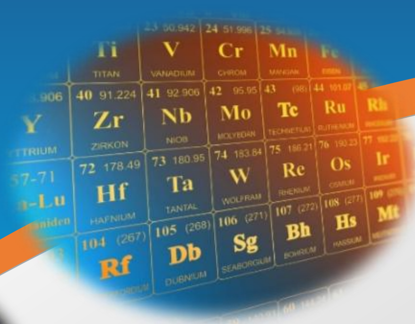
CARIBBEAN EXAMINATIONS COUNCIL

CAPE® INTEGRATED MATHEMATICS



Subject Report with Exemplars

May/June 2024

CARIBBEAN EXAMINATIONS COUNCIL

REPORT ON CANDIDATES' WORK IN THE
CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

MAY/JUNE 2024

INTEGRATED MATHEMATICS

Table of Contents

INTRODUCTION

This guide has been put together using candidate responses to the 2024 May/June examination in CAPE Integrated Mathematics. We have kept the answers according to the original design of the examination paper.

In 2024, approximately 1247 candidates registered for the Integrated Mathematics Proficiency examination and out of those, 960 sat the exam.

The examination targeted the content and specific objectives of the syllabus with the aim of ensuring coverage and simultaneously catering to the wide range of mathematical abilities of candidates. The examination consisted of a Paper 01 with multiple-choice items, a Paper 02 with extended response questions, a Paper 031 in the form of a school-based assessment (SBA) and a Paper 032 which is an alternative examination to the SBA. The questions were set at appropriate levels to test the following skills: Conceptual Knowledge, Algorithmic Knowledge and Reasoning.

Each paper of this examination tested the three modules: Module 1 — Foundations of Mathematics, Module 2 — Statistics and Module 3 — Calculus. The maximum possible mark for the examinations is 300, which represents 100 per cent. The composition and weighting of the associated papers are as follows.

- Paper 01: Forty-five multiple-choice items weighted as 90 marks and representing 30 per cent of the total examination
- Paper 02: Six extended response-type questions each at 25 marks, totaling 150 marks. This corresponded to 50 per cent of the total examination
- Paper 031/032: A total of 60 marks which represents 20 per cent of the total examination

Approximately 73.23 per cent of candidates earned acceptable grades, Grades I–V, compared with approximately 68.18 per cent in 2023 and approximately 65.35 per cent in 2022. Grade I performance in 2024 was 6.04 per cent compared with 3.95 per cent in 2023 and 4.54 per cent in 2022. Grade II performance in 2024 was 9.38 per cent compared with 5.30 per cent in 2023 and 7.80 per cent in 2022. Grade III performance in 2024 was 10.31 per cent compared to 12.20 per cent in 2023 and 10.17 per cent in 2022.

It is recommended that the subject be adequately timetabled in schools to ensure full syllabus coverage. In the classroom, emphasis should be placed on the interpretation of keywords in questions and more practice should be given to ensure the correct application of formulas needed for problem solving. There also needs to be greater exposure to the topics to reinforce understanding and mastery of concepts in each unit. In addition, candidates need to practise with past papers as part of their preparation for the exam. Specific topics need to be scrutinized in each module as a part of a success plan for the subject area.

PAPER 01 – MULTIPLE CHOICE

This paper tested all three modules of the syllabus and it comprised 45 items for a total of 90 marks. The maximum score obtained was 90 (100 per cent), the same as in the last three years. The mean score was 63.07 out of 90 (70.08 per cent) and the standard deviation was 18.34. In comparison, last year's mean was 56.49 out of 90 (62.77 per cent) and in 2022, it was 57.06 out of 90 (63.40 per cent).

PAPER 02 – EXTENDED RESPONSE QUESTIONS

This examination required candidates to answer all six questions for a maximum of 150 marks. There were two questions from each of the three modules and each question was worth 25 marks. The maximum score obtained was 146.67. The mean score was 29.68 out of 150 (19.79 per cent), much the same as last year when it was 30 out of 150 (20 per cent). In 2022, the mean score was 31.61 (21.08 per cent). The standard deviation scores were 30.74 in 2024, 26.89 in 2023 and 27.77 in 2022

Module 1: Foundations of Mathematics

Question 1

Candidate's Response to Part (a) (i)

(a) Given that $f(x) = 2 - x$ and $g(x) = 2^x$,

One to one

(i) define the term 'injective function' and state whether $f(x)$ is an injective function

An injective function is ~~one~~ a function where exactly one element is in the domain is mapped onto ~~on~~ one element in the codomain. $f(x)$ is an injective function

[2 marks]

Examiner's Comments
The candidate was able to define the term and state whether $f(x)$ is an injective function.

Candidate's Response to Part (a) (ii)

(ii) solve $gf(x) = 16$.

$gf(x) = g(2-x) = 2^{2-x} = 16$

$2^{2-x} = 2^4$.

Equate powers:

$2 - x = 4$

$x = -2$

[3 marks]

Examiner's Comments
The candidate was able to correctly solve the equation.

Candidate's Response to Part (b)

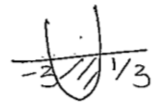
(b) Determine the range of values of x for which $3x^2 + 8x \leq 3$, where x is a real number.

$3x^2 + 8x - 3 \leq 0$; $-9, (-1, 9)$

$3x^2 - x + 9x - 3 \leq 0$

$x(3x-1)\,3(3x-1) \leq 0$

$(3x-1)(x+3) \leq 0$



$x = 1/3$; $x = -3$

$x: -3 \leq x \leq 1/3$

[4 marks]

Examiner's Comments
The candidate was able to correctly solve the inequality, determining the range of values for *x.*

Candidate's Response to Part (c)

(c) A board whose length is 258 cm is cut into 12 pieces. Calculate the length of the first piece of board, assuming that the lengths are in arithmetic progression and that the sum of the first 3 pieces of board is 24 cm.

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_3 = \frac{3}{2}[2a+(3-1)d]$

$S_3 = \frac{3}{2}[2a+2d]$

$24 = 3a+3d$

$d = \frac{24-3a}{3}$

$d = 8-a$

1) $u_1 = a$

2) $u_2 = a+d$

3) $u_3 = a+2d$

$S_3 = a + a+d + a+2d$

$S_3 = 3a+3d$

$S_{12} = 258$

$258 = \frac{12}{2}[2a+(12-1)d]$

$258 = 6[2a+11d]$

$258 = 12a+66d$

Substituting in $d = 8-a$

$258 = 12a + 66(8-a)$

$258 = 12a - 66a + 528$

$-270 = -54a$

$a = 5$ cm

[6 marks]

\

Examiner's Comments

The candidate was able to correctly use arithmetic progression to perform calculations for the first term and the sum of n terms.

Question 2

Candidate's Response to Part (a) (i)

(a) Let $f(x) = -x^3 + ax^2 - 5x - 12$. If $(x + 1)$ is a factor of $f(x)$,

(i) show that $a = 6$

$x + 1 = 0 \quad x = -1$

$f(-1) = -(-1)^3 + a(-1)^2 - 5(-1) - 12 = 0$

$1 + a + 5 - 12 = 0$

$a - 6 = 0$

$a = 6$

[2 marks]

Examiner's Comments

The candidate was able to use the factor theorem to show that a = 6.

Candidate's Response to Part (a) (ii)

(ii) hence or otherwise, determine the other two factors of the function.

$f(x) = -x^3 + 6x^2 - 5x - 12$

$$
\begin{array}{r|l}
 & -x^2 + 7x - 12 \\
x+1 & -x^3 + 6x^2 - 5x - 12 \\
 & -(-x^3 - x^2) \\
 & 7x^2 - 5x \\
 & -(7x^2 + 7x) \\
 & 0 \quad -12x - 12 \\
 & -(-12x - 12) \\
 & 0 \quad 0
\end{array}
$$

$(-x + 4)(x - 3)$

$f(x) = (x+1)(-x^2 + 7x - 12)$

$= (x+1)(-x+4)(x-3)$

[4 marks]

Examiner's Comments

The candidate was able to use the remainder theorem to find the other two factors.

Candidate's Response to Part (b)

(b) Solve the equation $\sqrt{2}\ \cos(\theta + \frac{\pi}{3}) = 1$ for $-\pi < \theta < \pi$.

$$\cos\left(\theta + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$$

S | A

T | C

$$\alpha = \theta + \frac{\pi}{3} = \cos^{-1}\left(\left|\frac{1}{\sqrt{2}}\right|\right)$$

$$\theta + \frac{\pi}{3} = \frac{\pi}{4}$$

$$\theta = -\frac{\pi}{12} \text{ rad}$$

$$\theta + \frac{\pi}{3} = 2\pi - \alpha$$

$$\theta + \frac{\pi}{3} = 2\pi - \frac{\pi}{4}$$

$$\theta = \frac{17\pi}{12}$$

In previous revolution:-

$$\theta = \frac{17\pi}{12} - 2\pi$$

$$= -\frac{7\pi}{12} \text{ rad}$$

$$\therefore \theta = \left\{-\frac{\pi}{12} \text{ rad}, -\frac{7\pi}{12} \text{ rad}\right\}$$

Examiner's Comments

The candidate was able to correctly solve the trigonometric equation showing the principal and secondary values.

Candidate's Response to Part (c)

(c) Jack buys 3 kg of apples, 4 kg of oranges and 2 kg of pears for a total of \$25. Jill buys 8 kg of apples, 6 kg of oranges and 3 kg of pears for a total of \$48. Mary buys 2 kg of apples, 3 kg of oranges and 4 kg of pears for a total of \$28. Using Cramer's rule, or otherwise, calculate the unit cost of a kg of apples.

[5 marks]

3kg = apple (x). 4kg = oranges (y). 2kg = pears (z).

$$3x + 4y + 2z = 25$$
$$8x + 6y + 3z = 48$$
$$2x + 3y + 4z = 28$$

$$\overset{A}{\begin{bmatrix} 3 & 4 & 2 \\ 8 & 6 & 3 \\ 2 & 3 & 4 \end{bmatrix}} \overset{x}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} \overset{B}{\begin{bmatrix} 25 \\ 48 \\ 28 \end{bmatrix}}$$

$$|A| = 3\begin{vmatrix} 6 & 3 \\ 3 & 4 \end{vmatrix} - 4\begin{vmatrix} 8 & 3 \\ 2 & 4 \end{vmatrix} + 2\begin{vmatrix} 8 & 6 \\ 2 & 3 \end{vmatrix}$$
$$= 3(24-9) - 4(32-6) + 2(24-12)$$
$$= 3(15) - 4(26) + 2(12)$$
$$= 45 - 104 + 24$$
$$= -35$$

$$x = \frac{\begin{vmatrix} 25 & 4 & 2 \\ 48 & 6 & 3 \\ 28 & 3 & 4 \end{vmatrix}}{|A|} \qquad y = \frac{\begin{vmatrix} 3 & 25 & 2 \\ 8 & 48 & 3 \\ 2 & 28 & 4 \end{vmatrix}}{|A|} \qquad z = \frac{\begin{vmatrix} 3 & 4 & 25 \\ 8 & 6 & 48 \\ 2 & 3 & 28 \end{vmatrix}}{|A|}$$

$$x = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{|A|} \qquad y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{|A|} \qquad z = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{|A|}$$

$$x = 25\begin{vmatrix} 6 & 3 \\ 3 & 4 \end{vmatrix} - 4\begin{vmatrix} 48 & 3 \\ 28 & 4 \end{vmatrix} + 2\begin{vmatrix} 48 & 6 \\ 28 & 3 \end{vmatrix}$$
$$= 25(24-9) - 4(192-84) + 2(144-168)$$
$$= 875 - 432 - 48$$
$$= \frac{-105}{-35} \Rightarrow 3$$

$$y = 3\begin{vmatrix} 48 & 3 \\ 28 & 4 \end{vmatrix} - 25\begin{vmatrix} 8 & 3 \\ 2 & 4 \end{vmatrix} + 2\begin{vmatrix} 8 & 48 \\ 2 & 28 \end{vmatrix}$$
$$= 3(192-84) - 25(32-6) + 2(224-96)$$
$$= 324 - 650 + 256$$
$$= \frac{-70}{-35} \Rightarrow 2$$

$$z = 3\begin{vmatrix} 6 & 48 \\ 3 & 28 \end{vmatrix} - 4\begin{vmatrix} 8 & 48 \\ 2 & 28 \end{vmatrix} + 25\begin{vmatrix} 8 & 6 \\ 2 & 3 \end{vmatrix}$$
$$= 3(168-144) - 4(224-96) + 25(24-12)$$
$$= 72 - 512 + 300$$
$$= -140$$
$$= \frac{-140}{-35}$$
$$= 4$$

Answer: $\therefore x = 3$, $y = 2$, $z = 4$

Examiner's Comments

The candidate was able to correctly solve the system of linear equations. The candidate correctly used Cramer's rule.

Question 3

Candidate's Response to Part (a) (i)

(a) The stem-and-leaf plot below gives the scores of several students who took an English examination. The maximum score for the examination was 80 marks.

21

Stem	Leaf
1	7
2	5 8
3	3 6 9
4	2 2 2 3 5 7 9
5	3 4
6	0 2 3 6
7	4 9

Key: 6|8 = 68 marks

(i) Determine the median score. $n = 21$

$Q_2 = \frac{1}{2}(21+1)^{th}$

$= 11^{th} = 4$

[1 mark]

Examiner's Comments

The candidate was able to correctly determine the median score.

Candidate's Response to Part (a) (ii)

(ii) Calculate the interquartile range.

$Q_3 = \frac{3}{4}(n+1)^{th}$ term

$= \frac{60+62}{2} = 61$ marks

$Q_1 = \frac{1}{4}(n+1)^{th}$ term $= \frac{36+39}{2} = 37.5$ marks

$IQR = Q_3 - Q_1$

$= 61 - 37.5$

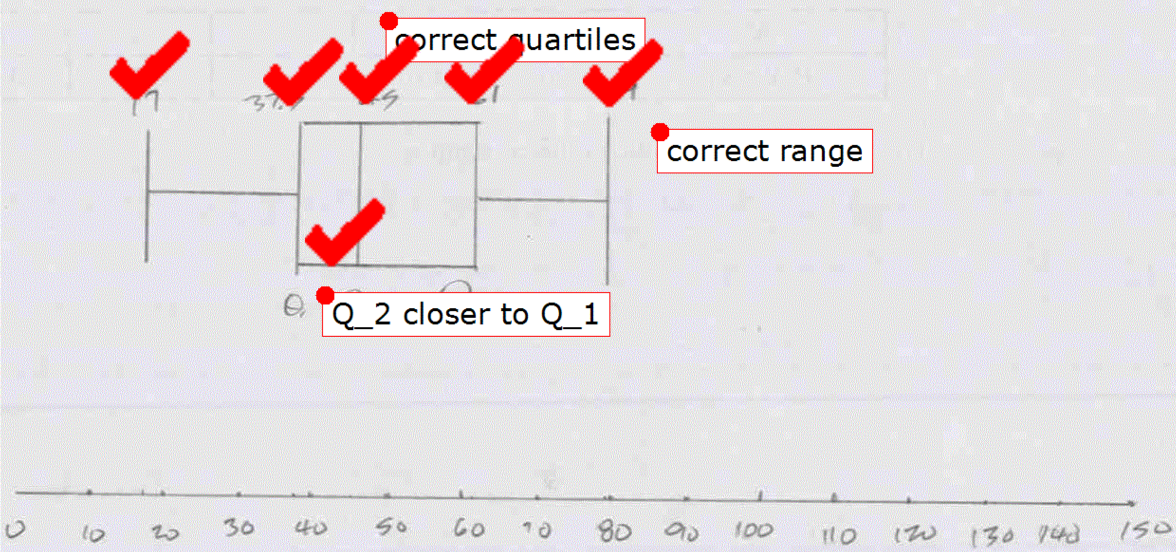
$= 23.5$ marks

[3 marks]

Examiner's Comments
The candidate correctly calculated the interquartile range.

Candidate’s Response to Part (a) (iii)

(iii) Sketch a box-and-whisker diagram to represent the data in the stem-and-leaf plot **on page 8.**



[3 marks]

Examiner’s Comments
The candidate was able to correctly sketch a box-and-whisker diagram.

Candidate’s Response to Part (a) (iv)

(iv) **State the shape of the distribution drawn in (a) (iii).**

Positively skewed

[1 mark]

Examiner’s Comments
The candidate was able to correctly state the shape of the distribution drawn.

Candidate's Response to Part (b) (i)

(b) The manager of a call centre has developed a probability distribution for the number of interruptions per day, X, which is a discrete random variable. The probability distribution is shown in the table below.

X	1	2	3	4	5
$P(X=x)$	0.20	0.30	n	0.25	0.20

(i) Calculate the value of the constant n.

$0.20 + 0.30 + n + 0.25 + 0.20 =$

$n = 1 - (0.20 + 0.30 + 0.25 + 0.20)$

~~$n = 1 - (0.95)$~~ $n = 1 - 0.95$

~~$n = 1.95$~~

$n = 0.05$

[2 marks]

Examiner's Comments

The candidate was able to correctly use the formula for discrete random variables to calculate the value of the constant *n*.

Candidate's Response to Part (b) (ii)

(ii) Determine the expected number of interruptions per day.

$E(X) = \sum x P(X = x)$

$= (1 \times 0.2) + (2 \times 0.3) + (3 \times 0.05) + (4 \times 0.25) + (5 \times 0.2$

$= 2.95$ interruptions

[2 marks]

Examiner's Comments
The candidate was able to correctly use the formula for discrete random variables to determine the expected number.

Candidate's Response to Part (b) (iii)

(iii) Determine the variance of the number of interruptions per day.

$Var(X) = \sum x^2 P(X=x) - [E(X)]^2$

$(1^2 \times 0.2) + (2^2 \times 0.3) + (3^2 \times 0.05) + (4^2 \times 0.25)$

$+ (5^2 \times 0.2) = 0.2 + 1.2 + 0.45 + 4 + 5$

$= 10.85$

$10.85 - (2.95)^2 = 2.15$

[3 marks]

Examiner's Comments

The candidate correctly applied the formula for discrete random variables to determine the variance.

Question 4

Candidate's Response to Part (a)

(a) An HR talent manager wishes to select 20 interviewees from among 500 applicants for internship positions in an organization.

Complete the table below by writing the appropriate sampling method for the descriptions.

Description	Sampling Method
Select equal numbers of male and female interviewees, even though there are twice as many female applicants as male applicants.	quota sampling
Group the applicants by age and select interviewees at random from within each group, in proportion to the number of applicants who make up that group.	stratified random
Assign a unique number from 001 to 500 to each applicant and use random numbers to select the interviewees.	simple random

[3 marks]

Examiner's Comments

The candidate was able to correctly identify the appropriate sampling method based on the descriptions.

Candidate's Response to Part (b) (i)

(b) A box contains 3 red, 4 blue, 6 white and 7 yellow marbles. A sample of 6 marbles is taken, without replacement, from the box.

(i) State the number of ways of selecting the 6 marbles.

since the order of events ~~doesn't~~ matter use ~~permutations~~ combinations

$\cancel{^{(3+4+6+7)}P_6 = {}^{20}P_6 = 27907200}$

[1 mark]

$^{(3+4+6+7)}C_6 = {}^{20}C_6 = 38760$ ways

Examiner's Comments The candidate was able to correctly apply combinations to state the number of ways of selecting the marbles.

Candidate's Response to Part (b) (ii)

(ii) If NO yellow marbles are chosen, show that the probability of obtaining 6 marbles is approximately 0.044.

$^{13}C_6 = 1716$

$\text{Probability} = \dfrac{1716}{38760}$ $\dfrac{P(\overline{Y})}{\text{total no. of ways}}$

$= 0.044$ to 2dp

[1 mark]

Examiner's Comments
The candidate was able to correctly apply combinations to show the probability.

Candidate's Response to Part (b) (iii)

(iii) Calculate the probability that EXACTLY 1 red marble, 1 blue marble, 2 white marbles and 2 yellow marbles are chosen.

$3C_1 \times 4C_1 \times 6C_2 \times 7C_2 =$

$3 \times 4 \times 15 \times 21 = 3780$

$\text{Probability} = \dfrac{\text{number of succesful outcomes}}{\text{the total number of outcoms}}$

$\text{Probability} = \dfrac{3780}{38760}$

$= 0.0975$

The probability is 0.0975

[2 marks]

Examiner's Comments

The candidate was able to correctly use combinations to calculate the probability required.

Candidate's Response to Part (c) (i)

(c) Ten students sat 2 Mathematics tests, 1 multiple choice test and 1 essay test. Their marks are recorded in the table below and a summary of the statistics given.

Marks in Multiple Choice Test, x	20	9	10	15	6	35	16	22	18	14
Marks in Essay Test, y	29	15	10	20	12	40	18	20	10	17

$n = 10$ $\sum x = 165$ $\sum y = 191$ $\sum xy = 3733$ $\sum x^2 = 3327$ $\sum y^2 = 4423$

(i) Determine the estimated linear regression line of y on x in the form $y = a + bx$.

$a = \bar{y} - b\bar{x}$ $\bar{y} = \frac{\sum y}{n} = \frac{191}{10} = 19.1$ $\bar{x} = \frac{\sum x}{n} = \frac{165}{10} = 16.5$

$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$

$a = 19.1 - 0.962(16.5)$

$19.1 - 15.873$

$b = \frac{10 \times 3733 - (165)(191)}{n\sum x^2 - (\sum x)^2}$

$\therefore a = 3.227$

$b = \frac{5815}{6045} = 0.962$

[4 marks]

Examiner's Comments

The candidate was able to correctly determine the estimated linear regression line of *y* on *x* in the form *y* = *a* + *bx*.

Candidate's Response to Part (c) (ii)

(ii) A student writes the multiple choice test and scores 25 marks but is absent for the essay test. Use the regression line found in (c) (i) to estimate an essay mark for the student.

FT $y = 3.86 + 0.96x$

$y = 3.86 + (0.96 \times 25)$

FT $y = 3.86 + 24$

$y = 27.86$

[2 marks]

Examiner's Comments
The candidate was able to correctly use the regression line to make an estimate.

Candidate's Response to Part (c) (iii)

(iii) Interpret the value of the correlation coefficient between multiple choice marks, *x*, and essay test marks, *y*, if its value is given as 0.850.

There is a very strong, positive, linear correlation between the multiple choice marks, x, and essay test marks, y.

Since ~~r~~ $0.8 \le r \le 1$ where r - correlation coefficient

ie $0.8 < 0.85 < 1$

[2 marks]

Examiner's Comments
The candidate was able to correctly interpret the value of the correlation coefficient given.

Module 3: Calculus

Question 5

Candidate's Response to Part (a)

(a) Let $f(x)$ be a function defined as $f(x) = \begin{cases} x^2 - 1 & x \neq 1 \\ 4 & x = 1 \end{cases}$.

Determine whether or not the function $f(x)$ is continuous at $x = 1$. Justify your answer.

$\lim_{x \to a} f(a) = f(a)$ $\quad \lim_{x \to 1} x^2 - 1$

$(1)^2 - 1 = 0$

hence the $f(x)$ is not continous at $x = 1$ and

the reason is that $\lim_{x \to a} f(a) \neq f(a)$

[2 marks]

Examiner's Comments

The candidate was able to correctly show the discontinuity of the function *f* (*x*) at *x* = 1. Correct justification was given.

Candidate's Response to Part (b) (i)

(b) Given a function $g(x) = x^3 - 4x^2 - 3x + 5$,

(i) calculate the x-coordinates of the stationary points of the function $g(x)$

$g'x = 3x^2 - 8x - 3$

$3x^2 - 8x - 3 = 0$

$(x - 3)(3x + 1) = 0$

$x = 3 \quad x = -\frac{1}{3}$

[4 marks]

Examiner's Comments

The candidate was able to correctly calculate the *x*-coordinates of the stationary points of the function *g*(*x*).

Candidate's Response to Part (b) (ii)

(ii) determine the nature of each stationary point.

$(-1/3, 4\frac{4}{15})$ $\quad$ $(3, -18)$ $\quad$ $y''(3x^2 - 8x - 3)$

$6x - 8$ $\quad$ $6x - 8$ $\quad$ $= 6x - 8$

when $x = -1/3$ $\quad$ when $x = -18$

$6(-1/3) - 8$ $\quad$ $6(3) - 8$

$= -10 < 0$ $\quad$ $= 10 > 0$

Max $\quad$ Min

[3 marks]

Examiner's Comments

The candidate was able to correctly determine the nature of each stationary point.

Candidate's Response to Part (c) (i)

(c) The volume of a spherical ball is given by the formula $V = \frac{4}{3}\pi r^3$, where r is the radius of the ball. The ball developed a leak and it's volume is decreasing at a rate of 2 cm^3/s.

(i) State the expression for $\frac{dV}{dr}$.

$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dr} = 4\pi r^2 \frac{dr}{dt}$ ISW

[1 mark]

Examiner's Comments

The candidate was able to correctly give the expression for $\frac{dV}{dr}$.

Candidate's Response to Part (c) (ii)

(ii) Derive the expression for the rate at which the radius is changing, $\frac{dr}{dt}$.

We know: $V = \frac{4}{3}\pi r^3$, $dV/dr = 4\pi r^2$, $\frac{dV}{dt} = -2$

$\frac{dr}{dt} = \frac{dV}{dr} \times \frac{dV}{dt} \Rightarrow \frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$

$= \frac{1}{4\pi r^2} \times -2$

$= \frac{-2}{4\pi r^2}$

$= \frac{-1}{2\pi r^2}$ cm/s

[3 marks]

Examiner's Comments The candidate was able to correctly apply the chain rule to derive the expression for the rate at which the radius is changing.

Candidate's Response to Part (c) (iii)

(iii) Calculate the rate at which the radius is changing when $r = 5$ cm.

Rate $= \frac{1}{2\pi r^2}$

$= \frac{1}{2 \times \pi \times 5^2}$

$= 0.00637$ (5 decimal places)

Negative sign missing

[2 marks]

Examiner's Comments The candidate was able to correctly calculate the rate at which the radius is changing.

Question 6

Candidate's Response to Part (a)

(a) Determine the following indefinite integral.

$$\int \frac{1}{(4x+7)^2}\,dx$$

$$\int (4x+7)^{-2}\,dx$$

$$= \frac{(4x+7)^{-1}}{-4} + C$$

$$= \frac{1}{-4(4x+7)} + C$$

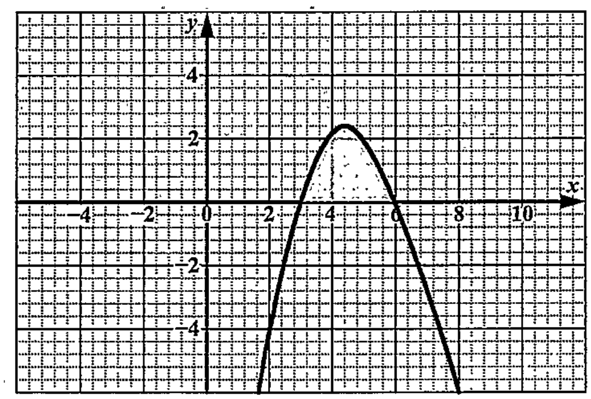
[3 marks]

Examiner's Comments

The candidate was able to correctly determine the indefinite integral.

Candidate's Response to Part (b)

(b) The diagram below shows the curve $f(x) = -x^2 + 9x - 18$.



$x = 6$

$-x^2+9x-18=0$

$x^2-9x+18=0$

$(x-6)(x-3)=0$

$x=6,\ x=3$

Determine the shaded area bounded between the curve and the x-axis.

$$\int_3^6 -x^2+9x-18\,dx$$

$$= \left[\frac{-x^3}{3} + \frac{9x^2}{2} - 18x\right]_3^6$$

$$\left[\frac{-6^3}{3} + \frac{9(6)^2}{2} - 18(6)\right] - \left[\frac{-3^3}{3} + \frac{9(3)^2}{2} - 18(3)\right]$$

$$\cancel{-18}\ -18 - \left(-\frac{45}{2}\right) = \frac{9}{2} \text{ square units.}$$

[5 marks]

Examiner's Comments

The candidate was able to correctly determine the shaded area bounded between the curve and the x-axis.

Candidate's Response to Part (c) (i)

(c) The rate of population growth per second for a species of bacteria, after an anti-bacterial solution is introduced, is modelled by the differential equation.

$$\frac{dP(t)}{dt} = -101.5e^{0.1\,t+3}$$

(i) If initially the size of the bacteria population was 10 000 specimens, derive a formula for the population size, P(t), at t seconds.

$$\frac{dP(t)}{dt} = -101.5\,e^{0.1t+3}$$

$$\int \frac{dP(t)}{dt} = \int -101.5\,e^{0.1t+3}$$

$$P = \frac{-101.5\,e^{0.1t+3}}{0.1} + c$$

if $t=0$

10000 specimens $= 10\,000 = \dfrac{-101.5\,e^{0.1(0)+3}}{0.1} + c$

$\therefore c = \dfrac{10\,000}{1} + \dfrac{101.5\,e^{3}}{0.1} = 30\,386.82$

$\therefore P = \dfrac{-101.5\,e^{0.1t+3}}{0.1} + 30\,386.82 = -1015\,e^{0.1t+3} + 30386.82$

[5 marks]

Examiner's Comments

The candidate was able to correctly use the method of separating the variables to solve the differential equation.

Candidate's Response to Part (c) (ii)

(ii) Determine the size of the bacteria population after 30 seconds. Give your answer to 2 decimal places.

at $t = 30s$

$$P(30) = -1015e^{0.1(30)+3} + 30399.82$$

$$= -379093.41$$

~~Due to the~~

~~-ve sign 101.~~

Due to the -ve coefficient of $e^{0.1(30)+3}$ the population calculated is ~~zero~~ -ve which cannot happen in real life

[2 marks]

Examiner's Comments

The candidate was able to correctly substitute to solve the equation.

PAPER 031 – SCHOOL-BASED ASSESSMENT (SBA)

The SBA is a research project worth a total of 60 marks; this represents 20 per cent of the total examination.

The mean score was 43.65 marks out of 60 (72.76 per cent) compared with 42.01 out of 60 (70.02 per cent) in 2023 and 39.09 out of 60 (65.15 per cent).

PAPER 032 – ALTERNATIVE TO THE SCHOOL-BASED ASSESSMENT

The questions on this paper were worth a total of 60 marks, which represents 20 per cent of the total examination.

The maximum score obtained on this paper was 21 out of 60 compared with 41 in 2023 and 55 in 2022. The mean score was 16 out of 60 (26.67 per cent) compared with 23.33 out of 60 (38.89 per cent) and 29.63 out of 60 (49.38 per cent).