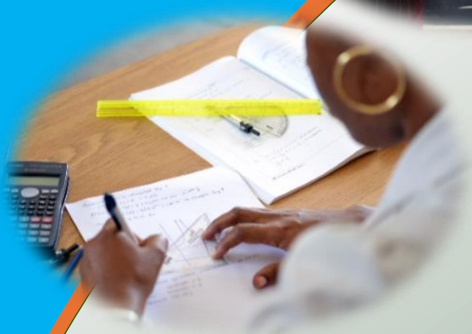
CARIBBEAN EXAMINATIONS COUNCIL

CAPE® INTEGRATED MATHEMATICS



Subject Report with Exemplars

May/June 2023

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE CARIBBEAN ADVANCED PROFICIENCY EXAMINATION

JUNE/JULY 2023

INTEGRATED MATHEMATICS

Table of Contents

INTRODUCTION

This guide has been put together using candidate responses to the 2023 May/June examination in CAPE Integrated Mathematics. The report reflects the original design of the examination paper.

In 2023, approximately 1276 candidates sat the Integrated Mathematics Proficiency examination.

The examination targeted the content and specific objectives of the syllabus with the aim of ensuring coverage and simultaneously catering to the wide range of mathematical abilities of candidates. The examination composed a Paper 01 with multiple-choice items, a Paper 02 with extended response questions, a Paper 031 in the form of a school-based assessment (SBA) and a Paper 032 which is an alternative examination to the SBA. The questions were set at appropriate levels to test the following skills: Conceptual Knowledge, Algorithmic Knowledge, and Reasoning.

Each paper of this examination tested the three modules: Module 1 — Foundations of Mathematics, Module 2 — Statistics, and Module 3 — Calculus. The maximum possible mark for the examinations is 300, which represents 100 per cent. The composition and weighting of the associated papers are as follows:

- Paper 01: Forty-five multiple-choice items weighted as 90 marks and representing 30 per cent of the total examination
- Paper 02: Six extended response-type questions each at 25 marks, totaling 150 marks. This corresponded to 50 per cent of the total examination
- Paper 031/032: A total of 60 marks which represents 20 per cent of the total examination

The following are comparative results spanning the five-year period 2019 to 2023. It must be noted that the results from 2020 are reflective of unusual circumstances and do not lend themselves to useful comparison with the other years.

In the 2023 sitting of the examination, 68.66 per cent of candidates earned acceptable Grades I–V compared with 65.36 per cent in 2022, 64.52 per cent in 2021 and 67.53 per cent in 2019. In 2020, the percentage was 72.48.

This year, the percentage of candidates who earned Grade I was 4.02 per cent compared with 4.54 per cent in 2022, 5.11 per cent in 2021 and 6.93 per cent in 2019. In 2020, the percentage was 9.10.

This year, the percentage of candidates who earned Grade II was 5.39 per cent compared with 7.80 per cent in 2022, 5.53 per cent in 2021 and 5.67 per cent in 2019. In 2020, the percentage was 8.80.

This year, the percentage of candidates who earned Grade III was 12.41 per cent compared with 10.17 per cent in 2022, 11.34 per cent in 2021 and 9.40 per cent in 2019. In 2020, the percentage was 12.29.

PAPER 01 – MULTIPLE CHOICE

This paper tested all three modules of the syllabus and it comprised of 45 items for a total of 90 marks. The maximum score obtained was 90 (100 per cent), with mean and standard deviation scores of 56.49 and 16.95 respectively. This represented a decrease from the previous year when the mean and standard deviation scores were 57.06 and 20.49 respectively.

PAPER 02 – EXTENDED RESPONSE QUESTIONS

This examination required candidates to answer all six questions for a maximum of 150 marks. There were two questions from each of the three modules and each question was worth 25 marks. The maximum score obtained was 146 marks. The mean and standard deviation scores were 30 and 26.89 respectively. This represented a decrease from the previous year when the mean and standard deviation scores were 31.61 and 27.77 respectively.

Module 1: Foundations of Mathematics

Question 1

Candidate's Response to Part (a) (i)

SECTION A

MODULE 1: FOUNDATIONS OF MATHEMATICS

1. (a) (i) Simplify $\frac{6-7i}{1-2i}$ in the standard form $a \pm bi$, where a, b$\in \mathbb{R}$. **[4 marks]**

$$\Rightarrow \frac{6-7i}{1-2i} \times \frac{1+2i}{1+2i}$$

$$= \frac{6(1)+6(2i)-7i(1)-7i(2i)}{1^2+(-2)^2}$$

$$= \frac{6+12i-7i-14(-1)}{1+4}$$

$$= \frac{6+14+5i}{5}$$

$$= \frac{20+5i}{5}$$

$$= 4+i.$$

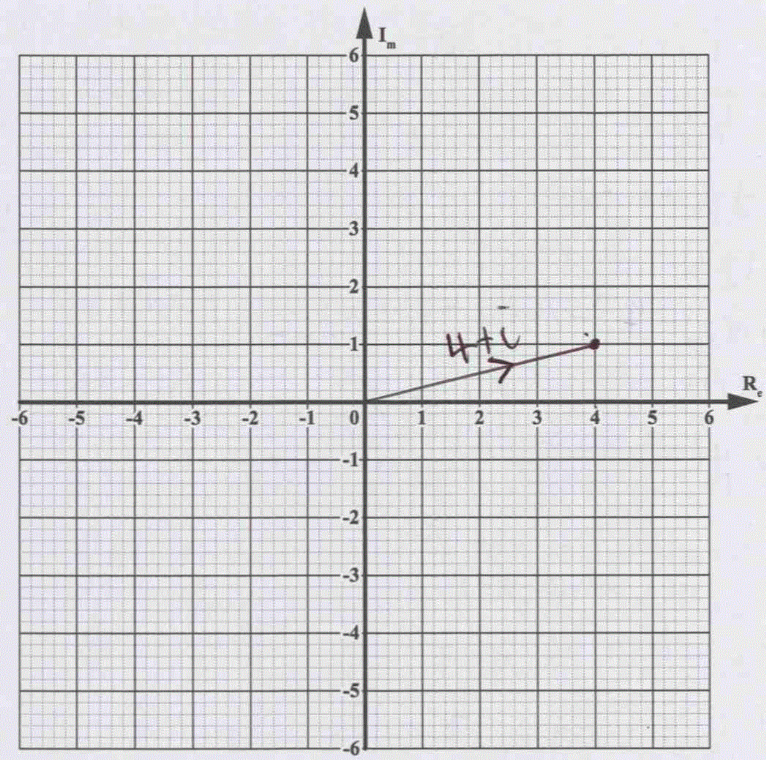
$\Rightarrow \frac{6-7i}{1-2i}$ in the form $a \pm bi$

$= 4+i$, $a = 4$ and $b = 1$.

Examiner's Comments
The candidate was able to correctly simplify the complex number in the correct format.

Candidate's Response to Part (a) (ii)

(ii) Represent the complex number from your solution in (a) (i) on the grid below. The labels I_m and R_e indicate the imaginary axis and the real axis respectively.



[2 marks]

Examiner's Comments

The candidate was able to correctly represent the complex number by plotting the point on the argand diagram and drawing a line from (0,0) to the point (4,1) with the arrow on the line directed towards the ending point.

Candidate's Response to Part (b)

(b) Given that $f(x) = -7x^2 - 5$ and $g(x) = 2x - 1$, solve $fg(x) = -68$.

$fg(x) = -68$

$fg(x) = -7(2x-1)^2 - 5$

$fg(x) = -7(4x^2 - 4x + 1) - 5$

$fg(x) = -28x^2 + 28x - 7 - 5$

$fg(x) = -28x^2 + 28x - 12$

$fg(x) = 4(-7x^2 + 7x - 3)$

$fg(x) = -68$

$\Rightarrow 4(-7x^2 + 7x - 3) = -68$

dividing throughout by 4 gives

$\rightarrow -7x^2 + 7x - 3 = -17$

$\Rightarrow 7x^2 - 7x + 3 = 17$

$7x^2 - 7x + 3 - 17 = 0$

$7x^2 - 7x - 14 = 0$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x+1)(x-2) = 0$

$\Rightarrow x = -1, \ x = 2.$

$\therefore x = -1$ or $x = 2.$

[5 marks]

Examiner's Comments
The candidate was able to correctly solve the composite function equation.

Candidate's Response to Part (c)

(c) Solve $|2x - 3| \leq 9$.

$|2x - 3| \leq 9$

$-9 \leq 2x - 3 \leq 9$

$-6 \leq 2x \leq 12$

$-3 \leq x \leq 6$

Examiner's Comments
The candidate was able to correctly calculate the solutions for x in the modulus inequality.

Candidate's Response to Part (d) (i)

(d) (i) Show that $x - 3$ is a factor of the function $f(x) = 2x^3 - 9x^2 + 10x - 3$.

$$f(3) = 2(3)^3 - 9(3)^2 + 10(3) - 3$$
$$= 54 - 81 + 30 - 3$$
$$= 0$$
$$\therefore \underline{(x-3) \text{ is a factor}}$$

Examiner's Comments
The candidate was able to correctly apply the factor theorem to show that x -3 is a factor of the given function.

Candidate's Response to Part (d) (ii)

(ii) Hence, solve the equation $f(x) = 0$.

$$\begin{array}{r}
2x^2 - 3x + 1 \\
x-3\overline{)\,2x^3 - 9x^2 + 10x - 3} \\
-(2x^3 - 6x^2) \qquad\qquad \\
\hline
0 \quad -3x^2 + 10x \qquad \\
-(-3x^2 + 9x) \qquad \\
\hline
0 \quad x - 3 \\
-(x - 3) \\
\hline
\underline{0}
\end{array}$$

$$\therefore (x-3)(2x^2 - 3x + 1) = 0$$
$$2x^2 - 3x + 1 = 0 \qquad \begin{matrix} -2 \\ 2 \;\; -1 \end{matrix}$$
$$(2x^2 - 2x)(-x + 1) = 0$$
$$(2x - 1)(x - 1) = 0$$
$$\therefore (x-3)(2x-1)(x-1) = 0$$
$$x = \underline{3, \tfrac{1}{2}, 1}$$

[5 marks]

Examiner's Comments
The candidate was able to correctly calculate the three values of x using long division and factorization.

Candidate's Response to Part (e)

(e) The coordinates of the midpoint of the line segment *PQ* are (–7, –2). If the coordinates of *P* are (6, 8), determine the coordinates of *Q*.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = (-7, -2)$$

$$\therefore \frac{6 + x}{2} = -7 \qquad \frac{8 + y}{2} = -2$$

$$6 + x = -14 \qquad 8 + y = -4$$

$$x = -20 \qquad y = -12$$

$$\therefore Q = \underline{(-20, -12)}$$

Examiner's Comments

The candidate was able to correctly apply the midpoint rule to obtain the coordinates of the endpoint.

Question 2

Candidate's Response to Part (a)

2. (a) Solve for x in the equation $5^{-(5x+8)} = 25^{2x+5}$.

$$5^{-(5x+8)} = 5^{2(2x+5)}$$

$$5^{-(5x+8)} = 5^{4x+10}$$

$$\log 5^{-(5x+8)} = \log 5^{4x+10}$$

$$-(5x+8)\log 5 = (4x+10)\log 5$$

$$-(5x+8) = 4x+10$$

$$-5x-8-4x-10 = 0$$

$$-9x-18 = 0$$

$$-9x = 18$$

$$x = -2$$

Examiner's Comments

The candidate was able to correctly apply the laws of indices and logarithms to determine the value for x.

Candidate's Response to Part (b)

(b) Solve $4\log_9(x-2) = \log_9 3 + 5$.

$$4\log_9(x-2) = \log_9 3 + 5$$

$$4\log_9(x-2) = \frac{1}{2} + 5$$

$$\log_9 (x-2)^4 = \frac{11}{2}$$

$$\cancel{9^{\frac{11}{2}} = (x-2)^4}$$

$$\cancel{177147 = (x-2)^4}$$

$$4\log_9(x-2) = \frac{11}{2}$$

$$\log_9(x-2) = \frac{11}{8}$$

$$x-2 = 9^{\frac{11}{8}}$$

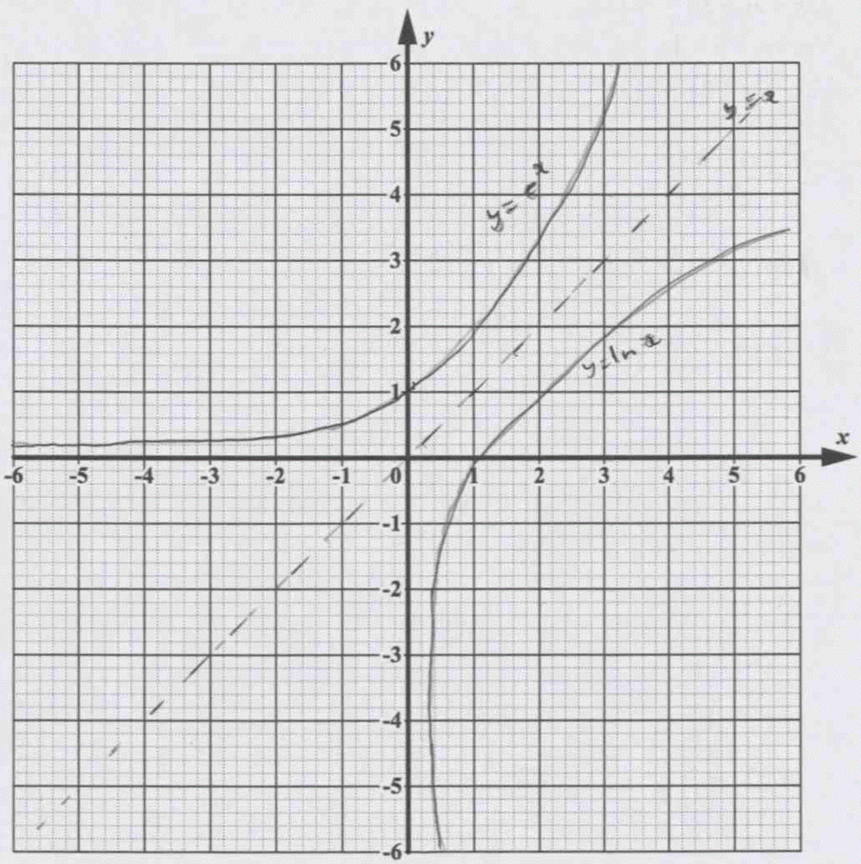
$$x = 9^{\frac{11}{8}} + 2$$

$$x \approx 22.5$$

Examiner's Comments
The candidate was able to correctly apply the laws of logarithms to solve for x.

Candidate's Response to Part (c)

(c) Sketch the graph of $y = e^x$ and its inverse function on the same set of axes belo showing the intercepts of EACH function.



Examiner's Comments

The candidate was able to correctly sketch the two graphs and label their points of intercept.

Candidate's Response to Part (d)

(d). Using Cramer's rule, or otherwise, determine the value of y in the following systems of equations.

$$\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix} \quad \begin{aligned} 5x-3y+5z&=-34 \\ -x+y+z&=4 \\ 2x-3y-3z&=-13 \end{aligned}$$

$$\begin{array}{cc} A & B \end{array}$$
$$\left(\begin{array}{ccc|c} 5 & -3 & 5 & -34 \\ -1 & 1 & 1 & 4 \\ 2 & -3 & -3 & -13 \end{array}\right) \qquad A_y \begin{pmatrix} 5 & -34 & 5 \\ -1 & 4 & 1 \\ 2 & -13 & -3 \end{pmatrix}$$

$$y = \frac{|A_y|}{|A|}$$

$$|A_y| = --34\begin{vmatrix} -1 & 1 \\ 2 & -3 \end{vmatrix} + 4\begin{vmatrix} 5 & 5 \\ 2 & -3 \end{vmatrix} - -13\begin{vmatrix} 5 & 5 \\ -1 & 1 \end{vmatrix}$$

$$|A_y| = 34(1) + 4(-25) + 13(10)$$
$$|A_y| = 64$$

$$|A| = 5\begin{vmatrix} 1 & 1 \\ -3 & -3 \end{vmatrix} - -1\begin{vmatrix} -3 & 5 \\ -3 & -3 \end{vmatrix} + 2\begin{vmatrix} -3 & 5 \\ 1 & 1 \end{vmatrix}$$

$$= 5(0) + 1(24) + 2(-8)$$
$$= 8$$

$$y = \frac{|A_y|}{|A|} = \frac{64}{8} = 8$$

Examiner's Comments
The candidate was able to correctly use Cramer's rule to calculate the value of *y*.

Candidate's Response to Part (e)

(e) Solve for x in the equation $5\tan(x+7) = 10$, where $-2\pi \le x \le 0$.

$$\tan(x+7) = 2 \qquad \alpha = \tan^{-1} 2$$
$$\alpha \approx 1.1071^c$$

In general

$$x+7^c = n\pi + 1.1071^c$$
$$x = n\pi + 1.1071^c - 7^c$$
$$x = n\pi - 5.8929^c$$

when $n=0$ $\quad x = -5.8929^c$ $\qquad$ when $n=1$ $\quad x = \pi - 5.8929^c = -2.7513^c$

$$x \approx -5.89^c, -2.75^c$$

Examiner's Comments
The candidate was able to correctly calculate the values of x for the given trigonometric equation.

Candidate's Response to Part (f)

(f) Adrianna bought a car at the beginning of 2016. The value of the car depreciates according to the following model.

$V = 70\,000 \times \left(\frac{6}{7}\right)^y$, where V is the value of the car, y years after it was purchased.

Calculate the value lost on the car at the end of 2019.

$y = \cancel{2020} - 2016 = 4$

Value at the end of 2019

$V = 70\,000 \times \left(\frac{6}{7}\right)^4 \approx \$37,784.26$

Value initially

$V = 70\,000 \times \left(\frac{6}{7}\right)^0 = \$70\,000$

$\therefore$ Value lost $= \$70\,000 - \$37,784.26$

$= \$32,215.74$

Examiner's Comments

The candidate was able to correctly calculate the loss of value after 4 years of depreciation.

Module 2: Statistics

Question 3

Candidate's Response to Part (a) (i)

3. (a) A code breaker has been challenged to overcome a six-element password. The password is a combination of the inputs A, S, T, U, V, 2, 5 and 9 without repetition.

(i) How many unique six-element passwords can be generated from these inputs?

# of passwords = $^8P_6$ = ~~60450~~ 20160

OR

$8 \times 7 \times 6 \times 5 \times 4 \times 3 = 20160$ unique passwords

[2 marks]

Examiner's Comments
The candidate was able to identify the number of unique six-element passwords can be generated.

Candidate's Response to Part (a) (ii)

(ii) How many unique six-element passwords can be generated if the code breaker knows that the password must begin and end with a number?

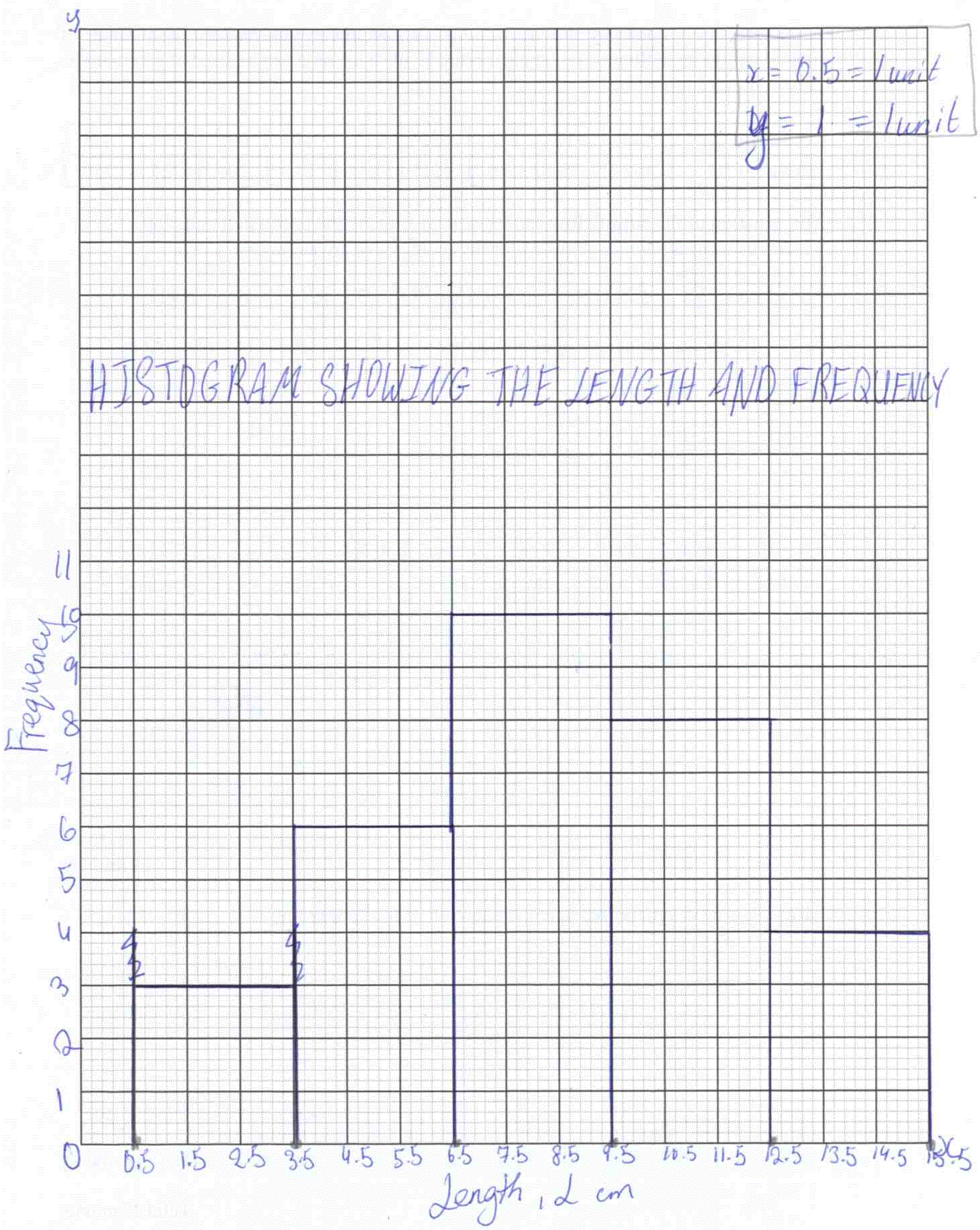
$3 \times 6 \times 5 \times 4 \times 3 \times 2$

$\boxed{\therefore \text{ ans} = 2,160}$

[2 marks]

Examiner's Comments
The candidate was able to identify the number of unique six-element passwords that can be generated given that the password must begin and end with a number.

Candidate's Response to Part (b) (i)



Examiner's Comments
The candidate was able to correctly draw and label the histogram.

Candidate's Response to Part (b) (ii)

(ii) Calculate the mean length of carrots gathered, giving your answer to 2 decimal places.

Recall: $\bar{L} = \dfrac{\sum f L_m}{\sum f}$

$\Rightarrow \bar{L} = \dfrac{260}{31} \approx 8.39$ cm (to 2 d.p.).

Examiner's Comments
The candidate was able to correctly calculate the mean length, giving the answer to 2 decimal places.

Candidate's Response to Part (c)

(c) It was assumed that the length of carrots follows a normal distribution with a mean length as calculated in (b) (ii) and a variance of 6.25 cm.

Calculate P (6 cm < L < 12 cm), giving your answer to 2 decimal places.

Recall: $Z = \dfrac{X - \mu}{\sigma}$ $\qquad L \sim N(\underset{\mu}{8.39}, \underset{\sigma^2}{6.25})$

$\Rightarrow P(6 < L < 12) = P(L < 12) - P(L < 6)$

$= P(Z < 1.444) - P(Z < -0.956)$

$= 0.9257 - P(Z > 0.956)$

$= 0.9257 - 1 + P(Z \leq 0.956)$ **[6 marks]**

$= 0.9257 - 1 + 0.8298$

$= 0.7555$

Examiner's Comments
The candidate was able to correctly calculate probability.

Candidate's Expected Response to Part (d) (i)

(d) A manufacturing company provided one of its customers with data on the number of units it would be willing to sell at each price level. This data can be used to estimate the equation of a supply line.

Quantity (x)	615	610	612	625	617	630	635	641	648
Price (y)	155	75	100	125	150	175	200	225	250

The equation of the **supply line** can be estimated using the regression equation $y = a + bx$. The summary statistics for the table above are as follows:

$\sum x = 5\,633$, $\sum x^2 = 3\,527\,093$, $\sum y = 1\,455$, $\sum y^2 = 261\,525$, $\sum xy = 916\,425$

(i) Using least squares regression estimation, calculate the values of a and b.

$$a = \bar{y} - b\bar{x}$$
$$= \frac{\sum y}{n} - \frac{b\sum x}{n}$$
$$= \frac{1455}{9} - 3.9405\frac{5633}{9}$$
$$= -2304.7$$

$$b = \frac{9(916{,}425) - 5633(1455)}{9(3527093) - 5633^2}$$
$$= 3.9405$$

Examiner's Comments

In the absence of a sample from a candidate's script, the preceding response shows how candidates were expected to correctly use least squares regression estimation.

Candidate's Expected Response to Part (d) (ii)

(d) A manufacturing company provided one of its customers with data on the number of units it would be willing to sell at each price level. This data can be used to estimate the equation of a supply line.

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The equation of the **supply line** can be estimated using the regression equation $y = a + bx$. The summary statistics for the table above are as follows:

$\sum x = 5\ 633$, $\sum x^2 = 3\ 527\ 093$, $\sum y = 1\ 455$, $\sum y^2 = 261\ 525$, $\sum xy = 916\ 425$

(ii) Hence or otherwise, state the equation of the supply line.

$$y = -2304.7 + 3.9405\,x$$

Examiner's Comments

In the absence of a sample from a candidate's script, the preceding response shows the correct way to state the equation of the supply line.

Question 4

Candidate's Response to Part (a) (i)

4. (a) A die and a coin are thrown one after the other in that order. While the die is fair, the coin is biased with the probability of attaining a 'head' outcome being $\frac{2}{3}$.

(i) Determine the probability that the coin shows 'tails', given that the die showed an even result.

Recall: Conditional Probability:

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

Let event T be a tail is obtained and event E be an even number is obtained:

$P(T|E) = \frac{\frac{1}{3} \times \frac{1}{2}}{\frac{1}{2}}$

$= \frac{1}{3}$

As the events are independant.

[2 marks]

Examiner's Comments
The candidate was able to correctly determine the conditional probability.

Candidate's Response to Part (a) (ii)

(ii) Determine the probability that the die shows an even result and the coin shows 'tails'.

$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

Examiner's Comments
The candidate was able to correctly calculate probability.

Candidate's Response to Part (b) (i)

(b) It is known that 40% of students at a large school live in poverty. In a sample of 10 randomly chosen students, calculate

(i) the probability that **exactly** 2 students live in poverty

$^{10}C_2 (0.4)^2 (0.6)^8$

$= 0.121$

Examiner's Comments
The candidate was able to correctly apply binomial distribution to determine the probability.

Candidate's Response to Part (b) (ii)

(ii) the probability that **at least** 3 students live in poverty.

$P(X \geq 3) = \cancel{1}\ 1 - [P(X=0) + P(X=1) + P(X=2)]$

$P(X=0) = {}^{10}C_0 (0.4)^0 (0.6)^{10}$

$= 0.00605$

$P(X=1) = {}^{10}C_1 (0.4)^1 (0.6)^9$

$= 0.0403$

[4 marks]

$P(X=2) = {}^{10}C_2 (0.4)^2 (0.6)^8$

$= 0.121$

$P(X \geq 3) = 1 - (0.00605 + 0.0403 + 0.121)$

$= 1 - 0.1674$

$= 0.8326$

Examiner's Comments
The candidate was able to correctly apply binomial distribution to determine the probability.

Candidate's Response to Part (c) (i)

(c) A principal intends to gather information about students' performance in Mathematics. A statistician suggested to her that either a sample survey or a population census could be conducted.

(i) Distinguish clearly between a sample survey and a population census.

A sample survey is ~~an portion of the population~~ experiment carried out using a portion of the population whereas a census is an experiment carried out using the entire population.

[2 marks]

Examiner's Comments

The candidate was able to distinguish clearly between a sample survey and a population census.

Candidate's Response to Part (c) (ii)

(ii) State a limitation of conducting a population census in the school.

The school may have a lot of students thus a population census may be time-consuming, tedious and impractical.

[1 mark]

Examiner's Comments

The candidate was able to provide a sound limitation of conducting a population census in the school.

Candidate's Response to Part (c) (iii)

(iii) The principal plans to build a sample of 100 students using either systematic random sampling or stratified random sampling.

Distinguish clearly between systematic and stratified random sampling techniques.

Systematic - sample is chosen by selecting every nth student depending on the sample size, and also the population. Usually selected in ascending order. eg: Selecting every 5th student in a line.

Stratified - sample is chosen by maintaining proportion of categories/strata from the population. eg: If 20% of the population is form 6 then 20% of the sample must be form 6.

[4 marks]

Examiner's Comments
The candidate was able to distinguish clearly between systematic and stratified random sampling techniques.

Candidate's Response to Part (d) (i)

(d) Let X be a random variable which records the number of blighted trees in a randomly selected park. The table below shows the probability distribution for X.

X	1	2	4	5	6
$P(X=x)$	$\frac{6}{27}$	$\frac{5}{27}$	$\frac{3}{27}$	$\frac{8}{27}$	$\frac{5}{27}$

Using the probability distribution table, calculate

(i) the expected number of blighted trees

Recall: $E(X) = \sum_{\text{all } x} xP(X=x)$

$\Rightarrow E(X) = (1)\left(\frac{6}{27}\right) + 2\left(\frac{5}{27}\right) + 4\left(\frac{3}{27}\right) + 5\left(\frac{8}{27}\right) + 6\left(\frac{5}{27}\right)$

$= \frac{98}{27}$

Examiner's Comments

The candidate was able to calculate the expected number using the probability distribution table provided.

Candidate's Response to Part (d) (ii)

(ii) the standard deviation in the distribution of blighted trees.

Recall: $\mathrm{Var}(X) = \sum_{\text{all } x} x^2 P(X=x) - [E(x)]^2$

$\Rightarrow \mathrm{Var}(X) = (1)^2\left(\frac{6}{27}\right) + (2)^2\left(\frac{5}{27}\right) + (4)^2\left(\frac{3}{27}\right) + (5)^2\left(\frac{8}{27}\right) + (6)^2\left(\frac{5}{27}\right) - \left(\frac{98}{27}\right)^2$

$\mathrm{Var}(X) = \frac{454}{27} - \left(\frac{98}{27}\right)^2$

$\mathrm{Var}(X) = \frac{2654}{729}$

$\therefore \sigma = 1.91$ (to 3 sig. fig.).

[4 marks]

Total 25 marks

Examiner's Comments
The candidate was able to calculate the standard deviation using the probability distribution table provided.

Module 3: Calculus

Question 5

Candidate's Response to Part (a)

5. (a) Evaluate the following limit.

$$\lim_{x \to -3} \frac{x^2+5x+6}{6x+18}$$

$$\frac{x^2+3x+2x+6}{6x+18}$$

$$\frac{x(x+3)+2(x+3)}{6(x+3)}$$

$$= \frac{(x+2)\cancel{(x+3)}}{6\cancel{(x+3)}} = \frac{x+2}{6} = \frac{-3+2}{6}$$

$$= \frac{-1}{6}$$

[4 marks]

Examiner's Comments
The candidate was able to correctly evaluate the limit.

Candidate's Response to Part (b)

(b) Determine the gradient of the curve $y = 6x^2 - 2x^2 \cos(x)$ at the point $x = 4$.

$y = 6x^2 - 2x^2 \cos x$, $x = 4$

grad $= \frac{dy}{dx} = 12x - [-2x^2 \sin x + 4x \cos x]$ u·v where $u = 2x^2$ $v = \cos x$

$\frac{du}{dx} = 4x$ $\frac{dv}{dx} = -\sin x$

$x = 4 = 12(4) - [-2[4^2] \sin 4 + 4(4)\cos(4)]$

$u\frac{dv}{dx} + v\frac{du}{dx} = -2x^2 \sin x + 4x \cos x$

$= 48 - 13.76$

$\therefore \frac{dy}{dx} = 34.24$

[4 marks]

$\therefore \frac{dy}{dx} = 34.2$

Examiner's Comments

The candidate was able to correctly calculate the gradient of the curve at the point.

Candidate's Response to Part (c)

(c) Differentiate $f(x) = \dfrac{\sin x}{\ln(x+2)+7}$.

$\ln f(x) = \dfrac{f'(x)}{f(x)}$

$u = \sin x$ $\quad\quad$ $v = \ln(x+2) + 7$

$\dfrac{du}{dx} = \cos x$ $\quad\quad$ $\dfrac{dv}{dx} = \dfrac{1}{x+2}$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{(\ln(x+2)+7)(\cos x) - \sin(x)\left(\frac{1}{x+2}\right)}{(\ln(x+2)+7)^2}$$

$$\therefore f'(x) = \frac{\cos x(\ln(x+2)+7) - \frac{\sin x}{x+2}}{(\ln(x+2)+7)^2}$$

[5 marks]

Examiner's Comments

The candidate was able to correctly differentiate the function, applying the rule.

Candidate's Response to Part (d) (i)

(d) The vertical displacement, in metres, of a particle on a ballistic trajectory is given as $y(t) = 15.7t - 5t^2$, where t is measured in seconds.

(i) Determine the displacement at $t = 0$.

$y(0) = 15.7(0) - 5(0)$

$= 0\text{m}$

Examiner's Comments

The candidate was able to correctly calculate the displacement at time zero.

Candidate's Response to Part (d) (ii)

(ii) Derive the velocity function, $v(t)$.

diff ↑ a
v
diff ↑ s

$V = \frac{dy}{dt}$

$a = \frac{dv}{dt}$

$v(t) = 15.7t - 5t^2 \quad \frac{dy}{dt}$

~~$v(t) = 15.7t - 10t$~~

$\therefore v(t) = 15.7 - 10t$

[2 marks]

Examiner's Comments
The candidate was able to correctly derive the velocity function.

Candidate's Response to Part (d) (iii)

(iii) Determine the time at which the particle reaches its maximum height.

At max height : $V = 0$

$15.7 - 10t = 0$

$10t = 15.7 \qquad t = 1.57\ s$

[2 marks]

Examiner's Comments
The candidate was able to correctly calculate the time at which the particle reaches the maximum height.

Candidate's Response to Part (d) (iv)

(iv) Determine the maximum height of the particle.

$y(1.57) = 15.7t - 5t^2$

$= 15.7(1.57) - 5(1.57)^2$

$= 12.32$ m

$\therefore$ max height $= 12.3$ m

[2 marks]

Examiner's Comments
The candidate was able to correctly calculate the maximum height of the particle.

Candidate's Response to Part (d) (v)

(v) State whether the acceleration, $a(t) = \dfrac{dv(t)}{dt}$, is a constant for the entire flight of the particle.

$a(t) = \dfrac{dv}{dt} = -10$

$\therefore$ a is constant for the entire flight

[2 marks]

Examiner's Comments
The candidate was able to state whether the acceleration was a constant.

Candidate's Response to Part (e)

(e) Given a function, f, such that $f(x,y) = 3\sqrt{x}y^2 - \frac{y}{2x}$, derive the partial derivative $\frac{df}{dx}$.

$\frac{df}{dx} = 3(x^{\frac{1}{2}})y^2 - y\left(\frac{1}{2x}\right)$ $\quad \frac{1}{2}\left(\frac{1}{x}\right)$

y is const $\quad 3y^2(x^{\frac{1}{2}}) - y\left(\frac{1}{2}x^{-1}\right)$

$3y^2\left(\frac{1}{2}x^{\frac{-1}{2}}\right) - y\left(\frac{-1}{2}x^{-2}\right)$

$= 3y^2\left[\frac{1}{2\sqrt{x}}\right] - y\left[\frac{-1}{2x^2}\right]$

$\therefore \frac{\partial f}{\partial x} = \frac{3y^2}{2\sqrt{x}} + \frac{y}{2x^2}$

[3 marks]

Total 25 marks

Examiner's Comments

The candidate was able to derive the partial derivative of the function with respect to x.

Question 6

Candidate's Response to Part (a) (i)

6. (a) A cylinder has a radius r cm and a height of h cm. The height is increasing at a constant rate of 2 cm/s. If the volume of a cylinder is given by $V = \pi r^2 h$ cm^3, determine the following.

(i) An expression for the rate at which the volume is changing per second

$$\frac{dh}{dt} = +2 \text{ cm/s}$$

$$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$$

$$\frac{dV}{dh} = \pi r^2 \text{ cm}^2$$

$$\Rightarrow \frac{dV}{dt} = \pi r^2 \text{ cm}^2 \times 2 \text{ cm/s}$$

$$\frac{dV}{dt} = 2\pi r^2 \text{ cm}^3/\text{s}$$

[4 marks]

Examiner's Comments

The candidate was able to correctly determine an expression for the rate at which the volume is changing per second.

Candidate's Response to Part (a) (ii)

(ii) The rate of change of the volume when $r = 10$ cm

$$\left.\frac{dV}{dt}\right|_{r=10\text{ cm}} = 2\pi(10)^2 \text{ cm}^3/\text{s}$$

$$\left.\frac{dV}{dt}\right|_{r=10\text{ cm}} = 200\pi \text{ cm}^3/\text{s}$$

rate of change of volume when $r = 10$ cm

$= 200\pi$ cm^3/s

[2 marks]

Examiner's Comments

The candidate was able to correctly calculate the rate of change of volume for the given value of radius.

Candidate's Response to Part (b) (i)

(b) Evaluate the following integrals.

(i) $\int\left[4\cos(4x+10)+\frac{7}{x}\right]dx$

$\int\left[4\cos(4x+10)+\frac{7}{x}\right]dx.$

$= 4\int\cos(4x+10)\,dx + 7\int\frac{1}{x}\,dx$

$= \frac{4\sin(4x+10)}{4} + 7\ln|x| + c$

$= \sin(4x+10) + 7\ln|x| + c.$

$= \sin(4x+10) + \ln|x^7| + c.$

[3 marks]

Examiner's Comments

The candidate was able to correctly find the indefinite integral of the given function.

Candidate's Response to Part (b) (ii)

(ii) $\int_2^4(8x^3+4x-1)\,dx$

$\int_2^4(8x^3+4x-1)\,dx$

$= \frac{8x^4}{4}+\frac{4x^2}{2}-x \Big|_2^4$

$= \left[2x^4+2x^2-x\right]_2^4$

$= \left[2(4)^4+2(4)^2-4\right]-\left[2(2)^4+2(2)^2-2\right]$

$= \left[512+32-4\right]-\left[32+8-2\right]$

[4 marks]

$= 540 - 38$

$= 502$ squared units.

Examiner's Comments

The candidate was able to correctly evaluate the definite integral of the function.

Candidate's Response to Part (c)

(c) Determine the area bounded between the curve of $f(x) = -x^2 + 9x - 18$ and the x-axis.

① $f(x) = -x^2 + 9x - 18$

$f(x) = 0$

$\Rightarrow -x^2 + 9x - 18 = 0$

$x^2 - 9x + 18 = 0$

$(x - 3)(x - 6) = 0$

$x = 3, \; x = 6$

Curve cuts x-axis at $x = 3$ and at $x = 6$.

② Area

$= \int_3^6 f(x)\,dx$

$= \int_3^6 -x^2 + 9x - 18\,dx$

$= \left[\frac{-x^3}{3} + \frac{9x^2}{2} - 18x\right]_3^6$

$= \left[-\frac{6^3}{3} + \frac{9(6)^2}{2} - 18(6)\right] - \left[-\frac{3^3}{3} + \frac{9(3)^2}{2} - 18(3)\right]$

$= -18 - \left(-\frac{45}{2}\right)$

$= -18 + 45/2 = 9/2$ squared units.

[7 marks]

Examiner's Comments
The candidate was able to correctly calculate the area bounded between the curve and the x-axis.

Candidate's Response to Part (d)

(d) Solve the following first-order differential equation with the given initial condition

$x\frac{dy}{dx} = 4x^4 + x^3 + 3x$, when $x = 0, y = 4$.

$$x\frac{dy}{dx} = 4x^4 + x^3 + 3x.$$

$$\frac{1}{\cancel{x}} \cdot \cancel{x}\frac{dy}{dx} = \frac{1}{x}(4x^4 + x^3 + 3x).$$

$$\frac{dy}{dx} = \frac{4x^4}{x} + \frac{x^3}{x} + \frac{3x}{x}$$

$$\frac{dy}{dx} = 4x^3 + x^2 + 3$$

$$\int \frac{dy}{dx}\, dx = \int 4x^3 + x^2 + 3 \, dx$$

$$y = \frac{4x^4}{4} + \frac{x^3}{3} + 3x + c.$$

$$y = x^4 + \frac{x^3}{3} + 3x + c.$$

$y = 4, x = 0$

$$\Rightarrow 4 = 0^4 + \frac{0^3}{3} + 3(0) + c$$

$$c = 4$$

$$\Rightarrow y = x^4 + \frac{x^3}{3} + 3x + 4.$$

[5 marks]

Total 25 marks

END OF TEST

Examiner's Comments The candidate was able to solve the first-order differential equation with the given initial condition.

PAPER 032 – ALTERNATIVE TO THE SCHOOL BASED ASSESSMENT

The questions on this paper were worth a total of 60 marks, which represents 20 per cent of the total examination.

Samples of correct answers are shown below except for Parts 2 (b) (ii) and 4 (a) for which no correct responses were observed.

Question 1

Candidate's Response to Part (a)

TABLE 1: NUMBER OF NEW CASES OVER FIRST SEVEN DAYS

Day	1	2	3	4	5	6	7
Number of New Cases	4	8	16	32	No data	128	256

1. (a) State the type of progression that the data represents. Justify your answer.

The above data shows a geometric progression. It is observed that each day, the # of infections ~~data~~ is doubled (common ratio of 2) from the previous day, which means everyday one new infected person is responsible for infecting one more.

[2 marks]

Examiner's Comments

After stating that the type of progression is a *geometric progression*, the candidate then showed that a common ratio exists.

Candidate's Response to Part (b)

(b) The data on the number of new cases for Day 5 was unavailable due to the overwhelming situation at the hospital. Using the information in the table, calculate the number of new cases which occurred on Day 5.

$$r = \frac{d_2}{d_1} = \frac{8}{4} = 2$$

$$d_5 = d_4 \times r = 32 \times r$$
$$= 32 \times 2$$
$$= 64$$

Examiner's Comments
The candidate first showed that the correct value for the ratio is 2 and then correctly calculated the number of new cases that occurred on Day 5.

Candidate's Response to Part (c)

(c) Calculate the total number of cases that would be recorded at the end of Day 12.

$\cancel{d_n = ar^{n-1} \quad a = 4 \quad r = 2}$

$\cancel{d_{12} = 4(2^{11})}$

$\cancel{= 4 \times 2048}$

$\cancel{= 8192}$

Total # of new cases $= S_{12}$

as $r > 1$

$$S_{12} = \frac{a(r^n - 1)}{r-1} = \frac{4(2^{12}-1)}{1} = 16380$$

[2 marks]

Examiner's Comments
The candidate showed the correct substitution and calculated the correct answer.

Candidate's Response to Part (d)

(d) The country's prime minster will declare a national emergency when the number of new cases on a given day exceeds 20 000. Assuming the rate of infection does not change, on which day will the prime minister be forced to declare a national emergency?

$d13 = 4(2^{12}) = 16384 \quad d^{14} = 4(2^{13}) = 32768$

$d_n = ar^{n-1} > 20\,000$

$4(2^{n-1}) > 20\,000$

$2^2\,2^{n-1} > 20\,000$

$2^{n+1} > 20000$

[4 marks]

$\boxed{\begin{array}{l} d_n = ar^{n-1} \\ d_n = 4(2^{n-1}) \end{array}}$

$n > 13$

$\therefore n = 14$

Examiner's Comments

The candidate correctly stated the equation for nth term went on to provide the correct answer.

Question 2

Candidate's Response to Part (a)

2. It is presumed that some of the residents in a particular parish in the country have contracted the illness. The mayor of the parish informed residents that if more than 1 out of 5 residents tests positive for the illness, then all sporting events would be cancelled. Let X be a random variable which represents the number of residents in the parish who have the illness.

(a) State the type of probability distribution of X. Give TWO reasons for your answer.

Binomial probability

(1) There is a definite Number of

Succes $1/5 = .20$

failure $4/5 = .80$

(2) The number is also discrete

[3 marks]

Examiner's Comments

The candidate correctly stated the type of probability as binomial and provided reasons which indicated that there is a definite number of trials with only two outcomes and that each must be independent/discrete.

Candidate's Response to Part (b) (i)

(b) Calculate the probability that

(i) no resident from the parish has the illness

$\cancel{P(x=0) = {}^5C_0}$

$P(\cancel{\text{zero res}}\ X < 1) =$

Let x be the # infected

n be the ~~population~~ sample size of the parish

p be the ~~number~~ probability that anyone is infected

q be $(1-p)$

$$P(x=0) = {}^{n}C_{x}\,(p)^{x}\,(1-p)^{n-x}$$

[3 marks]

$$= p^{x}(1-p)^{n-x}$$

$$= p^{0}(1-p)^{5-0}$$

$$= (1-p)^{5}$$

Let $p = 0.2$ & $q = 0.8$

$$= (0.8)^{5}$$

$$= 0.32768 = 32.768\%$$

GO ON TO THE NEXT PAGE

Examiner's Comments
The candidate used the correct formula, substituted correctly and calculated the correct answer.

Candidate's Response to Part (b) (ii)

(ii) all sporting events will have to be cancelled.

~~If the infection rate~~

$P(\text{cancelled}) = P(x > 1)$

$= 1 - [P(x=0) + P(x=1)]$

$= 1 - (1-p)^n - {}^nC_1\, p(1-p)^{n-1}$

$= 1 - (1-p)^n - np(1-p)^{n-1}$

$= 1 - (1-p)^{n-1}(1-p-np)$

If $n = 5$, $= 1 - (1-p)^4(1-p-5p)$

$= 1 - (1-p)^4(1-4p)$ **[5 marks]**

~~$= 1 - 0.327$~~

when $p = 0.2$

$= 1 - (0.8)^4(0.2)$

$= 0.91808$

$= 91.808\%$

Examiner's Comments

There was no correct response for this part of the question. An example of the correct use of complement, correct interpretation of X≤1, correct calculation of probability X = 1, correct substitution and correct answer are shown below.

$P(X > 1) = 1 - P(X \leq 1)$

$= [1 - P(X = 0) - P(X = 1)]$

$P(X = 1) = {}^5C_1 \left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^4$

$= 0.4096$

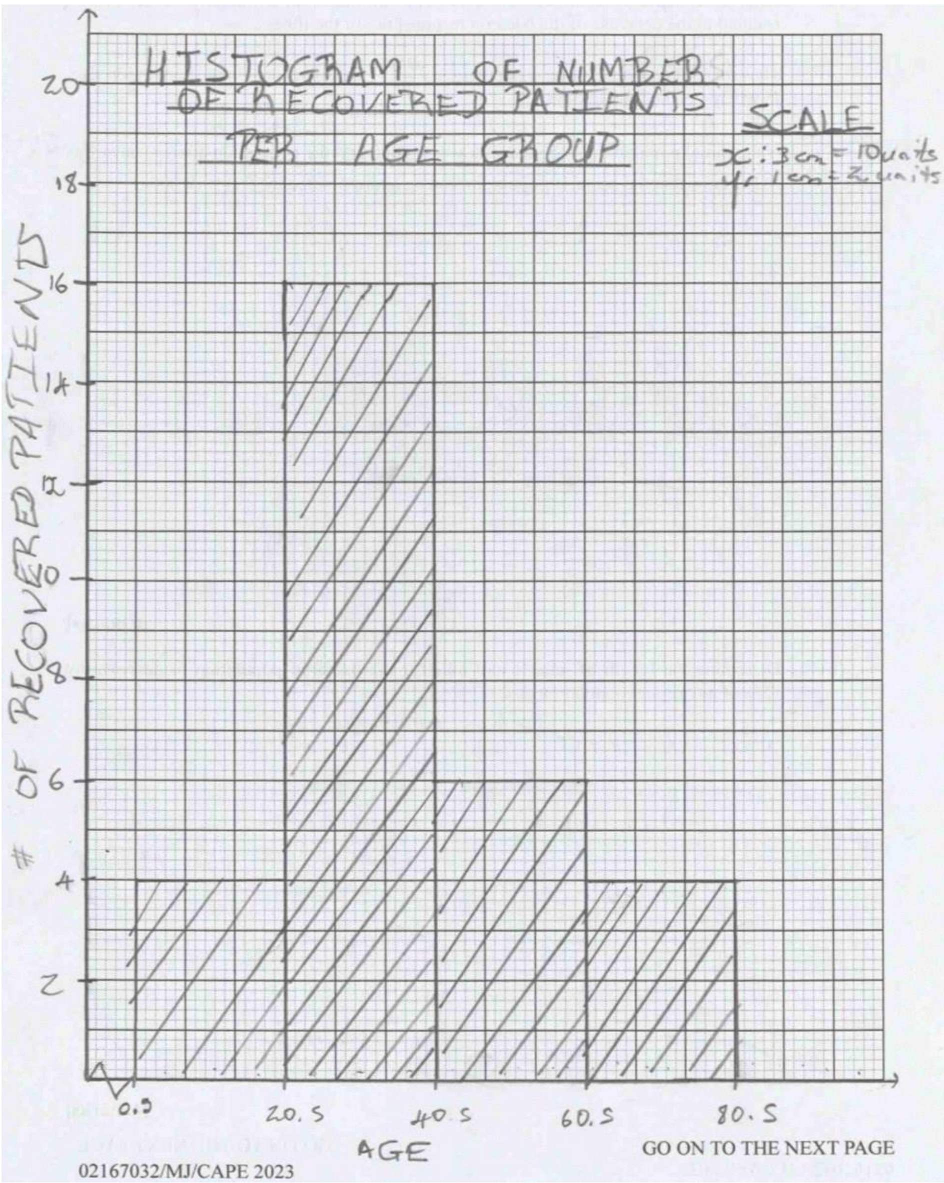
$P(X > 1)$

$= [1 - 0.32768 - 0.4096]$

$= 0.26272$

Question 3

Candidate's Response to Part (a)



Examiner's Comments

The candidate constructed a correct histogram showing the number of patients from each age range who would have recovered.

Candidate's Response to Part (b)

(b) Calculate the average age of patients who recovered.

$\bar{x} = \frac{\Sigma x}{n}$

$= \frac{4(10.5) + 16(30.5) + 6(50.5) + 4(70.5)}{30}$

$= \frac{42 + 488 + 303 + 282}{30} = \frac{1115}{30} = 37.1\overline{6}$

[5 marks]

Sol^n: $37.1\overline{66}$

Examiner's Comments
The candidate calculated the average correctly.

Candidate's Response to Part (c)

(c) State the modal age group of patients who are still in treatment.

The modal age group is 61 – 80.

Examiner's Comments
The candidate provided the correct modal age.

Candidate's Response to Part (d)

(d) Determine the median age group of patients who are still in treatment.

The median age group is ~~21 – 40~~ 41 – 60.

Examiner's Comments
The candidate provided the correct median age.

Question 4

Candidate's Response to Part (a)

4. (a) After several trials, scientists developed and introduced an antibacterial solution that resulted in the decrease of the bacteria responsible for the illness.

The rate of decrease of bacteria population, $\frac{dP(t)}{dt}$, is proportional to the population size, P, where t is the time in days.

If the initial population of the bacteria was 10 000 and the population after 30 days was 6110, show that the bacteria population can be approximated by $P(t) = 10\,000e^{-0.0164t}$.

Examiner's Comments

There was no correct response for this part of the question. An example of the correct equation, and correctly separating the variables, integrating LHS, integrating RHS, calculating *A* and calculating *k* are shown below.

$$\frac{dP(t)}{dt} = kP$$

$$\int \frac{dP}{P} = \int k\,dt$$

$$\ln P = kt + C$$

$$P(t) = Ae^{kt}$$

$$Using \;\; P(0) = 10\,000$$

$$P(0) = Ae^{0}$$

$$A = 10\,000$$

$$P(t) = 10\,000e^{kt}$$

$$Using \;\; P(30) = 6\,110$$

$$P(30) = 10\,000e^{30k}$$

$$10\,000e^{30k} = 6\,110$$

$$e^{30k} = \frac{6\,110}{10\,000}$$

$$30k = \ln\frac{6\,110}{10\,000}$$

$$k = \frac{1}{30}\ln\frac{6\,110}{10\,000}$$

$$k = -0.0164$$

$$hence\ P(t) = 10\,000e^{-0.0164t}$$

Candidate's Response to Part (b)

(b) Determine the length of time it will take for the bacterial population to decrease by 75% of its original population size.

$$2500 = 10000e^{-0.0164t}$$

$$\ln 2500 = \ln 10000 + \ln e^{-0.0164t}$$

$$\ln 2500 = \ln 10000 - 0.0164t$$

$$\frac{\ln 2500 - \ln 10000}{-0.0164} = t$$

$$t \approx 84.53$$

$\therefore$ Approximately 85 days

Examiner's Comments

The candidate provided the correct answer.

Question 5

Candidate's Response to Parts (a) to (c)

5. Two new treatments, A and B, have been developed and are in trials throughout the country. The first round of results is presented in Table 3.

TABLE 3: RESULTS OF TREATMENT A AND TREATMENT B

	Recovered	No Change	Adverse Reaction	Total
Treatment A	120	25	2	**147**
Treatment B	50	2	1	**53**
Total	**170**	**27**	**3**	**200**

When comparing the effectiveness of the two treatments, you were asked to compute the following.

(a) The probability that a patient recovered

$P(\text{recovered}) = \frac{170}{200} = \frac{\# \text{ recov}}{\# \text{ infected}}$

$= 0.85$

$= 85\%$

(b) The probability that a patient recovered using Treatment A

$P(\text{recovered})_A : \frac{120}{200} = 0.60 = 60\%$

(c) Given that the patient experienced an adverse reaction, determine the probability that the patient was given Treatment B.

$\frac{1}{3} = 0.33$ / ~~or~~ 33.3%

Examiner's Comments

The candidates correctly computed the probabilities.

Candidate's Response to Part (d)

(d) Determine if the events 'Treatment A' and 'no change' are mutually exclusive. Justify your answer.

They are not mutually exclusive because they can occur at the same time. The occurrence of one does affect the occurrence of the other because (if one was treated with B they might have had change ~~therefore~~)

Examiner's Comments

The candidate correctly determined that they are not mutually exclusive. The justification could have been shown as follows.

$$P(A \cap no\ change) \neq 0$$

Question 6

Candidate's Response to Question 6

6. Design a questionnaire instrument that could be used to collect the summarized information recorded in Table 2 and Table 3.

1 How old are you?

☐ 1-20 ☐ 21-40 ☐ 41-60 ☐ 61-80

2 Which treatment was administered to you?

☐ A ☐ B

3 Have you recovered with use of the treatment

☐ yes ☐ No

4 If the above was answered as no which of the following describe your feeling

☐ No changes ☐ Other Reaction

5 State the reaction to the treatment if above was answered accordingly

Examiner's Comments

The candidate's response consisted of appropriate questions and answer options. An introduction, worth one mark, was missing. An example of an appropriate introduction is provided below.

```
This questionnaire investigates patients and their
reactions to the treatments. All responses will be
recorded and treated with strict confidence. Your
answers are important to the success of this study and
we thank you for your assistance. Please tick your
choice for the appropriate questions or where necessary
fill in the blanks.
```

Overall Recommendations

- Emphasis should be placed on the interpretation of keywords in questions and the use of exact values where applicable or three significant figures.
- More emphasis is needed on recognition of laws and theories in questions, and more practice is required to ensure their correct application.
- Algebra skills should also be reinforced in every unit, to ensure through understanding and mastery.