = a + (n-1)d$

$864\,000 = 720\,000 + (n-1)5000$

$864\,000 = 720\,000 + 5000n - 5000$

$864\,000 - 720\,000 + 5000 = 5000n$

$149\,000 = 5000n$

$\therefore n = \dfrac{149\,000}{5000}$

$= 29.8$

$= 30$

It would take $\cong$ 30 years for her annual salary to be 20% greater than her salary in the first year.

(4 marks)

Total 15 marks

Examiner's Comments

Candidates responded to Part (c) using different approaches. Many candidates used deductive reasoning to solve the problem rather than the arithmetic progression (AP) method. For example, one candidate used a percentage increase approach to determine the number of years. This approach was well thought out and mathematically correct. Nevertheless, while novel approaches are welcome, candidates are encouraged to use the conventional mathematical approach.

Recommendations

Students should be encouraged to explore and understand the concepts of sum and product of roots and what they tell us about a quadratic. One possible strategy that teachers could use is providing students with more examples that have known values for $a, b \text{ and } c$ of the quadratic. Students can then be given the opportunity to solve questions similar to these examples to find the roots, the sum and the product, and then compare them with the original quadratic.

Candidates need more practice in describing sequences and series when answering questions. Additionally, the ability of candidates to use correct mathematical jargon and symbols also needs to improve.

Another area of weakness was candidates' inability to differentiate between a sum and a sequence. Teachers can use ICT and digital tools to teach students about the specific objectives related to quadratic equations. This may possibly improve this area of weakness. A good resource for interactive activities related to these concepts can be found at geogebra.org and can be accessed using https://www.geogebra.org/t/quadratic-equation?lang=en.

Question 2: Section 1

_____

This question tested candidates' ability to

- apply the remainder theorem to determine values of constants in a given polynomial
- perform operations on a fractional expression with surds, that is, rationalization of the denominator to show equivalence
- express a given quadratic in the form $a(x + h)^2 + k$ and, hence, determine its roots and sketch its graph.

The maximum possible score was 15 marks. The mean mark was 6.94 and the standard deviation 4.92. Three hundred and seventy-six candidates scored full marks, while 520 candidates scored zero.

Candidate's Response to Part (a)

When the polynomial expression $2x^3 - 3x^2 - cx + d$ is divided by $(x + 1)$ and $(x - 2)$, the same remainder of 64 is obatined.

Determine the value of c and d.

$$f(-1) \Rightarrow 2(-1)^3 - 3(-1)^2 - c(-1) + d = 64$$
$$\Rightarrow -2 - 3 + c + d = 64$$
$$\Rightarrow -5 + c + d =$$

$$f(2) \Rightarrow 2(2)^3 - 3(2)^2 - c(2) + d = 64$$
$$\Rightarrow 16 - 12 - 2c + d = 64$$
$$\Rightarrow 4 - 2c + d = 64 \quad ✔$$
$$\Rightarrow 4 - 64 =$$
$$\Rightarrow -2c + d = 64 - 4$$
$$\Rightarrow -2c + d = 60 \quad ✔$$

$$f(-1) \Rightarrow 2(-1)^3 - 3(-1)^2 - c(-1) + d = 64$$
$$\Rightarrow -2 - 3 + c + d = 64$$
$$\Rightarrow -5 + c + d = 64$$
$$\Rightarrow c + d = 64 + 5$$
$$\Rightarrow c + d = 69 \quad ✔$$
$$\therefore d = 69 - c$$

$$-2c + (69 - c) = 60$$
$$-3c + 69 = 60$$

$$-3c = 60 - 69$$
$$-3c = -9$$
$$c = \frac{-9}{-3}$$
$$c = 3$$

$$\therefore d = 69 - 3$$
$$d = 66 \quad ✔$$

(4 marks)

Candidate's Response to Part (b)

(b)   Show that the expression $\dfrac{\sqrt{25}}{\sqrt{45}}$ is the same as $\dfrac{\sqrt{5}}{3}$.

① $\sqrt{45}$

$\sqrt{45} = 3\sqrt{5}$

$\sqrt{25} = 5$

② $\dfrac{\sqrt{25}}{\sqrt{45}} = \dfrac{5}{3\sqrt{5}} \times \dfrac{(3\sqrt{5})}{(3\sqrt{5})}$

③ $(3\sqrt{5})(3\sqrt{5}) = 15$

$9(5)(\sqrt{5}) = 5\sqrt{5}$

(2 marks)

④ $\dfrac{5\sqrt{5}}{15_{3}} = \dfrac{\sqrt{5}}{3}$ (Q.E.D)

Candidate's Response to Part (c) (i)

(i) Given $g(x) = 6x^2 + 12x - 18$, express $g(x)$ in the form $a(x+h)^2 + k$.

$$g(x) = 6x^2 + 12x - 18$$
$$= (6x^2 + 12x) - 18$$
$$= 6(x^2 + 2x + (1)^2) - 18 - 6(1)^2$$
$$= 6(x+1)^2 - 18 - 6$$
$$= 6(x+1)^2 - 24$$

(3 marks)

Examiner's Comments

Candidates had challenges completing the square for the given quadratic. Many candidates did not use the classical approach shown above. Instead, they attempted to use a formula approach. Several candidates did not use the right formula for either one or both constants, h and k, and so did not obtain correct values.

Candidate's Response to Part (c) (ii)

(ii)   Using the expression derived in (c) (i), determine the roots of $g(x)$.

$$6(x+1)^2 - 24 = 0$$
$$6(x+1)^2 = 24$$
$$(x+1)^2 = 4$$
$$x+1 = \pm\sqrt{4}$$
$$x+1 = \pm 2$$

$$\therefore x_1 = -1 + 2$$
$$= 1$$

$$x_2 = -1 - 2$$
$$= -3$$

(3 marks)

Examiner's Comments

There were many candidates who obtained the correct completed square expression in Part (c) (i) but did not use it to determine the roots of $g(x)$. Instead, they used factorization or the quadratic formula on the original expression. This suggested that some candidates were unaware of what they should do. Since this part was dependent on using the completed square expression from Part (c) (i), these candidates lost points.

Candidate's Response to Part (c) (iii)

(iii)   Hence, sketch the graph of $g(x)$ on the following grid.
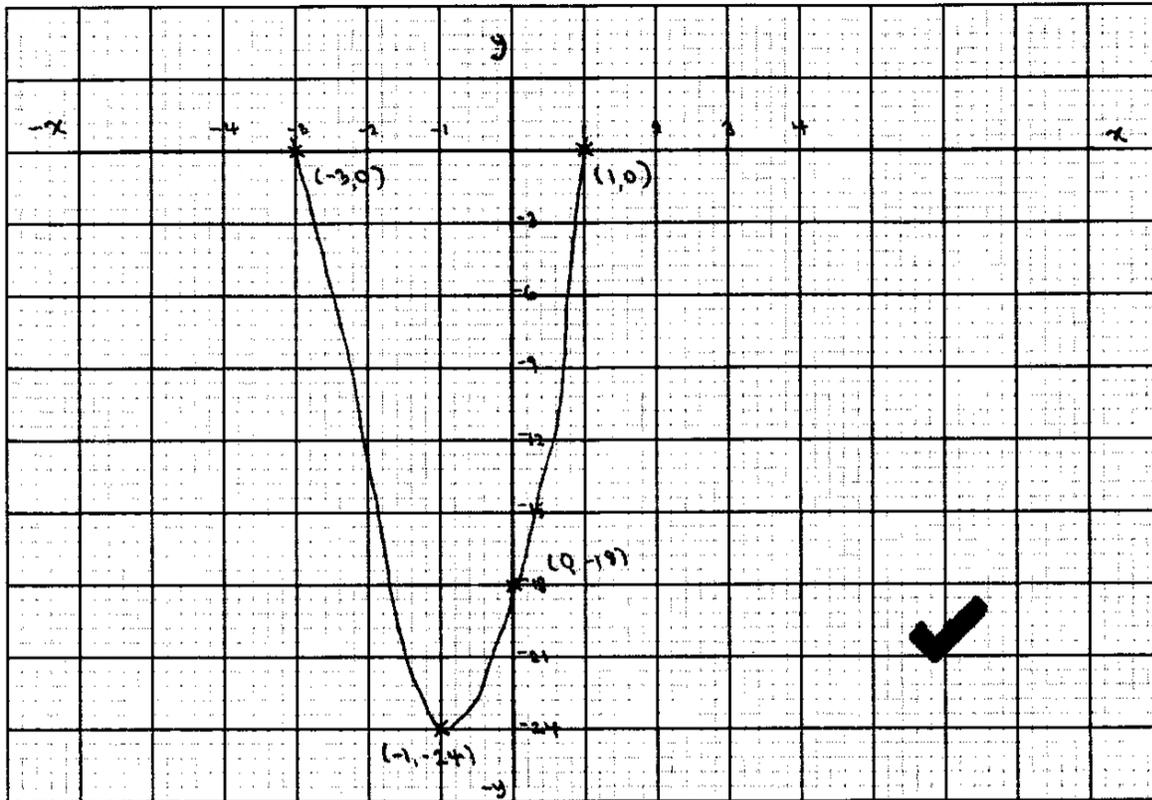
(3 marks)

$a = 6 \Rightarrow$ min

Total 15 marks

x-intercepts $\Rightarrow (1,0) \ (-3,0)$

y-intercept $\Rightarrow 6(0+1)^2 - 24 = -18 \Rightarrow (0,-18)$

max/min point $\Rightarrow (-h, k) \Rightarrow (-1, -24)$

Examiner's Comments

Candidates generally sketched the quadratic curve fairly well. This included those candidates who made errors in their previous answers. It must be noted, however, that some candidates did not use a high degree of accuracy to represent the points on the graph.

Recommendations

To improve candidates' performance on this section it is recommended that teachers

- emphasize the difference between the remainder theorem and the factor theorem and provide sufficient examples of when each theorem should be applied.

- ensure that students know the formula approach and that they are competent in using it if it is the preferred method for completing the square. Students should also be encouraged to memorize the formula since it is not included on the formula sheet.

- ensure that students know how to check that their obtained expression is equivalent to the one originally given.

- teach students completing the square as an approach to solving quadratic equations and allow them to practice this method.

- spend time teaching students the classical approach to completing the square. Arguably, this skill should be part of the repertoire of any successful Additional Mathematics student as there is a likelihood that they will undertake further studies in mathematics.

- emphasize that students should use a high degree of accuracy to represent key coordinates when sketching a curve or other graphs on a grid.

Question 3: Section 2

This question tested candidates' ability to

- determine the centre and radius of a given circle, and, hence, determine the equation of a tangent to the circle at the given point on its circumference
- use properties of vectors to show that two given vectors are perpendicular
- apply knowledge of area of a circle's sector to determine the area of a shaded part of a circle, the product of two vectors and the angle between them
- prove a simple trigonometric identity using the double angle formula and then use this identity to solve a trigonometric equation for possible values of π.

The maximum possible score was 20 marks. The mean mark was 6.45 and the standard deviation 6.60. Only 167 candidates scored full marks while 1228 candidates scored zero.

Candidate's Response to Part (a) (i)

The equation of a circle is $x^2 + y^2 + 4x - 8y + 10 = 0$.

(i)   Determine the coordinates of its centre AND the length of the radius of the circle.

$$\text{centre} \left[-\tfrac{1}{2} \text{ coeff. of } x , -\tfrac{1}{2} \text{ coeff. of } y \right]$$

$$\left(-\tfrac{1}{2} \times \tfrac{4}{1} , -\tfrac{1}{2} \times \tfrac{-8}{1} \right)$$

$$\text{centre} (-2, 4)$$

$$c = 10$$
$$c = a^2 + b^2 - r^2$$
$$10 = (-2)^2 + (4)^2 - r^2$$
$$10 = 4 + 16 - r^2$$
$$10 - 4 - 16 = -r^2$$
$$-10 = -r^2$$

$$\therefore r^2 = 10$$
$$r = \sqrt{10} \text{ , units}$$

(3 marks)

Candidate's Response to Part (a) (ii)

(ii)   Determine the equation of the tangent to the circle at the point P (–5, 5).

gradient of normal $\neq p$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{5 - 4}{-5 - (-2)}$

$= \dfrac{1}{-3}$

$\therefore$ gradient of tangent $= -1\left(\dfrac{-3}{1}\right)$

$= 3$

equation of tangent $\Rightarrow y - y_1 = m(x - x_1)$

$y - 5 = 3(x - (-5))$

$y - 5 = 3(x + 5)$

$y - 5 = 3x + 15$

$y = 3x + 20$

(3 marks)

Candidate's Response to Part (b)

(b) The vectors $\overline{OX}$ and $\overline{OY}$ are such that $\overline{OX} = 4i + j$ and $OY = i - 4j$. Show that the vectors $\overline{OX}$ and $\overline{OY}$ are perpendicular.

① $\cos\theta = \dfrac{a \cdot b}{|a||b|}$ ∴ $\dfrac{0}{|a||b|}$ NB: Since the numerator is 0, then the entire fraction is 0.

$a \cdot b = (4i + j)(i - 4j)$ ②
$= 4 - 4$
$= 0$ ⟹ $\cos\theta = 0$

③ $\theta = \cos^{-1}(0)$

Thus, $\overrightarrow{OX}$ and $\overrightarrow{OY}$ are perpendicular. $\theta = 90°$

(5 marks)

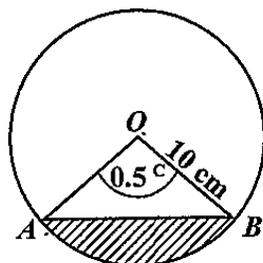**Examiner's Comments**

Candidates who knew one of the strategies used to determine if two vectors are perpendicular performed well on Part (b). However, some candidates answered poorly or did not provide a response since they could not find an approach to answering the question. In the exemplar, the candidate's method was particularly welcome given that it was logical and succinct.

The diagram below, **not drawn to scale**, shows a chord AB which subtends an angle of 0.5^c (0.5 radians) at the centre, O, of a circle of radius 10 cm. Given that the area of triangle $AOB = \dfrac{1}{2} r^2 \sin \theta$, calculate the area of the shaded region.

$A_{\text{shaded region}} = A_s - A_T$

$A_s = \frac{1}{2} r^2 \theta$

$\cancel{= \frac{1}{2} (0.5^c)^2}$

$= \frac{1}{2} (10)^2 (0.5^c)$

$= 25 \cancel{\pi} \, cm^2$

$A_T = \frac{1}{2} r^2 \sin \theta$

$= \frac{1}{2} (10)^2 \sin (0.5^c)$

$= 23.97 \, cm^2$

$\therefore A_{\text{shaded region}} = 25 - 23.97$

$= 1.03 \, cm^2$

(3 marks)

Examiner's Comments

Generally, Part (c) was well answered by those candidates who provided a response. However, a common error made by candidates was to perform calculations without switching their calculators to radian mode. Candidates should have used radian mode because the angle was given in radians.

Candidate's Response to Part (d) (i)

(i) Show that $\cos 2\theta = 2\cos^2\theta - 1$.

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta = \cos\theta\cos\theta - \sin\theta\sin\theta$

$\qquad = \cos^2\theta - \sin^2\theta$

$\qquad = \cos^2\theta - (1 - \cos^2\theta)$

$\qquad = \cos^2\theta - 1 + \cos^2\theta$

$\cos 2\theta = 2\cos^2\theta - 1 \quad (Q.E.D)$

(3 marks)

Examiner's Comments

Part (d) proved to be the most challenging part. In many cases, it remained unanswered. Some candidates began to complete the proof but could go no further as they seemingly did not recognize that the double angle formula should have been used.

Candidate's Response to Part (d) (ii)

(ii)   Hence, solve the equation $\cos 2\theta + \cos \theta + 1 = 0$ for $0 < \theta < 2\pi$.

$\cos 2\theta + \cos \theta + 1 = 0$

~~$2\cos^2\theta + \cos\theta + 1 = 0$~~

~~let $x = \cos\theta$~~

~~$2x^2 + x + 1 = 0$~~

$2\cos^2\theta - 1 + \cos\theta + 1 = 0$

$2\cos^2\theta + \cos\theta = 0$

let $x = \cos\theta$

$2x^2 + x = 0$

$x(2x + 1) = 0$

$x = 0$ or $2x + 1 = 0$

                   $2x = -1$

                   $x = -\frac{1}{2}$

Since $x = \cos\theta$,

$\cos\theta = 0$ or $\cos\theta = -\frac{1}{2}$

quad. I $\perp$ IV

$\text{I} \Rightarrow \theta = \beta = 0° = 0$

$\text{IV} \Rightarrow \theta = 360 - \beta$

$\therefore \beta_1 = \cos^{-1}(0) = 90°$

$\beta_2 = \cos^{-1}(\frac{1}{2}) = 60°$

$\text{I} \Rightarrow \theta = \beta_1 = 90° = \frac{1}{2}\pi$

$\text{IV} \Rightarrow \theta = 360 - \beta_1$

         $= 360 - 90$

         $= 270°$

         $= \frac{3}{2}\pi$

$\text{II} \Rightarrow \theta = 180 - \beta_2$

         $= 180 - 60$

         $= 120°$

         $= \frac{2}{3}\pi$

                         (3 marks)

$\text{III} \Rightarrow \theta = 180 + \beta_2$

         $= 180 + 60$ **Total 20 marks**

         $= 240°$

         $= \frac{4}{3}\pi$

Examiner's Comments

Candidates who were able to answer up to this point were usually able to substitute correctly for the double angle term, factorize and obtain a quadratic equation. There were a few candidates who were unable to complete the proof in Part (d) (i) but they were able to use the identity to solve the equation given in Part (d) (ii). Candidates did not always list all possible values of θ in the given range as solutions.

Recommendations

It is recommended that teachers spend some time teaching the classical approach to completing the square as its use extends beyond algebraic manipulations. Additionally, teachers should spend some time ensuring that students know the correct calculator input for various calculations, including operations with fractions, logarithms and trigonometry (switching from degrees to radians, etc.). Students should also be given more practice in completing proofs, including showing why particular given mathematical results are obtained.

Question 4: Section 3

This question tested candidates' ability to
- determine the second derivative of a function
- use rules of differentiation, including the chain rule, to determine the derivative of trigonometric functions and products of simple polynomials
- determine the stationary points on a given curve and determine the nature of those points.

The maximum possible score was 15 marks. The mean mark was 5.06 and the standard deviation 4.76. Eighty-six candidates scored full marks while 1190 did not score.

Candidate's Response to Part (a)

(a)   Given that $f(x) = x(5-x)^2$, determine $f''(x)$.

$$f(x) = x(5-x)^2$$
$$= x(x^2 - 10x + 25)$$
$$= x^3 - 10x^2 + 25x$$
$$f'(x) = 3x^2 - 20x + 25$$
$$f''(x) = 6x - 20$$
$$f''(x) = 6x - 20$$

$(5-x)(5-x)$
$= 25 - 5x - 5x + x^2$
$= x^2 - 10x + 25$

(3 marks)

Examiner's Comments

Generally, Part (a) was poorly done. In some instances, candidates attempted to find the inverse of the given function rather than determining its second derivative. Other candidates only differentiated once. It seemed that some candidates may not have understood what was required.

Candidate's Response to Part (b) (i)

(b)   Differentiate EACH of the following expressions with respect to x, simplifying your answer where possible.

(i) $2 \sin 3x + \cos x$

$$2 \sin 3x + \cos x$$

$$\frac{d}{dx}(2\sin 3x + \cos x) = (2\cos 3x)(3) + (-\sin x)$$

$$= 6\cos 3x + -\sin x$$

$$\frac{d}{dx}(2\sin 3x + \cos x) = 6\cos 3x - \sin x$$

(3 marks)

Examiner's Comments

Candidates were generally able to provide an answer. However, several candidates made errors when applying the chain rule, while others chose the incorrect rule (product rule) to answer the question.

Candidate's Response to Part (b) (ii)

(ii) $(1 + 2x)^3 (x + 2)$

$$f'(x) = 3(1+2x)^2(2)$$
$$= 6(1+2x)^2$$
$$g'(x) = 1$$

$$y = (1+2x)^3(x+2)$$

$$\frac{dy}{dx} = f(x)g(x) + f(x)g'(x)$$

$$\frac{d}{dx}(1+2x)^3(x+2) = 6(1+2x)^2(x+2) + (1+2x)^3(1)$$

$$= (1+2x)^2 \left[6(x+2) + (1+2x) \right] = (1+2x)^2(8x+13)$$

$$\frac{d}{dx}(1+2x)^3(x+2) = (1+2x)^2(8x+13)$$

(3 marks)

Examiner's Comments

Candidates made several errors in their attempts to answer Part (b) (ii). One approach was to apply the product rule to find the derivative. However, many of the candidates who used this approach were then unable to use factorization or expansion to correctly simplify the expression for the derivative. Alternatively, several candidates attempted to expand before finding the first derivative, but many were unable to do this accurately or made errors in simplification. Some candidates did not even attempt to find the derivative but stopped after expanding the function.

Candidate's Response to Part (c) (i)

(i)   Determine the stationary points on the curve $y = x^3 - 4x^2 + 4x$.

$$y = x^3 - 4x^2 + 4x$$

$$\frac{dy}{dx} = 3x^2 - 8x + 4$$

when x is stat. $\frac{dy}{dx} = 0$

$$\therefore 3x^2 - 8x + 4 = 0$$

$$(3x - 2)(x - 2) = 0$$

$$x = \frac{2}{3}, 2$$

$$x = \frac{2}{3} \text{ and } 2$$

when $x = \frac{2}{3}$

$$y = \left(\frac{2}{3}\right)^3 - 4\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right)$$

$$= \frac{8}{27} - \frac{16}{9} + \frac{8}{3}$$

$$= \frac{32}{27}$$

when $x = 2$

$$y = 2^3 - 4(2)^2 + 4(2)$$

$$= 0$$

$$y = \frac{32}{27} \text{ and } 0$$

$\therefore$ the stationary points are

$$\left(\frac{2}{3}, \frac{32}{27}\right) \text{ and } (2, 0)$$

·(4 marks)

Examiner's Comments

Part (c) (i) was the subpart of Part (c) on which candidates performed best. Candidates were aware that differentiation was needed and that they also needed to equate the derivative to zero for the stationary $x - values$ to be found. However, a common error candidates made was not finding the $y - value$. This meant that they did not find the actual coordinate of the stationary point. Some candidates also made errors in factorizing to find the $x - values$ or substituting incorrectly when trying to find the $y - values$.

Candidate's Response to Part (c) (ii)

(ii)   Providing details, determine the nature of EACH stationary point in (c) (i).

$$\frac{d^2y}{dx^2} = 6x - 8$$

when $x = 2$

$$\frac{d^2y}{dx^2} = 4$$

when $x = \frac{2}{3}$

$$\frac{d^2y}{dx^2} = -4 = 1$$

when $\frac{d^2y}{dx^2} > 0$

$\therefore x$ is minimum.

when $\frac{d^2y}{dx^2} < 0$

$\therefore (2, 0)$ is a minimum point

x is maximum

$\therefore \left(\frac{2}{3}, \frac{32}{27}\right)$ is a maximum point

(2 marks)

Total 15 marks

Examiner's Comments

Generally, Part (c) (ii) was also well answered. Some of the common errors included substitutions of $x - values$ into the given function for y instead of the second derivative or not knowing what maximum and minimum points mean.

Recommendations

It is evident that emphasis needs to be placed on ensuring that students are familiar with mathematical notation related to the Additional Mathematics syllabus. Students also need more practice in simplifying expressions, including using factorization methods. In relation the teaching of stationary points, instruction should include the graphical approach and the algebraic processes so that students have a better understanding of the concept, including maximum and minimum points and what they look like on the curve. Teachers may find the following websites useful in covering the concepts assessed in this question.

- https://mathsmadeeasy.co.uk/a-level-maths-revision/product-rule/
- https://medium.com/mathadam/the-product-rule-an-intuition-ba7955950228

Question 5: Section 3

This question tested candidates' ability to

- integrate simple trigonometric functions
- compute a definite integral
- use integration to find the area of a region in the first quadrant which is bound by a curve and a line parallel to the y-axis
- use integration to determine velocity from acceleration.

The maximum possible score was 15 marks. The mean mark was 4.91 and the standard deviation 5.52. Three hundred fifty-eight candidates scored full marks while 1657 candidates scored zero.

Candidate's Response to Part (a)

(a)   Determine $\int (4 \cos \theta - 6 \sin \theta)\, d\theta$.

$\int (4\cos\theta - 6\sin\theta)\,d\theta = 4\sin\theta - (-6\cos\theta) + c$

$= 4\sin\theta + 6\cos\theta + c$

$\int (4\cos\theta - 6\sin\theta)\,d\theta = 4\sin\theta + 6\cos\theta + c$

(2 marks)

Examiner's Comments

Some candidates were able to obtain the correct answer, although many did not include the constant of integration.

Candidate's Response to Part (b)

(b) Evaluate $\int_1^2 (3-x)^2\, dx$.

$$\dfrac{(3-x)(3-x)}{9-3x-3x+x^2}$$

$$\int_1^2 (3-x)^2\, dx = \int_1^2 (x^2 - 6x + 9)\, dx$$

$$= \left[\dfrac{x^{2+1}}{2+1} - \dfrac{6x^{1+1}}{1+1} + \dfrac{9x^{0+1}}{0+1} \right]_1^2$$

$$= \left[\dfrac{x^3}{3} - 3x^2 + 9x \right]_1^2$$

$$= \left[\dfrac{2^3}{3} - 3(2)^2 + 9(2) \right] - \left[\dfrac{1^3}{3} - 3(1)^2 + 9(1) \right]$$

$$= \dfrac{26}{3} - \dfrac{19}{3} = \dfrac{7}{3}$$

$$\int_1^2 (3-x)^2\, dx = \dfrac{7}{3}$$

(3 marks)

Examiner's Comments

Generally, candidates were able to obtain the correct answer. Many candidates expanded the expression while others integrated the binomial as given. Some candidates who attempted this part knew that they needed to substitute values of x over the region $x = 2$ and $x = 1$ to obtain the answer for the definite integral.

Candidate's Response to Part (c)

(c) Determine the area of the region bounded by the curve $y = 5 + 5x - x^2$, the x-axis, the y-axis and the line $x = 2$.

$$\text{area} = \int_0^2 (5 + 5x - x^2)\, dx$$

$$= \left[\frac{5x^{0+1}}{0+1} + \frac{5x^{1+1}}{1+1} - \frac{x^{2+1}}{2+1} \right]_0^2$$

$$= \left(5x + \frac{5x^2}{2} - \frac{x^3}{3} \right)_0^2$$

$$= \left[5(2) + \frac{5(2)^2}{2} - \frac{2^3}{3} \right] - \left[5(0) + \frac{5(0)^2}{2} - \frac{0^3}{3} \right]$$

$$= \frac{52}{3} \text{ units}^2$$

$$\text{area} = \frac{52}{3} \text{ units}^2$$

(4 marks)

Examiner's Comments

Candidates did not always know what limits to include in the integral to find the area, although they were mostly able to integrate correctly.

Candidate's Response to Part (d)

(d)   A particle moves in a straight line so that t seconds after passing through a fixed point, O, its acceleration, a, is given by $a = (3t - 1)$ m s^{-2}. When $t = 2$, the particle has a velocity, v, of 4 m s^{-1}, and a displacement of 6 m from O.

Determine the velocity when $t = 4$.

$$v = \int (3t - 1)\, dt$$

$$= \frac{3t^{1+1}}{1+1} - \frac{1t^{0+1}}{0+1} + c$$

$$= \left(\frac{3t^2}{2} - t + c\right) \text{ms}^{-1}$$

$$\therefore \boxed{v = \left(\frac{3t^2}{2} - t\right) \text{ms}^{-1}}$$

when $v = 4$, $t = 2$

$$4 = \frac{3(2)^2}{2} - 2 + c$$

$$= 6 - 2 + c$$

$$4 - 4 = c$$

$$0 = c$$

when $t = 4$

$$v = \left(\frac{3(4)^2}{2} - 4\right) \text{ms}^{-1}$$

$$= (24 - 4) \text{ms}^{-1}$$

$$= 20 \text{ms}^{-1}$$

when $t = 4$

velocity $= 20 \text{ms}^{-1}$

(6 marks)

Recommendations

More work is required to help students understand why a constant of integration is included when integrating an indefinite integral and why this is different to integrating a definite integral. More work needs to be done with students on calculus applications, including kinematics. Teachers may find the information on the websites associated with Khan Academy, Edutopia as well as DeltaMath useful.

Question 6

This question tested candidate's ability to

- use laws of probability related to set theory
- use the addition rule to determine probability
- calculate conditional probability
- identify mutually exclusive and independent events
- construct a stem and leaf diagram from given data
- identify an advantage of using a stem-and-leaf diagram to display data
- determine the range of a given data set
- interpret a box-and-whisker plot.

The maximum possible score was 20 marks. The mean mark was 8.71 and the standard deviation 4.96. Twenty-seven candidates scored full marks while 224 scored zero.

Candidate's Response to Part (a) (i)

At a school canteen, 80% of the students (S) purchase chips (C) and 55% purchase chicken (K). Of the students who purchase chicken, 11% do not purchase chips.

(i)   Complete the following Venn diagram to illustrate this information.
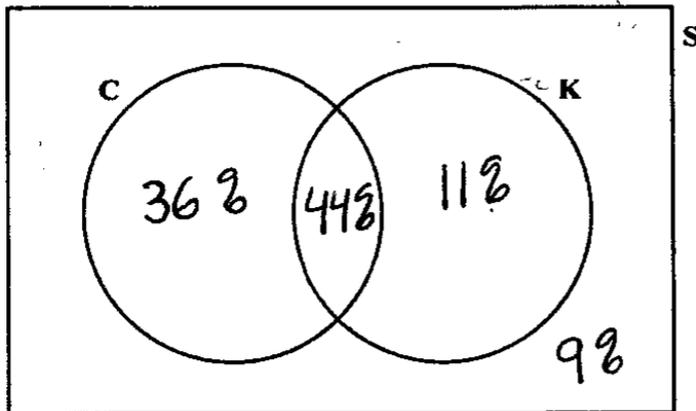
(4 marks)

Candidate's Response to Part (a) (ii)

(ii) Determine the probability that a student chosen at random purchases ONLY chicken or ONLY chips.

$P(K \text{ only} \cup C \text{ only})$

$= P(K \text{ only}) + P(C \text{ only})$

$= 11\% + 36\%$

$= 47\%$

$P(K \text{ only} \cup C \text{ only}) = 47\%$

(1 mark)

Candidate's Response to Part (b) (i)

(b)   The probabilities of the occurrence of two events, A and B, are given by

$P(A) = \frac{1}{4}$, $P(B) = \frac{3}{5}$ and $P(A \cup B) = \frac{7}{10}$. Determine

(i) $P(A \cap B)$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{7}{10} = \frac{1}{4} + \frac{3}{5} - P(A \cap B)$$

$$\frac{7}{10} - \frac{17}{20} = -P(A \cap B)$$

$$-\frac{3}{20} = $$

$$\frac{3}{20} = P(A \cap B)$$

$$P(A \cap B) = \frac{3}{20}$$

(3 marks)

Examiner's Comments

Many of the candidates obtained the desired result of $\frac{3}{20}$. Even though the required formula is given on the formula sheet, some candidates responded using an incorrect formula — $P(A \cap B) = P(A) \times P(B)$. While both the correct formula and the incorrect formula yielded the same result, the incorrect formula assumes that events A and B are independent events. Another area of concern was that candidates gave negative answers or answers greater than 1 for probability.

Candidate's Response to Part (b) (ii)

(ii) P($A|B$).

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{3/20}{3/5} = \frac{1}{4}$$

$$P(A|B) = \frac{1}{4}$$

(3 marks)

Examiner's Comments

Some candidates stated and applied the conditional probability accurately to get the required answer, $\frac{1}{4}$. However, there were a few candidates who did not recognize that their answer from Part (b) (i) was to be used as the numerator in the formula. In some cases, even though candidates may have gotten Part (b) (i) correct, they made errors that caused them to obtain the wrong answer.

Candidate's Response to Part (c)

State, with a reason, whether Events A and B are independent.

Events A and B are independent as the probability of A does not depend on that of B. P(A) is the same whether or not B occurs. Also, $P(A \cap B) \neq P(A|B)$

(1 mark)

Examiner's Comments

Only a few candidates made correct use of the fact that two events A and B are independent if $P(A \cap B) = P(A) \times P(B)$ or that $P(A|B) = P(A)$ and showed this. Candidates were required to use part of the calculated result from Part (b) in the response. However, many candidates opted to give a reason using words which meant that they provided a definition for independent events without any reference to the context and values.

Candidate's Response to Part (d) (i)

(d)   The following table shows the marks obtained by 27 students in a Mathematics test.

30	35	39	42	45	45	52	59	61
61	65	69	70	71	75	77	79	81
83	83	85	87	89	90	90	95	98

(i)   Construct a stem-and-leaf diagram to display this data.

```
3 | 0   5   9          Key
                       
4 | 2   5   5          3 | 0  represents 30
                       
5 | 2   9
                       
6 | 1   1   5   9
                       
7 | 0   1   5   7   9
                       
8 | 1   3   3   5   7   9
                       
9 | 0   0   5   8
```
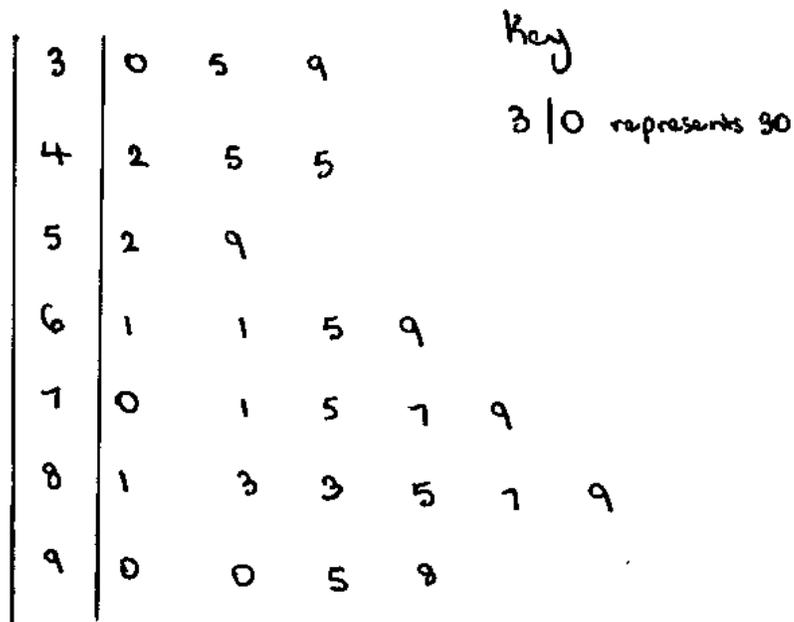
(4 marks)

Examiner's Comments

Apart from candidates who drew actual trees with leaves, Part (d) (i) was generally well done. One noticeable area of weakness was that some candidates omitted a key for the diagram.

Candidate's Response to Part (d) (ii)

(ii)   State ONE advantage of using a stem-and-leaf diagram to display the data.

A stem-and-leaf diagram is shows the exact values of the data, thereby making in possible to calculate the mean.

(1 mark)

Examiner's Comments

Generally, most candidates stated a descriptive aspect or feature of stem-and-leaf diagrams. An example of this was candidates stating that the stem-and-leaf diagram was easy to read. Candidates were, however, required to give an advantage of using this type of diagram such as it retains all the original data.

Candidate's Response to Part (d) (iii)

(iii)   State the range of values of the marks obtained by the students.
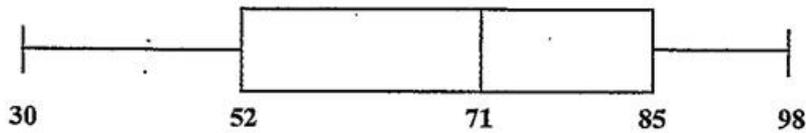
Range = 98 - 30

68

(1 mark)

Examiner's Comments

Many candidates answered Part (d) (ii) correctly. A few candidates gave 30 to 98 as an interval rather than working out the number value.

Candidate's Response to Part (d) (iv)

(iv.) The data are displayed in the following box-and-whisker plot.

State TWO distinct observations about the data as seen in the box-and-whisker plot.

① The lower quartile and upper quartile is 52 and 85, respectively.

② The data is negatively skewed.

(2 marks)

Examiner's Comments

In Part (d) (iv), many candidates were able to give at least one distinct observation. An example of this was stating that the distribution was negatively skewed. Another distinct observation frequently made by candidates was stating that the median was 71. However, candidates often repeated the same observation or failed to indicate the observation in relation to the data and the numbers given in the box-and-whisker plot. Special mention must be made of a few candidates who recognized that the data had no outliers which implies that they were familiar with the representation of outliers on the diagram.

Recommendations

Teachers should ensure that students are aware of which formulas are given on the formula sheet for each section of the syllabus as this is an indication of the formulas that will be given on the examination papers. Teachers should also emphasize the range of possible values for probability so that students can recognize when an answer for probability is impossible and be prompted to recheck their solution. Additionally, teachers should ensure that students are familiar with key aspects of statistical diagrams and what ought to be included in such sketches/drawings.

PAPER 031 – SCHOOL-BASED ASSESSMENT (SBA)

In 2022, the mean mark was 36.12 and the standard deviation 4.37. There was a slight decline in the mean compared with 2021 when it was 37.18. This decline in the mean stood in contrast to the improvements noted on Paper 01 and Paper 02. Teachers are asked to continue to note the areas of concern from previous years as they remain relevant, and they are encouraged to continue the good work to improve student performance in those areas.

This was the second year in which an Additional Mathematics project could be used as the project for General CSEC Mathematics. However, this created an issue related to the level of mathematics at which some Additional Mathematics SBAs were done. Some projects were written at a CSEC level rather than the level of difficulty required for Additional Mathematics. Teachers should note that an Additional Mathematics project may be used for a CSEC Mathematics project, but the projects cannot be used vice versa.

Comments on Project A

Generally, projects of this nature explore the topics: Differentiation (Maxima and Minima) and Series and Sequences.

In 2022, the projects of this type covered similar concepts with many examining the maximizing and/or minimizing of some area of space or profit and costings to determine economic sustainability. There were some projects which proffered successful attempts at using indices and logarithms. Comments related to each part of the scoring rubric are presented in the following table.

PROJECT DESCRIPTORS	COMMENTS
Project Title	For most projects, this was clear and concise, letting the reader know what was to be expected from the project.
Purpose	Most projects had a purpose which outlined why the project was being done.
Mathematical Formulation	Generally, most projects identified the mathematical concepts and formulas to be used. However, in the economic profit and loss projects, explanations were given for the cost and revenue functions with no mention of where these functions originated. In projects of this type, students can use assumptions for these functions.
The Problem Solution	Most projects had the correct problem solution based on the information presented in The Purpose section. Students showed that they understood the project by properly developing the ideas and formulas to determine the solution to their problem.

PROJECT DESCRIPTORS	COMMENTS
	However, diagrams for this type of project remain an area of weakness. The use of an exponential or parabolic curve is suggested to illustrate differential functions. It was observed that for some projects, concepts from other disciplines such as Economics or Advanced Physics were employed. In such cases, the theory or concept should be referenced so that the examiner can verify the information.
Application of Solution	In many projects, students applied their solution by carrying out a cost factor analysis or by choosing the best shape to maximize the benefits. Some of the projects used more than two comparisons of different 2D-shapes, comparing the surface area and perimeter to determine which would be the best fit for their solution. There were projects in which students also compared the volumes of various 3D-figures to determine the best fit for their solution.
Discussion of Findings/Conclusion	In most projects, students provided findings that were easy to follow. Suggestions for future use were also given. These suggestions usually entailed applying the project concept to other ideas. However, including suggestions for future use remains an area of weakness of students or it is possibly overlooked as many projects continue to exclude discussion on this aspect.
Overall Presentation	Projects made appropriate use of mathematical symbols as well as grammar and spelling.

Comments on Project B

Generally, projects of this nature explore the topics under Statistics & Probability.

In 2022, projects of this type covered many of the same ideas. There were many projects that compared data between performances of different groups of students or different ways the COVID-19 pandemic affected students. Comments related to each part of the scoring rubric are presented in the following table.

PROJECT DESCRIPTORS	COMMENTS
Project Title	For most projects, the project title was clear and concise, letting the reader know what was to be expected from the project.
Purpose	In most projects, the purpose was appropriate.
Method of Data Collection	Most projects included the sample details of the project. Some also included the questionnaire used to collect the data.

PROJECT DESCRIPTORS	COMMENTS
	A fair proportion of SBAs, however, lacked further details related to the methods of data collection. Most students used an online survey (Google Forms) as the method, but they did not provide either a screenshot of the survey or a brief description of the survey to clearly detail the variables and type of data being collected.
Presentation of Data	Most students represented the data collected in a stem-and-leaf diagram and a box-and-whisker plot. Careful consideration should be taken of the following. • The original data collected can be put in either the presentation of data or the appendix. • Students should be complimented for their knowledge of inserting bar charts, line graphs and pie charts using electronic means. However, representing data using these formats is associated with CSEC Mathematics. • Students often presented box-and-whisker plots with no calculations. Also, standard deviation values were stated without working. Because these are a part of the specific objectives of the syllabus, examiners are expecting to see the associated calculations to award marks for calculation. • In many cases, the stem-and-leaf diagrams were inaccurately constructed. When there are too many leaf entries for a particular stem, students continued on the line below. Constructing the leaf diagram in this way is inaccurate because it does not allow the skewness of the data to be seen. Teachers should teach students to slit the stems, as in the case of grouped data, so that a more accurate stem-and-leaf diagram is presented.
Mathematical Knowledge/Analysis	Overall, students submitted valid analyses based on their findings. Careful consideration should be taken of the following. • For a more logically ordered project, the box-and-whisker plots should be placed under Mathematical Knowledge/Analysis rather than in the Presentation of Data. The reason for this is that calculations of the raw data must be done for the box-and-whisker plots to be constructed. • There were a few projects submitted where the Analysis of Data was placed in the Appendix. This should be placed in the Mathematical Knowledge/Analysis section in the body of the project.

PROJECT DESCRIPTORS	COMMENTS
Discussion of Findings/Conclusion	Generally, most students attempted to discuss some of their SBA findings. However, there were many cases in which there were no suggestions given for future analysis. Students often mentioned increasing the sample size as a suggestion. However, suggestions for future analysis should show ways in which the project could be developed and/or used in a related area.
Overall Presentation	Projects generally made appropriate use of mathematical symbols as well as grammar and spelling.

PAPER 032 – ALTERNATIVE TO THE SCHOOL-BASED ASSESSMENT (SBA)

The question on this paper tested candidates' ability to
- use notation for indefinite integrals and simple rules of integration to find the integral of a function for a real-world problem
- compute a definite integral
- use the addition rule to calculate probabilities
- calculate conditional probability and identify mutually exclusive events for a real-world problem
- draw graphs of functions related to $cos\ kx$ and $sin\ kx$ where $k\ is\ 1\ or\ 2$
- find solutions to simple trigonometric equations using the graphical method.

This paper continued to be challenging for candidates. The mean mark was 11.00 and the standard deviation was 8.45. No candidate scored full marks.

Candidate's Response to Part (a) (i)

Mark starts painting at noon. He can paint $(140 - kt)$ square feet per hour where t is the number of hours since he started painting and k is a constant accounting for the fact that Mark slows down as he gets tired.

On Friday, Mark paints 100 square feet between 2 p.m. and 3 p.m. Determine the value of k.

Question No. **1** **a)**

starts: 12:00 pm   140 - kt /hour

between 2:00 — 3:00 p.m = ~~140-kt~~ 100 square feet

$\frac{140}{140} = 2(9)$

$\therefore (140 - kt) - (140 - kt) = 100$

at 3   at 2

$V = 140 - kt$ t at 3   3:00 — 12:00 t at 2:00

$v = f/h$ $= 3$ hours $= 2$ hours

$S = \int_2^3 v$ $140 - K$

$S = 140t - \dfrac{kt^2}{2}$ when $t = 3$ $140 - kt$ $\left[140 - kt = 100 \right]_2$

when $t = 3$ $140 - 3k = 100$ per hour $140 - 2k = 100$

$k = \dfrac{40}{3}$

$\left[140t - \dfrac{kt^2}{2} \right]_2^3 = 100$ $V = 140 - kt$ $\dfrac{40}{3}$

$S = \int v$ $2.5k + 140 = 100$

$\left[420 - k\dfrac{9}{2} \right] - \left[280 + \dfrac{2k}{2} \right] = 100$ $-\dfrac{5}{2}k + 140 = 100$

$140 - 100 = \dfrac{5}{2}k$

$k = 16$

$S = \int_2^3 140 - kt = 100$

$140 - \dfrac{5}{2}k = 100$

$140 - 100 = \dfrac{5}{2}k$

$k = 16$

Examiner's Comments

Only a few candidates recognized that the use of integration was required to solve the given problem. Of these candidates, a few recognized the need to find the definite integral for the period between 2 p.m. to 3 p.m. Generally, candidates used simple algebra and substituted 1 hour into the given function.

(b)　The following table shows the results of a survey distributed to 400 adult participants to determine their opinions about receiving a COVID-19 vaccination. The tabulation summarizes the distribution of males/females (M/F) in the sample, as well as whether they were for vaccination(Y) or against vaccination(N)

Gender	For Vaccination (Y)	Against Vaccination (N)	TOTAL
Male (M)	98	102	200
Female (F)	136	64	200
TOTAL	234	166	400

(i)　One of the researchers claims that the probability of a randomly selected participant being female or being for vaccinations is about 0.75. Determine whether the researcher's claim is **true**.

$$P(F \cup \text{ or } Y) = P(F) + P(Y) - P(F \cap Y)$$

$$P(F \cap Y) = \frac{136}{400} \quad\quad P(F) = \frac{200}{400} = \frac{1}{2}$$

$$= \frac{17}{50}$$

$$P(M) = \frac{200}{400} =$$

$$P(Y) = \frac{234}{400}$$

$$= \frac{117}{200}$$

$$P(F \cup Y) = \left(\frac{1}{2} + \frac{117}{200}\right) - \left(\frac{17}{50}\right)$$

$$= \frac{217}{200} - \frac{17}{50} \quad \cancel{\frac{149}{200}}$$

$$= \frac{149}{200}$$

(5 marks)

$$P(F \cup Y) = 0.75.$$

∴ the researcher's claim is true;

The probability of being female or being
for vaccination is about 0.75.

Candidate's Response to Part (b) (ii)

(ii)   Using appropriate steps, show that being **male** and being **against** vaccinations are not independent nor mutually exclusive events.

$\cancel{P(M \cup N) = P(A|B) \times P(B} \quad P(M|N) \times P(N)}$

$P(M \cap N) = \frac{102}{400} = \frac{51}{200} = 0.255 \simeq 0.26$

For independent events,

$P(M \cap N) = P(M) \times P(N)$

$\qquad = \frac{200}{400} \times \frac{166}{400}$

$\qquad = \frac{83}{400}$

$\qquad = 0.2075$

$\qquad \simeq 0.21$

Since $0.21 \neq 0.26$, being male and being against vaccinations are not independent events.

For mutually exclusive events,

$P(M \cap N) = 0$

Since $0 \neq 0.26$, being male and being against vaccinations are **(6 marks)** not mutually exclusive events.

Examiner's Comments

Many candidates were not successful in proving that the events were neither independent nor mutually exclusive. More candidates attempted to complete a proof for the events not being independent, however only a few did so accurately. There were fewer attempts at proving that the events were not mutually exclusive. A number of these proofs began with an incorrect formula or premise for mutually exclusive events. These candidates multiplied any two probabilities rather than showing that $P(M \cap N) \neq 0$.

(c)   (i)   Use a graphical method to determine the solutions (in terms of π) to the following
             simultaneous equations over the range $0 \le x \le 2\pi$:

$y = 2 \sin 2x$

$y = \cos x$ (1,0,

Scale : x-axis : 1 cm = $\pi/4$ units
       y-axis : 2 cm ≡ 1 unit

x-axis

y-axis
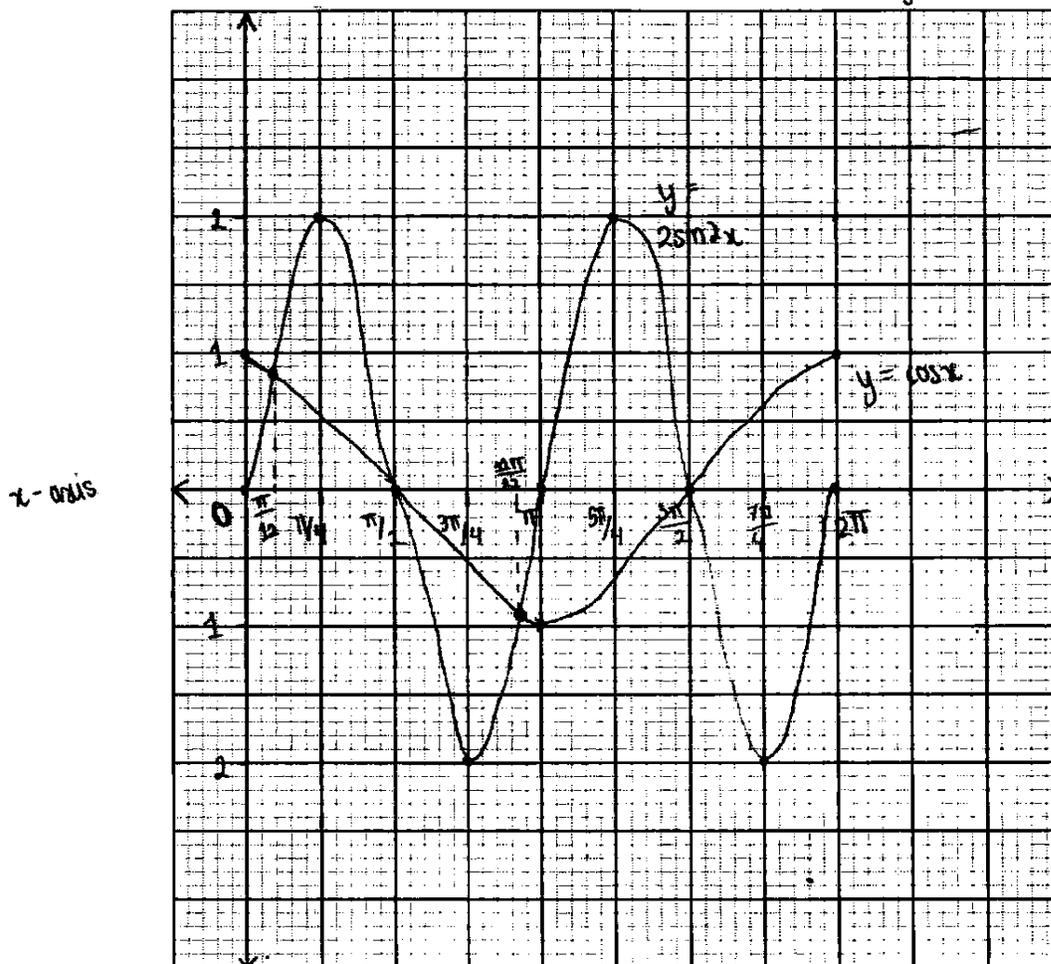
(9 marks)

Using the graph, solutions are $\frac{\pi}{2}$ (90°) and $\frac{3\pi}{2}$ (270°)

and at approx. $\frac{11\pi}{12}$ (165°) in the range $0 \le x \le 2\pi$

and at approx. $\frac{\pi}{12}$ (15°)

$\{ x : x = \frac{\pi}{12}, \frac{\pi}{2}, \frac{11\pi}{12}, \frac{3\pi}{2} \}$

Candidate's Response to Part (c) (ii)

(ii)   Describe the periodicity and the amplitude of EACH graph.

For $y = 2\sin 2x$, the periodicity $= \pi$ units (values repeats every $180°$)

amplitude $= 2$

For $y = \cos x$, the periodicity $= 2\pi$ units (values repeats every $360°$)

amplitude $= 1$

(2 marks)

Recommendations

Instructors should provide more examples and exercises which apply mathematical concepts to real-world situations for candidates. This would help candidates make sense of questions and provide suitable responses.

General Recommendations

Teachers are encouraged to seek out opportunities for collaboration and learning from each other. They should also seek opportunities for professional development to enhance the general teaching and learning of mathematics. Holistic strategic planning for mathematics from primary to post-secondary learning can potentially go a long way towards raising the bar on students' understanding and knowledge of the subject.